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BY
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BULLETIN NO. 11

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BULLETIN NO. 11 APRIL 1907

EFFECT OF SCALE ON THE TRANSMISSION OF HEAT THROUGH LOCOMOTIVE BOILER TUBES

BY EDWARD C. SCHMIDT, M. E., ASSOCIATE PROFESSOR OF RAILWAY ENGINEERING, AND
JOHN M. SNODGRASS, B. S., INSTRUCTOR IN RAILWAY ENGINEERING

During the past twenty or thirty years there has been considerable discussion in railroad circles as to the effect of scale upon the heat-transmitting properties of tube surfaces, and the consequent effect upon the consumption of fuel. Statements as to the extent to which deposits of scale affect the conductivity of a tube or sheet have been made from time to time and have differed widely.

In a committee report on boiler incrustation in the Proceedings of the American Railway Master Mechanics Association of 1872, we find the following quotation from a paper by Dr. Joseph G. Rodgers before the American Association for the Advancement of Science, given as the best information which the committee had been able to obtain: "The evil effects of scale are due to the fact that it is relatively a non-conductor of heat. Its conducting power compared with that of iron is as 1 to 37.5. This known, it is readily appreciated that more fuel is required to heat water through scale and iron than through iron alone. It has been demonstrated that a scale $\frac{1}{16}$ in. thick requires the extra expenditure of 15% more fuel. As the scale thickens the ratio increases. Thus when it is $\frac{1}{4}$ in. thick, 60% more is required; ...."

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The report continues as follows: "On most western roads incrustations will form to a thickness of from $\frac{1}{8}$ in. to $\frac{3}{16}$ in. in the course of one year, and will increase at a still greater ratio as long as the engine is kept in service. Thus after four months' time, there will have accumulated in our engines nearly $\frac{1}{16}$ in. of scale. If Dr. Rodgers' theory be correct, after one month's service our engines will consume 3¼% more fuel than at first; after two months' service 7½% and so on, making an average for the year of over 20% more fuel than they would have consumed if using pure water."

In a report before the same Society in the year 1877 upon "Feed Water" a committee under the sub-heading, "The Effect of Incrustations on the Consumption of Fuel," reports in part as follows: "The increase in the consumption of fuel, on account of incrustations on the heating surfaces of boilers, varies with the thickness and density of the deposit. When porous the water will penetrate it, but when hard and compact it presents a complete barrier to the contact of the water with the heating surfaces. As incrustations are poor conductors of heat, an increased consumption of fuel is inevitable where they exist." The committee then cites a number of cases for which sufficient data were collected to estimate the per cent loss that was occasioned due to scale deposits. A table showing the average miles run to one ton of coal by engines upon the Illinois Central Railroad for three months prior to and for three months after the removal of incrustations, including 120 such cases and extending over a period of three years, showed as a general average an increase of 11% in the consumption of coal for three months prior to the cleaning of the boilers, as compared with the three months immediately succeeding. The result of 11% loss due to scale is, of course, entirely a general result, as individual cases often showed less miles run per ton of coal after cleaning than before. This difference from the general result could in most cases be accounted for by weather differences. A second case is cited for two passenger engines, which were of the same size and pattern, and which were run with the same trains on alternate days. Records were kept for the six months preceding and six months following the cleaning of the boilers. Both engines had previously been cleaned at the same time and had made an average of 34,047 miles before the test began. The tests as run showed a difference in favor of clean heating surfaces of 17.5%. Mr. Wells, the master mechanic making this test, however, concludes, on account of the scaled tubes being run more often during the winter months than the cleaned tubes, that of the 17.5% difference in consumption of fuel between clean and incrusted heating surfaces about 2% was due to temperature and 15½% to the effects of incrustation. Similar tests with two freight engines gave a difference of 26% in favor of clean heating surfaces. A correction of 4% was applied to this on account of different atmospheric temperatures under which the tests were made, "thus giving a net saving of 22% in favor of clean boilers on two freight engines."

Other cases might be mentioned giving results more or less similar, also cases in which little or no loss was found to be occasioned by the presence of scale. Likewise, the opinion has been not uncommonly expressed that there is either no fuel loss due to the presence of scale in the usual amounts or that loss is so small as to be of little practical importance.

During the last few years there have been made by the Railway Engineering department of the University of Illinois four series of experiments to determine the relative conductivities for heat of clean and scale-covered locomotive boiler heating surfaces. A fifth series of tests is now being carried on along the same general lines as the others. It is the purpose of this bulletin to report upon the results of the first four series of these tests.

The tests were planned with the purpose of determining, not only the actual transmission loss due to scale in individual cases, but also the relation of this loss to the scale thickness. The last three series were arranged especially to try to determine whether there is any regularity of variation of heat transmission loss with scale thickness and to study at the same time the effects of chemical composition on this loss.

It was recognized from the outset that in any series of comparative tests for the purpose of determining the loss in heat transmission due to the presence of scale, practically exact similarity of conditions was essential for trustworthy results. A study of previous work done along this line, such as the cases and results already referred to, also served to emphasize the necessity of such care. In all of the work hereinafter reported the greatest stress has been laid upon this point, i. e. the elimination of variations in conditions except the scale itself. The difficulties which have been encountered while prosecuting this work have still further emphasized this necessity. The difficulties attending road-testing are well understood both as to exact measurements and similarity of conditions. These considerations were of weight in determining that the tests herein reported should be largely of laboratory character rather than road tests.

The first series* of tests was made during May and June 1898. The method employed in making the tests was as follows:—

A Mogul freight locomotive, which had been in service 21 months and which was about to be sent to the shops for repairs and new tubes, was set in the roundhouse and the boiler tested by the standard method. The locomotive was then sent to the shops and the boiler carefully cleaned and retubed. All the scale was removed and samples analyzed from nine different parts of the boiler. It was then sent back and again tested for evaporation under the same conditions as before cleaning. Before making the trials with the clean tubes the locomotive was allowed to make one or two trips on the road so as to insure its being thoroughly clean. The tests were made in the round-house at Champaign, Illinois.

The locomotive was set in the roundhouse over a pit and the tender removed. A car of coal was then run in back of the engine and on this were arranged the scales for weighing the coal. All of the feed water was weighed and then delivered into a tank placed on a platform by the side of the car, and connected with the suction pipe of the injector.

The slide valve on one side of the locomotive was moved back far enough, by disconnecting the valve rod, so that the steam generated could pass directly into the exhaust, and thus out through the nozzle and produce the necessary draft as usual. A 2-in. pipe was also run from the dome to the atmosphere, a valve in the pipe furnishing additional means of disposing of the steam generated. The tests were started by the standard method, i. e., raising steam to the running pressure, drawing the fire and starting with weighed wood.

At the end of the tests the ashes were all weighed. One of the regular road firemen fired for all the tests and the boiler and furnace were operated under the usual road conditions. A series of observations was made during these tests, to determine the re-

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*This test constituted the thesis for graduation of Messrs. F. H. Armstrong and J. N. Herwig.

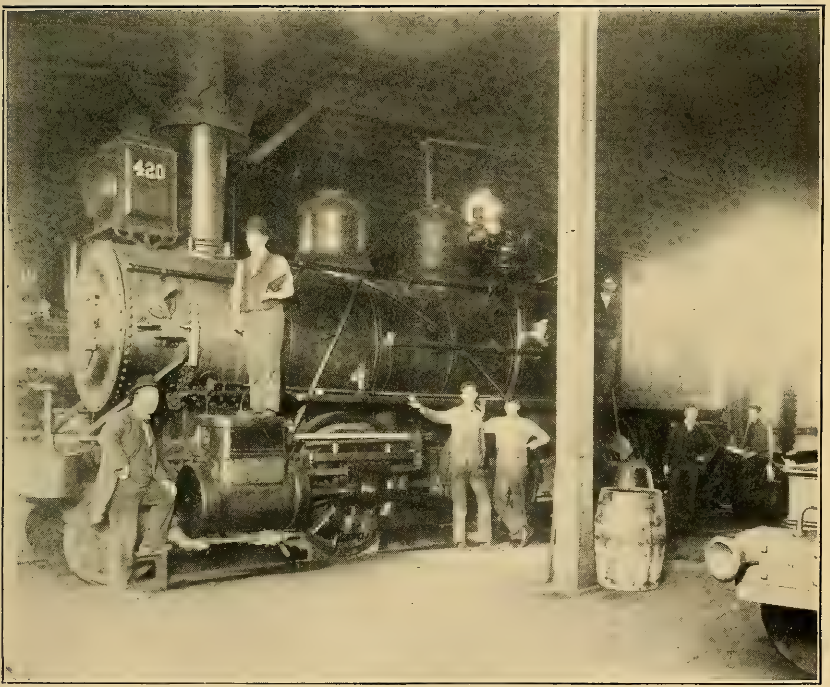

Fig. No. 1   Locomotive No. 420

lation between the blast-pipe pressures and vacuum in smoke box and furnace, as well as the velocity of the gases in the stack at various points along two diameters at right angles to each other. The locomotive upon which the tests were made was a Mogul freight engine made by the Rogers Locomotive Works, and was one of nineteen in use at that time on the Chicago division of the Illinois Central Railroad between Champaign and Centralia, Illinois. Fig. 1 shows the arrangements just described and gives a general view of the locomotive.

Leading dimensions:	
No. of Locomotive	420
Diameter of cylinder	19 in.
Stroke	26 in.
Diameter of drivers	56½ in.
Weight on drivers	106,400 lbs.
Weight on trucks	19,600 lbs.
Total weight of engine	126,000 lbs.
Diameter of boiler	62 in.
Number of tubes	236
Diameter of tubes	2 in.
Length of tubes	11 ft. 1 in. over tube sheets
Length of firebox	114⅛ in.
Width of firebox	33⅜ in.
Depth of firebox, front end	67⅞ in.
Depth of firebox, back end	59⅞ in.
Length of grate	114⅛ in.
Width of grate	33⅜ in.
Diameter of dry pipe	8 in. outside
Diameter of steam dome	29⅛ in. inside
Height of steam dome	28 in.
Kind of lagging	Magnesia sectional
Governing proportions:	
Grate area	26.45 sq. ft.
Total heating surface	1531.6 sq. ft.
Area of draft through tubes	573.5 sq. in.
Ratio of grate to heating surface	57.9
Fuel used:	
Commercial name	Odin
Commercial size	Mine run
Lumps per cent	75
Small coal per cent	20
Slack per cent	5
Heat units per lb. of dry coal (by calorimeter)	12,240

The results of these tests are exhibited in the accompanying tables.

TABLE 1

LOG OF OBSERVATIONS GIVING AVERAGE VALUES

LOCOMOTIVE NO. 420 ILLINOIS CENTRAL RAILROAD

	First Series Scale in Boiler		Second Series Cleaned Boiler	
Date of Trial (1898)	May 2	May 3	May 31	June 1
Duration of trial, hours	8.33	8.17	8.03	8.16
Steam pressure by gage	143	140	116.40	114
Vacuum in smoke box (in. of water)	2	2	2.9	2.8
Temperature of roundhouse (degrees F.)	72	62	79	89
Temperature of feed water in tank (degrees F.)	57	54	58.5	59.4
Temperature of escaping gases (degrees F.)	623	670	621	687
Temperature of steam (degrees F.)	362	360	348	346
Moisture in coal, per cent	4.0	4.0	4.0	4.0
Percentage of ash (from ash pan)	15.6	15.6	16.6	18.7
Percentage of moisture in steam	2.25	2.25	2.85	2.85

TABLE 2

RESULTS OF EVAPORATION TEST OF LOCOMOTIVE BOILER ENGINE NO. 420, ILLINOIS CENTRAL RAILROAD

First Series: After running 21 months and accumulating a scale deposit $\frac{1}{32}$ to $\frac{3}{64}$ inch thick.

Second Series: After cleaning and putting in new tubes.

		First Series Scale in Boiler			Second Series Clean Boiler		
	Date of Trial (1898)	May 2	May 3	Mean	May 31	June 1	Mean
		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Evaporative Performance	Water actually evaporated per lb. of dry coal	5.21	5.27	5.24	5.81	5.85	5.83
	Equivalent water from and at 212° F. per lb. of dry coal	6.29	6.39	6.34	6.99	7.04	7.01
	Water actually evaporated per lb. of combustible	6.17	6.25	6.21	6.95	7.16	7.05
	Equivalent water from and at 212° F. per lb. of combustible	7.46	7.59	7.53	8.36	8.61	8.48
Rate of Combustion	Dry coal burned per hour per sq. ft. of grate surface	57.45	58.51	57.95	59.80	60.00	59.90
	Per sq. ft. of tube opening	394.80	402.10	398.40	411.00	412.80	411.90
	Per sq. ft. of water heating surface	.93	.95	.94	.97	.98	.97
Rate of Evaporation	Water evaporated per hour from and at 212° F. per sq. ft. of grate surface	361.80	374.40	368.10	418.00	416.00	417.00
	Per sq. ft. of tube opening	2486.00	2573.00	2529.00	2874.00	2857.00	2865.00
	Per sq. ft. of water heating surface	5.89	6.09	5.99	6.81	6.76	6.79

The loss due to scale in this boiler was (7.01 minus 6.34) divided by 7.01 or 9.55 %.

The water used in the locomotive tested was taken from tanks at Centralia, Kinmundy, Little Effingham, Neoga, Dorans, Galton and Champaign. From the thickness of scale deposited during the 21 months it is evident that these waters are comparatively good for this section of the country.

The average thickness of the scale on the principal heating surfaces was $\frac{3}{64}$ in. The total weight of scale removed on cleaning was 485 lbs. The boiler had been in regular service during the 21 months.

The locomotive was cleaned and retubed at the Burnside shops of the Illinois Central Railroad. When the boiler was opened all the scale removed was carefully weighed, the scale on the tubes being determined by weighing the tubes before and after cleaning them. The scale from the shell and firebox sheets that could be removed was carefully collected. The total weight of scale was as follows:—

Weight of scale from flues	360 lbs.
Weight of scale from shell	125 lbs.
Total weight of scale	485 lbs.

At nine different points in the boiler the thickness of the scale was determined by the average of many measurements, and samples were secured for analysis as follows:—

Point 1. Near injector discharge, hard and soft scale $\frac{1}{8}$ in. thick.
2. On upper tubes, hard smooth scale uniform thickness $\frac{1}{32}$ in.
3. On lower tubes, hard scale near middle, $\frac{1}{16}$ in. thick.
4. Mud covering hard scale at No. 3, $\frac{3}{32}$ in. thick.
5. Scale from side sheet, flue sheet, and tubes rough and scaly.
6. From bottom of barrel, 4 ft. from flue sheet.
7. On crown stays, 3 in. to 6 in. from crown sheet.
8. On crown sheet, rivet heads and base of stays.
9. From stay bolts at water line.

The results of the analyses of these scales calculated to compounds are shown in Table 3.

TABLE 3

Results of the Analyses of Boiler Scale from Engine No. 420

Scale Constituents Calculated to Compounds and Expressed in per cent

Point No.	Silica SiO_2	Iron and Aluminum Oxides Fe_2O_3 and Al_2O_3	Calcium Sulphate $CaSO_4$	Calcium Carbonate $CaCO_3$	Calcium Oxide CaO	Magesium Carbonate $MgCO_3$	Magnesium Oxide MgO	Organic Matter and Undetermined
1	7.70	3.20	10.86	65.81	....		9.55	2.78
2	25.20	7.10	16.45	20.92	....	3.05	19.52	7.67
3	8.00	4.99	21.22	48.90	1.90	....	4.48	10.51
4	7.84	3.27	4.38	61.17	....	8.14	5.47	9.73
5	15.89	4.30	21.38	30.36	....	8.71	7.66	11.70
6	11.25	7.70	1.97	67.08	....	....	9.29	2.71
7	18.25	6.90	1.95	45.51	5.69	....	16.77	4.93
8	13.05	7.85	40.03	24.33	1.14	....	9.12	4.48
9	22.70	12.75	11.73	28.32	....	5.86	18.45	0.11

The loss, as found by these trials, due to the presence of scale, was 9.55 % of the fuel.

Experiments with Single Tubes

The last three series of experiments to determine the loss due to scale have been laboratory experiments entirely. They were made during the years 1901, 1904 and 1905, and are referred to as the series of 1901, 1904 and 1905 respectively.*

The locomotive boiler tubes upon which the experiments were made in 1901 were furnished by the Peoria and Eastern division of the Cleveland, Cincinnati, Chicago and St. Louis, the Illinois Central, the Chicago, Burlington and Quincy, and the Chicago, Milwaukee and St. Paul Railways. The tubes used in 1904 and 1905 were furnished by the first two railroad companies mentioned above. Table No. 4 gives information concerning these tubes. Fig. 2 shows some of the tubes tested.

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*These experiments were conducted by the following men as theses for graduation: Series of 1901, by F. L. McCune; Series of 1904, by W. A. Miskimen and C. N. Stone; Series of 1905, by H. F. Godeke and A. A. Hale.

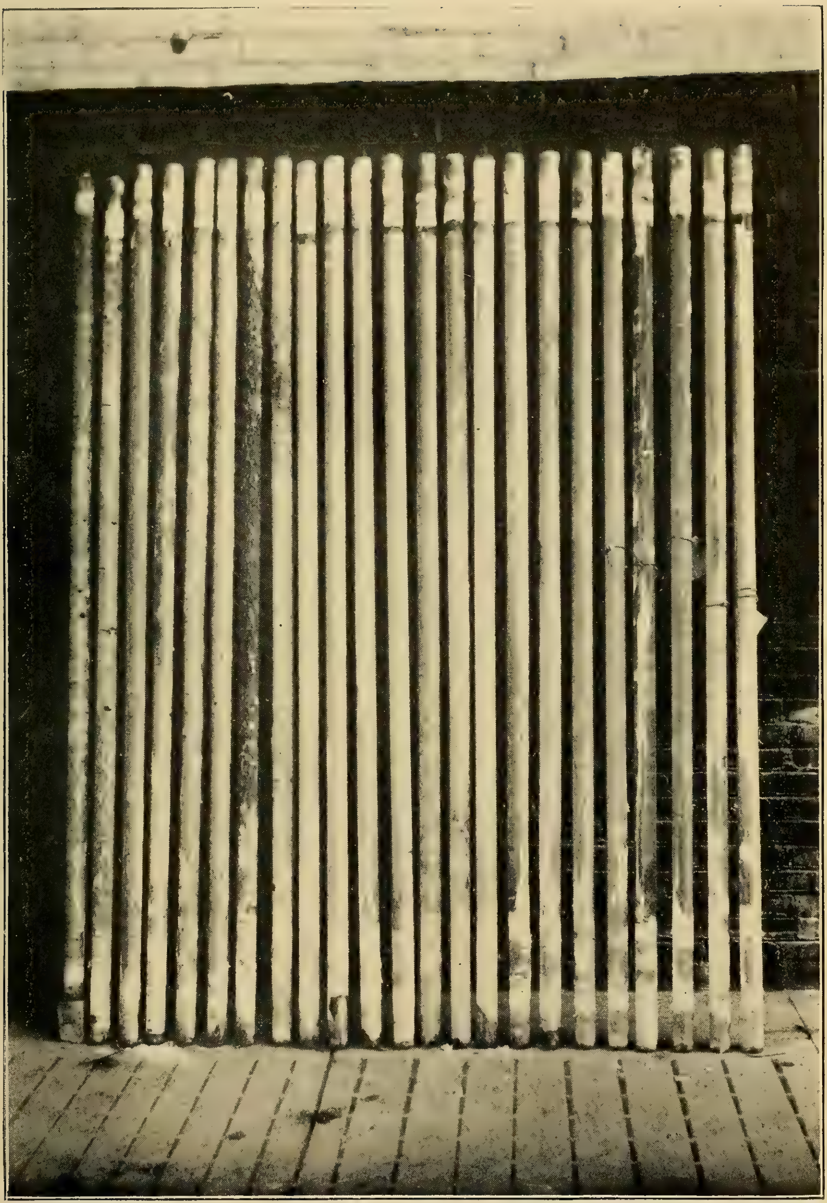

FIG. NO. 2 SCALED BOILER TUBES

TABLE 4

THE TRANSMISSION OF HEAT THROUGH SCALE-COVERED BOILER TUBES
RAILWAY ENGINEERING DEPARTMENT—UNIVERSITY OF ILLINOIS

Tube Number	Furnished by	No. of Engine from which Tube was taken	Length of time in service Months	Outside Diameter of Tube Inches	Average Thickness of Scale Inches	REMARKS General Character of Scale, Etc.
1	2	3	4	5	6	7
SERIES OF 1901						
1	I. C. R. R.	311	10.5	2	0.06	Even, hard, dense
2	P. & E. RY.	526	13.5	2	0.04	Soft, porous. Removed in places
3	P. & E. RY.	536	5.5	2	0.02	Hard, dense, white
4	C. M. & ST. P.	126	....	2	0.03	Hard, dense, white
5	C. M. & ST. P.	1337	....	2	0.13	Hard, dense
6	I. C. R. R.	820	5.5	2	0.07	Mileage during service, 19690
7	P. & E. RY.	513	37.5	2	0.04	Hard, dense, rough, one end. Soft, porous at the other
9	C. B. & Q.	1179	....	2	0.11	Hard, porous, gray. Mileage, 50889
11	I. C. R. R.	1107	21.	2	0.09	Soft, porous
14	P. & E. RY.	....	....	2	....	New and clean tube
SERIES OF 1904						
1	I. C. R. R.	41	16	2	0.04	Hard, gray. In bad condition
2	I. C. R. R.	41	16	2	0.07	Loose, gray
3	I. C. R. R.	41	16	2	0.08	Loose, gray
4	I. C. R. R.	141	15	2	0.05	White, porous. Removed in places
5	I. C. R. R.	141	15	2	0.04	White, porous. Removed in places
6	I. C. R. R.	141	15	2	0.08	White, porous
7a	C.C.C. & ST. L.	540	..	2	0.06	White, soft, irregular
7b	C.C.C. & ST. L.	540	..	2	0.06	White, soft, irregular
8a	C.C.C. & ST. L.	540	..	2	0.05	Hard, white, irregular
8b	C.C.C. & ST. L.	540	..	2	0.04	Hard, white
9	I. C. R. R.		..	2	0.06	Hard
10	I. C. R. R.	440	..	2	0.03	Hard, gray
11	I. C. R. R.	440	..	2	0.09	Gray, porous
12	I. C. R. R.	440	..	2	0.03	Gray, porous
13	I. C. R. R.	...	..	2	....	Clean tube
SERIES OF 1905						
3	I. C. R. R.	136	18	2	0.07	Medium
4	I. C. R. R.	802	8	2	0.05	Hard
8	C.C.C. & ST. L.	533	10	2	0.03	Soft
9	C.C.C. & ST. L.	233	14	2	0.09	Very soft
10	I. C. R. R.	1424	10	2	0.07	Soft
11	C.C.C. & ST. L,	233	14	2	0.04	Very soft
12	I. C, R. R.	140	21	2	0.07	Hard
13	I, C, R R.	303	18	2	0.02	Hard
14	I. C. R. R.	1004	21	2	0.04	Medium
15	I. C. R. R.	1012	12	2	0.03	Very hard
7	....	....	..	2	....	Clean tube

These tests were made as laboratory tests on account of the desire to make comparative tests under entirely similar conditions except in regard to the scale itself and in order that more exact measurements might be made than were found possible with road or roundhouse tests. The apparatus used in all of these tests has been practically the same from year to year. It is shown in Fig. 3, 4, and 5 and consists of a long water chamber through which the tube to be tested was passed, and in which water was circulated. On one end of this water chamber was fastened a combustion chamber, at the forward end of which was placed a burner. This burner was supplied with gas and air. Combustion took place in the chamber, which served the purpose of the firebox. The hot gases passed through the boiler tube to the air. The water entered the water chamber at the right, leaving it at the left as indicated in Fig. 3, at both of which points its temperature was read upon the thermometers there shown. The water tank received the water from the city mains. It was provided with an overflow, as shown, and the water was led directly down from the bottom of this tank to the water chamber. This was used in order to give a constant pressure at the inlet and thus to avoid variations in the rate of flow of the water. The gas and air tanks were arranged to give constant pressures of gas and air. The inner vessels, open at the bottom, float in water contained in the outer tank and confine the air or gas in the space above the water level. These inner tanks can be weighted at will to give any desired pressure to the air or gas contained within them.

During the tests of 1901 a copper ball pyrometer was employed to obtain the temperatures of the gases entering the tube being tested. For the series of 1904 and 1905 a Le Chatelier pyrometer was employed for this purpose. The location of the pyrometer is shown in Fig. 3. The temperature of the gases as they left the flue was read on the thermometer shown at the end of the tube.

The purpose of the tests was to measure the number of heat units transmitted per hour through the different tubes. This was accomplished by weighing the water which circulated around the tube in the water chamber and measuring its rise in temperature. The attempt was made to maintain a constant furnace temperature at the entrance to the tubes throughout all experiments of each

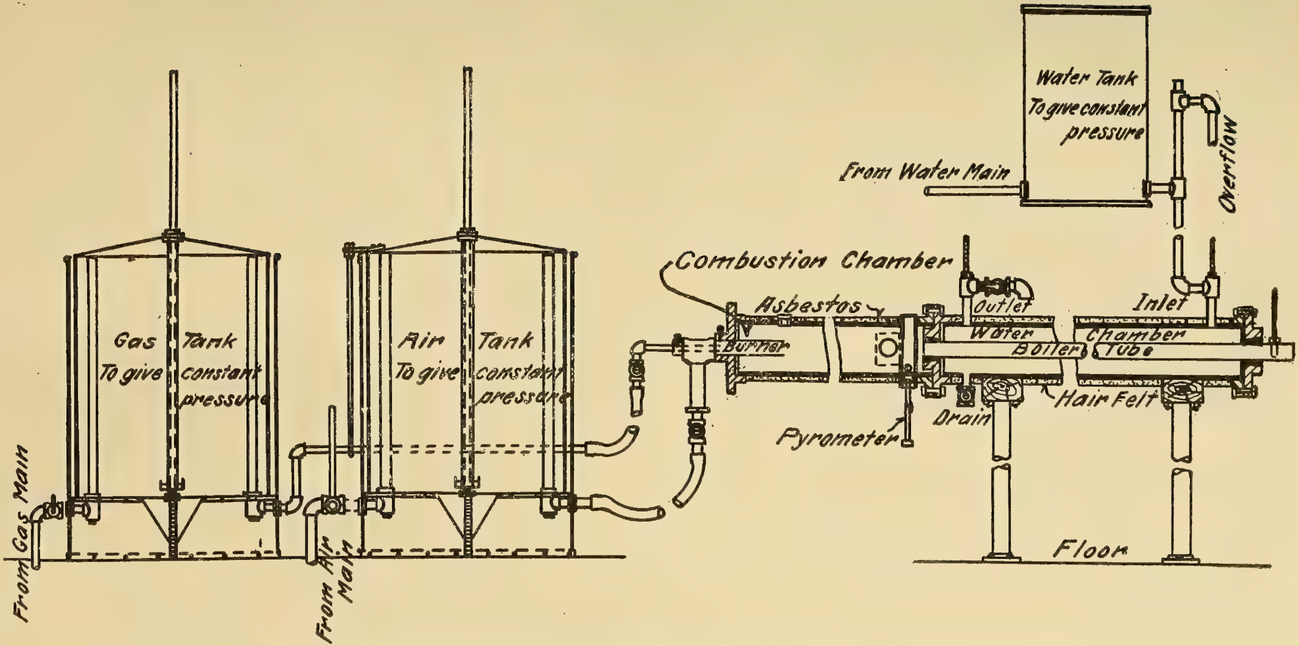

FIG. NO. 3 APPARATUS FOR THE DETERMINATION OF THE EFFECT OF SCALE ON HEAT TRANSMISSION THROUGH TUBES

series and thereby have available for transmission the same amount of heat, since the amounts of gas and air supplied to the burner were continually the same.

TABLE 5

The Transmission of Heat through Scale-covered Boiler Tubes

Railway Engineering Department—University of Illinois

Tube Number	Test Number	Duration of Test—Hours	Average Temperatures During Tests Degrees Fahr.							Weight of Water Used During Test—In Pounds	Difference Between Temperature of Water and Temperature of Gases—"Range of Temperature"—Column 6 Minus Column 9	B. T. U. Transmitted Through Tube During Test	B. T. U. Which Would Have Been Transmitted Had The Range Of Temperature Been The Same As For Clean Tubes	Decrease in Conductivity Due to Scale Per Cent Loss
			Of Furnace Gases			Of Circulating Water								
			In Combustion Chamber	Of Escaping Gases	Average Temperature of Gases	At The Inlet	At The Outlet	Average Temperature of Water	Rise in Temperature of Water					
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
						Series of 1901								
1	11	1	1657	256	957	66.9	112.7	89.8	45.8	651.3	867.2	29830	30063	*0.6
1	12	1	1702	252	977	66.8	112.6	89.7	45.8	643.3	887.3	29463	29021	2.9
2	16	1	1650	260	955	65.0	113.9	89.5	48.9	526.5	865.5	25746	25999	13.0
2	18	1	1631	269	950	64.1	110.3	87.2	46.2	584.0	862.8	26981	27331	8.5
3	1	1	1693	239	966	64.7	110.4	87.6	45.7	593.5	878 4	27123	26987	9.7
3	2	1	1690	238	964	65.7	111.0	88.4	45.3	600.7	875.6	27212	27162	9.1
4	20	1	1639	259	949	67.1	111.2	89.2	44.1	676.0	859.8	29812	30304	*1.4
4	21	1	1622	252	937	66.8	109.7	88.3	42.9	713.5	848.7	30609	31522	*5.5
5	6	1	1559	248	904	65.9	110.6	88.3	44.7	583.0	815.7	26060	27923	6.5
5	22	1	1702	265	984	63.9	109.3	87.1	45.4	624.0	896.9	28330	27606	7.6
6	3	1	1682	265	974	65.3	110.4	87.9	45.1	685.5	886.1	30916	30494	*2.1
6	23	1	1718	284	1001	63.6	110.7	87.2	47.1	646.0	913.8	30427	29101	2.6
7	8	1	1695	252	974	67.1	113.3	90.2	46.2	612.0	883.8	28274	27961	6.4
7	9	1	1693	256	975	66.9	112.0	89.5	45.1	669.5	885.5	30194	29802	0.2
9	4	1	1535	249	892	65.3	109.9	87.6	44.6	546.0	804.4	24351	26459	11.4
9	15	1	1597	252	925	65.8	110.9	88.4	45.1	505.0	836.6	22776	23794	20.4
11	7	1	1659	262	961	63.9	110.0	87.0	46.1	589.3	874.0	27167	27167	9.1
11	24	1	1683	271	977	64.1	109.5	86.8	45.4	590.5	890.2	26809	26321	11.9
14	5	1	1595	266	931	64.0	109.4	86.7	45.4	644.0	844.3	29238	Clean Tubes *Increase	
14	13	1	1690	267	979	67.8	114.0	90.9	46.2	650.2	888.1	30039		
14	14	1	1693	268	981	68.5	114.2	91.4	45.7	664.0	889.6	30345		
Average			1659	267	964	66.8	112.5	89.7	45.8	652.7	874.0	29874		

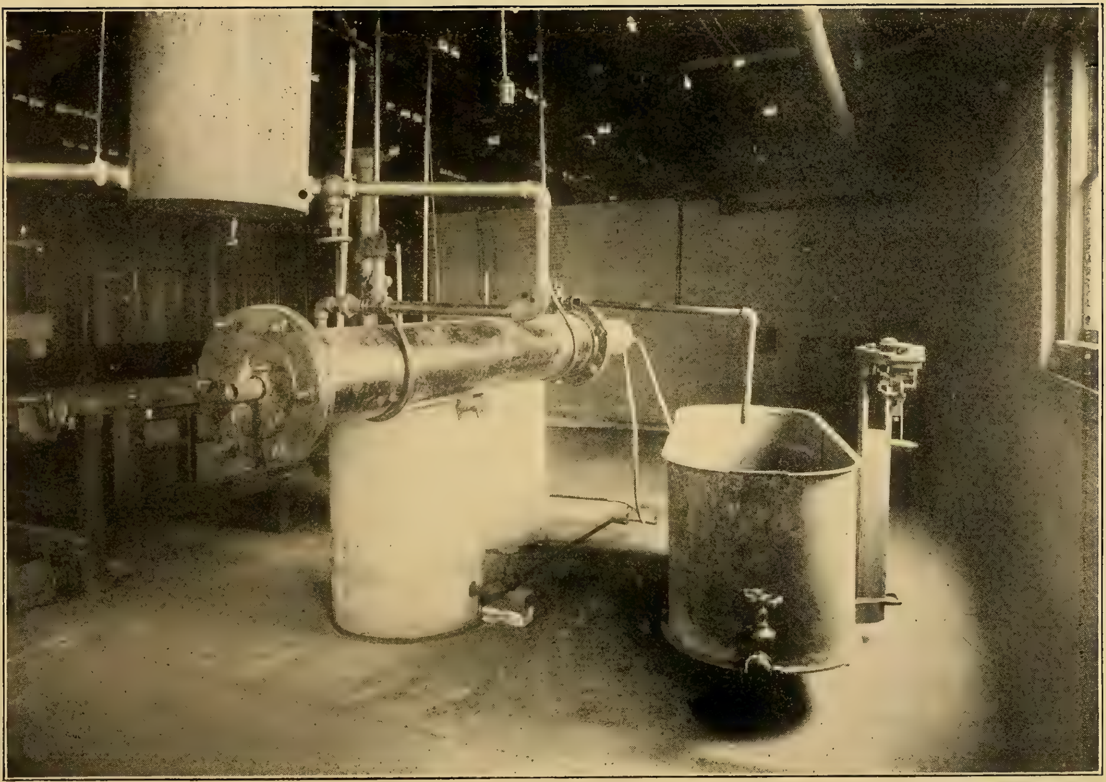

FIG. NO. 4 APPARATUS FOR THE DETERMINATION OF THE EFFECT OF SCALE ON HEAT TRANSMISSION THROUGH TUBES

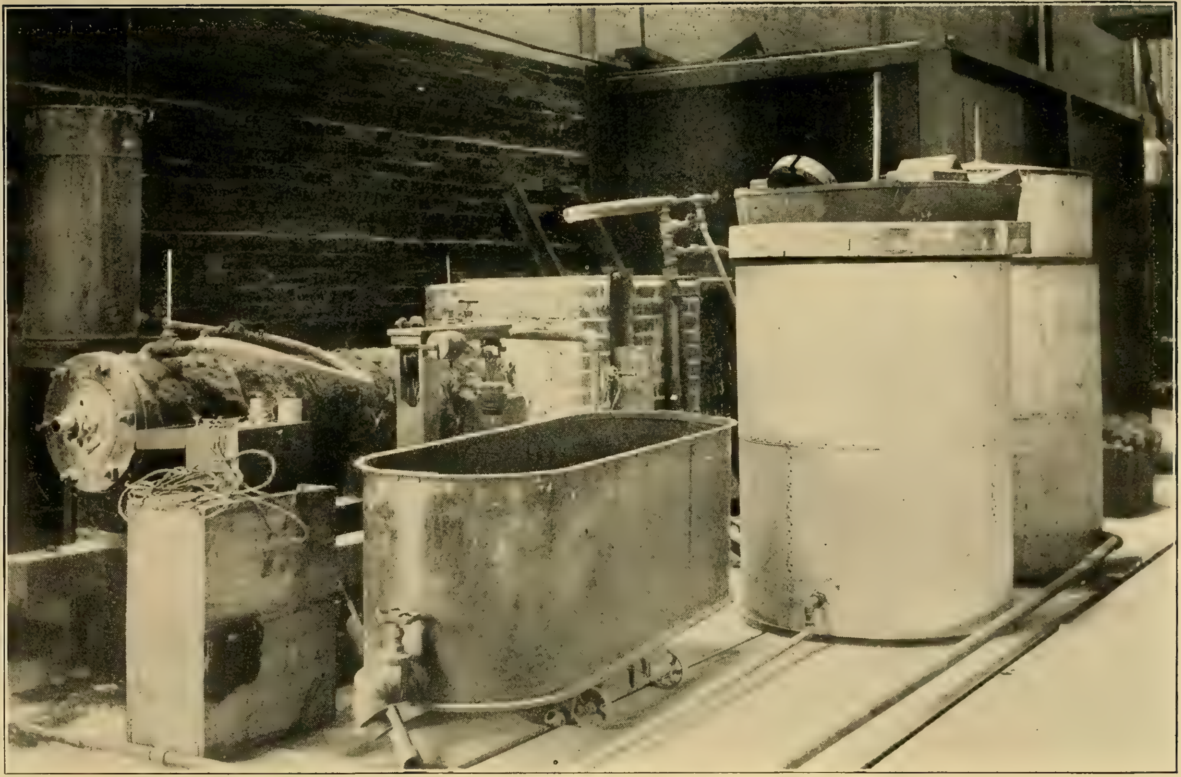

Fig. No. 5 Apparatus for the Determination of the Effect of Scale on Heat Transmission Through Tubes

SERIES OF 1904

1	15	1	1432	648	1040	56.8	84.7	70.8	27.9	829	969.2	23129	22227	5.1
2	16	1	1439	659	1049	57.9	84.0	71.0	26.1	850	978.0	22185	21128	9.8
3	17	1	1436	667	1052	58.0	83.2	70.6	25.2	895	981.4	22554	21405	8.6
4	18	1	1418	515	967	57.7	80.4	69.1	22.7	975	897.9	22133	22958	2.0
5	19	1	1438	523	981	57.1	83.4	70.3	26.3	814	910.7	21408	21895	6.5
6	20	1	1439	511	975	58.3	82.7	70.5	24.4	865	904.5	21106	21734	7.2
7a	13	1	1439	647	1043	57.0	81.9	69.5	24.9	875	973.5	21788	20845	11.0
7b	14	1	1437	523	980	56.5	81.5	69.0	25.0	803	911.0	20075	20525	12.4
8a	12	1	1443	623	1033	57.0	89.4	73.2	32.4	662	959.8	21449	20812	11.1
8b	11	1	1414	593	1004	58.9	91.3	75.1	32.4	654	928.9	21190	21247	9.3
9	10	1	1423	547	985	57.6	83.8	70.7	26.2	767	914.3	20095	20471	12.6
10	7	1	1431	517	974	59.1	84.8	72.0	25.7	808	902.0	20766	21442	8.5
11	8	1	1415	536	976	58.8	85.1	72.0	26.3	735	904.0	19331	19916	15.0
12	9	1	1426	551	989	59.0	82.9	71.0	23.9	896	918.0	21414	21727	7.2
13a	6	1	1441	554	998	56.5	82.5	69.5	26.0	891.	928.5	23166	Clean	
13b	5	1	1439	563	1001	56.7	85.1	70.9	28.4	825.	930.1	23430	Tubes	
13c	4	1	1440	569	1005	56.3	82.6	69.5	26.3	900.	935.5	23670		
Average			1440	563	1001	56.5	83.4	70.0	26.9	872.	931.4	23422		

SERIES OF 1905

3	48	1	1783	873	1328	62.4	103.6	83.0	41.2	717	1245.0	29540	28722	2.8
3	49	1	1781	879	1330	62.4	102.1	82.3	39.7	753	1247.7	29894	29002	1.9
3	50	1	1798	867	1333	62.0	99.2	80.6	37.2	800	1252.4	29760	28764	2.7
4	45	1	1816	733	1275	62.2	102.6	82.4	40.4	699	1192.6	28240	28663	3.0
4	46	1	1809	715	1262	63.2	102.0	82.6	38.8	707	1179.4	27432	28155	4.8
4	47	1	1806	705	1256	63.3	102.4	82.9	39.1	692	1173.1	27057	27920	5.6
8	24	1	1788	740	1264	59.7	102.7	81.7	43.0	641	1182.3	27563	28220	4.5
8	25	1	1790	743	1267	60.1	102.4	81.3	42.3	660	1185.7	27918	28502	3.6
8	26	1	1805	743	1274	60.1	101.7	80.9	41.6	656	1193.1	27290	27688	6.3
9	27	1	1752	791	1272	59.9	101.1	80.5	41.2	683	1191.5	28140	28588	3.3
9	28	1	1753	800	1277	60.0	100.6	80.3	40.6	682	1196.7	27690	28009	5.2
9	29	1	1763	793	1278	59.7	100.6	80.2	40.9	695	1197.8	28426	28727	2.8
10	30	1	1772	786	1279	59.0	101.2	80.1	42.2	674	1198.9	28443	28718	2.9
10	31	1	1782	788	1285	58.8	101.8	80.3	43.0	663	1204.7	28509	28646	3.1
10	32	1	1783	792	1288	59.1	101.4	80.3	42.3	671	1207.7	28383	28449	3.8
11	34	1	1776	805	1291	67.1	103.1	85.1	36.0	767	1205.9	27612	27717	6.2
11	35	1	1785	803	1294	67.0	102.4	84.7	35.4	753	1209.7	26656	26674	9.8
12	37	1	1776	804	1290	64.5	102.1	83.3	37.6	747	1206.7	28087	28176	4.7
12	38	1	1785	790	1288	64.0	102.1	83.1	38.1	721	1204.9	27470	27598	6.6
13	39	1	1766	740	1253	60.2	101.4	80.8	41.2	679	1172.2	27975	28889	2.3
13	40	1	1754	748	1251	60.7	102.9	81.8	42.2	668	1169.2	28190	29190	1.3
13	41	1	1747	736	1242	61.2	101.5	81.4	40.3	695	1160.6	28009	29213	1.2
14	43	1	1805	808	1307	58.5	102.1	80.3	43.6	657	1226.7	28645	28267	4.4
14	44	1	1792	803	1298	58.8	102.9	80.9	44.1	645	1217.1	28445	28290	4.3
15	51	1	1804	759	1282	61.1	102.1	81.6	41.0	688	1200.4	28208	28445	3.8
15	52	1	1808	736	1272	60.8	100.9	80.9	40.1	701	1191.1	28110	28568	3.4
7	21	1	1792	794	1293	64.5	102.1	83.3	37.6	782	1209.7	29403	Clean	
7	22	1	1796	787	1292	64.2	103.6	83.9	39.4	754	1208.1	29708	Tubes	
7	23	1	1808	784	1296	63.3	101.4	82.4	38.1	776	1213.6	29566		
Average			1799	788	1294	64.0	102.4	83.2	38.4	770.7	1210.5	29559		

The method of conducting a test was as follows:

The burner was first lighted and the gas and air pressures adjusted, then the flow of water through the water chamber was regulated and the apparatus allowed to run until all conditions had become uniform. This usually occupied about one hour, at the end of which time the test was started.

At the beginning of a test for the series of 1901 a determination of temperature was made by the copper ball pyrometer and readings were taken on all three thermometers, which readings were also taken at intervals of five minutes throughout the test. At the end another determination was made of the furnace temperature, and the water which had flowed through the chamber was weighed.

Observations for the tests of 1904 and 1905 were taken in a similar manner except that all temperature readings including that of the furnace were taken at regular intervals of 10 minutes.

Table No. 5 gives a summary of the data of the various tests and also the calculated results. The furnace temperature was not maintained quite the same throughout the tests, as an inspection of Table No. 5 will show. It was likewise impossible to maintain the average temperature of the circulating water the same during all the tests. Consequently the range of temperature between the gases in the tube and the water varied somewhat. Since the rate of transmission of heat through the tube varies directly with this range in temperature it is necessary, in order to compare the conductivity of the different tubes, to reduce the actual amounts of heat transmitted to what they would have been for one standard range of temperature. This standard range was assumed the same as the range existing during the test of a new clean tube, such a tube being tested with each series of tests.

These derived figures are given in column 14, Table No. 5, and they show the amounts of heat which would have been transmitted in each case had the difference between the temperature of the gases and the temperature of the water been the same in all tests of that particular series, i. e., the same as during the tests of the clean tube then tested. It is from the figures in this column that the losses due to scale are computed. This loss expressed as a per cent is exhibited in column 15 of Table No. 5.

Table No. 6 gives the chemical analyses of the scale found upon the tubes tested in the series of 1901, 1904 and 1905. The constituents of the scale are calculated to compounds and expressed as per cent. These analyses were made by the Chemical department of the University of Illinois.

TABLE 6

THE TRANSMISSION OF HEAT THROUGH SCALE-COVERED BOILER TUBES RAILWAY ENGINEERING DEPARTMENT—UNIVERSITY OF ILLINOIS

CHEMICAL ANALYSES OF SCALE

Tube Number	Constituents of Scale —— Amount in per cent								
	SiO_2 Silica	Fe_2O_3 Al_2O_3 Oxides of Iron and Aluminum	$CaSO_4$ Calcium Sulphate	$CaCO_3$ Calcium Carbonate	CaO Calcium Oxide	$MgCO_3$ Magnesium Carbonate	MgO Magnesium Oxide	Moisture	Organic Matter and Alkali
1	2	3	4	5	6	7	8	9	10
SERIES OF 1901									
1	7.00	8.32	16.78	47.16	....	1.30	11.20	1.18	7.06
2	5.68	8.98	27.30	22.27	10.45	....	15.82	1.14	8.34
3	9.24	10.98	2.04	59.75	0.62	....	10.18	0.64	6.55
4	9.80	15.92	25.86	23.20	3.02	....	13.16	1.42	7.62
5	10.00	7.00	14.35	50.30	....	8.40	7.30	0.84	1.81
6	7.42	4.26	16.08	51.44	....	1.53	11.25	1.19	6.83
7	12.22	5.38	17.70	37.02	....	0.79	16.76	1.63	8.50
9	12.46	11.24	36.85	21.37	....	3.13	0.72	1.02	13.21
11	17.82	10.32	5.06	37.50	6.70	....	13.92	2.25	6.43
SERIES OF 1904									
1	26.04	10.80	1.36	27.50	3.84	.....	23.07	7.39	
2	17.21	8.02	18.50	11.52	....	15.61	21.74	7.40	
3	15.10	3.95	21.93	10.48	....	11.06	25.28	12.20	
4	8.99	1.90	22 81	49.11	....	14.11	0.50	2.58	
5	9.11	2.20	47.96	16.85	....	2.46	11.76	9.66	
6	12.10	2.60	54.99	10.78	....	4.63	9.57	5.33	
7a	12.93	1.35	21.83	31.14	....	5.14	15.94	11.67	
7b	12.93	1.35	21.83	31.14	....	5.14	15.94	11.67	
8a	12.70	2.33	35.05	17.03	....	3.99	17.85	11.05	
8b	12.70	2.33	35.05	17.03	....	3.99	17.85	11.05	
9	23.52	7.20	6.56	37,56	3.48	.....	5.59	16.09	
10	10.05	6.47	11.71	50.98	....	7,33	4.94	8.52	
11	9.75	2.08	6.05	54.71	....	5.72	6.43	15.26	
12	7.98	4.68	8.89	55.61	....	11.36	3.75	7.73	
SERIES OF 1905									
3	7.09	5.05	17.16	19.45	.....	0.77	34.10	0.58	15.80
4	6.92	3.57	21.57	3.61	25.61	....	1.85	0.56	36.31
8	6.61	1.34	0.62	74.26	.....	0.15	10.87	0.68	5.47
9	8.44	2.52	12.59	56.80	.....	0.33	11.43	0.87	7.02
10	3.33	1.43	5.82	67.99	1.64	....	11.11	0.48	8.20
11	7.09	2.30	14.87	58.18	.....	2.22	8.26	0.75	6.33
12	27.72	9.53	12.11	10.05	0.24	....	29.90	1.07	9.38
13	9,54	10.38	8.41	9.67	3.87	....	39.09	0.71	18.33
14	16.87	4.73	2.22	40.30	6.06	....	19.25	0.83	9.74
15	24.03	12.69	1.46	31.37	0.35	....	20.91	1.45	7.74

Table No. 7 gives a summary of the data and results. In it are given for each tube the corresponding average loss as determined by the several experiments, as well as the thickness and some of the results of analyses. From this table there have been plotted five diagrams—Fig. 6, 7, 8, 9 and 10, which exhibit the loss due to the scale with reference to thickness, hardness and chemical composition. Fig. 6 shows the loss due to scale plotted with reference to its thickness. Fig. 7 is identical with Fig. 6, except that the letters H, S or M have been added at the various points to indicate the scale as being either hard, soft, or medium. In Fig. 8, 9 and 10 the loss due to the scale is plotted with reference to the amount of its chemical constituents; in Fig. 8 with reference to the sum of the percentages of calcium carbonate and magnesium carbonate; in Fig. 9 with reference to the percentage of calcium sulphate; and in Fig. 10 with reference to the percentage of silica.

In the series of 1901 there are a few tests which indicate an increase of conductivity of the scaled tube as compared with the clean tube. These are perhaps to be accounted for by errors in conducting the experiments, although they could not be detected at the time the experiments were made. The apparatus used in 1904 and 1905 was improved in some particulars, the most important change being in the means for the measurement of furnace temperatures. Such discrepancies disappear in the latter series.

When the experiments were planned it was considered probable that the transmission of heat through the scale was principally dependent upon two of its characteristics, namely, its thickness and its mechanical structure and that probably, for such thicknesses as are usually met with, thickness had greater influence than structure. Thickness was therefore carefully determined and structure approximately designated as in Table 4, as hard, soft or medium, no more exact characterization of structure being possible with tubes collected from different sources as these were.

It was hoped that the experiments might develop, if perhaps only approximately, some law of variation of conductivity with thickness. After making allowance for probable errors due to the method of conducting the tests, consideration of Fig. 6 shows perhaps a decrease of conductivity with thickness; but certainly no regularity of variation. In Fig. 7 the loss in heat transmission is again plotted with reference to thickness; and the structure of the scale, in so far as it was determined, is indicated as previously explained. No regularity of variation is observable with respect to hardness or softness.

TABLE 7

SUMMARY

THE TRANSMISSION OF HEAT THROUGH SCALE-COVERED BOILER TUBES

RAILWAY ENGINEERING DEPARTMENT—UNIVERSITY OF ILLINOIS

SERIES OF 1901

Tube No.	Test No.	Loss per cent	Ave. Loss per cent	Character of Scale	Thickness of Scale	$CaCO_3+MgCO_3$ in Scale	Ca SO_4 in Scale	Si O_2 in Scale
1	11	*0.6	1.2	H	0.06	48.46	16.78	7.00
1	12	2.9						
2	16	13.0	10.8	S	0.06	22.27	27.30	5,68
2	18	8.5						
3	1	9.7	9.4	H	0.02	59.75	2.04	9.24
3	2	9.1						
4	20	*1.4	*3.5	H	0.03	23.20	25.86	9.80
4	21	*5.5						
5	6	6.5	7.1	H	0.13	58.70	14.35	10.00
5	22	7.6						
6	3	*2.1	0.3	✓	0.07	52.97	16.08	7.42
6	23	2.6						
7	8	6.4	3.3	M	0.04	37.81	17.70	12.22
7	9	0.2						
9	4	11.4	15.9	H	0.11	24.50	36.85	12.46
9	15	20.4						
11	7	9.1	10.5	S	0.09	37.50	5.06	17.82
11	24	11.9						

*Increase

SERIES OF 1904

Tube No.	Test No.	Loss per cent	Ave. Loss per cent	Character of Scale	Thickness of Scale	$CaCO_3+MgCO_3$ in Scale	Ca SO_4 in Scale	Si O_2 in Scale
1	15	5.1	....	H	0.04	27.50	1.36	26.04
2	16	9.8	....	S	0.07	27.13	18.50	17.21
3	17	8.6	....	S	0.08	21.54	21.93	15,10
4	18	2.0	....	S	0.05	63.22	22.81	8.99
5	19	6.5	....	S	0.04	19.31	47.96	9.11
6	20	7.2	....	S	0.08	15.41	54.99	12,10
7a	13	11.0	....	S	0.06	36.28	21.83	12.93
7b	14	12.4	....	S	0.06	36.28	21.83	12.93
8a	12	11.1	....	H	0.05	21.02	35.05	12.70
8b	11	9.3	....	H	0.04	21.02	35.05	12.70
9	10	12.6	....	H	0.06	37.56	6.56	23.52
10	7	8.5	....	H	0.03	58.31	11.71	10.05
11	8	15.0	....	S	0 09	60.43	6.05	9,75
12	9	7.2	....	S	0.03	66.97	8.89	7,98

SERIES OF 1905

Tube No.	Test No.	Loss per cent	Ave. Loss per cent	Character of Scale	Thickness of Scale	$CaCO_3+MgCO_3$ in Scale	Ca SO_4 in Scale	Si O_2 in Scale
3	48	2.8	2.5	M	0.07	20.22	17.16	7.09
3	49	1.9						
3	50	2.7						
4	45	3.0	4.5	H	0.05	3.61	21.57	6.92
4	46	4.8						
4	47	5.6						
8	24	4.5	4.8	S	0.03	74.41	0.62	6.61
8	25	3.6						
8	26	6.3						
9	27	3.3	3.8	S	0.09	57.13	12.59	8.44
9	28	5.2						
9	29	2.8						
10	30	2.9	3,3	S	0.07	67.99	5.82	3.33
10	31	3.1						
10	32	3.8						
11	34	6.2	8.0	S	0.04	60.40	14.87	7.09
11	35	9.8						
12	37	4.7	5.7	H	0.07	10.05	12.11	27.72
12	38	6.6						
13	39	2.3	1.6	H	0.02	9.67	8,41	9.54
13	40	1.3						
13	41	1.2						
14	43	4.4	4.4	M	0.04	40.30	2.22	16,87
14	44	4.3						
15	51	3.8	3.6	H	0.03	31.37	1.46	24.30
15	52	3.4						

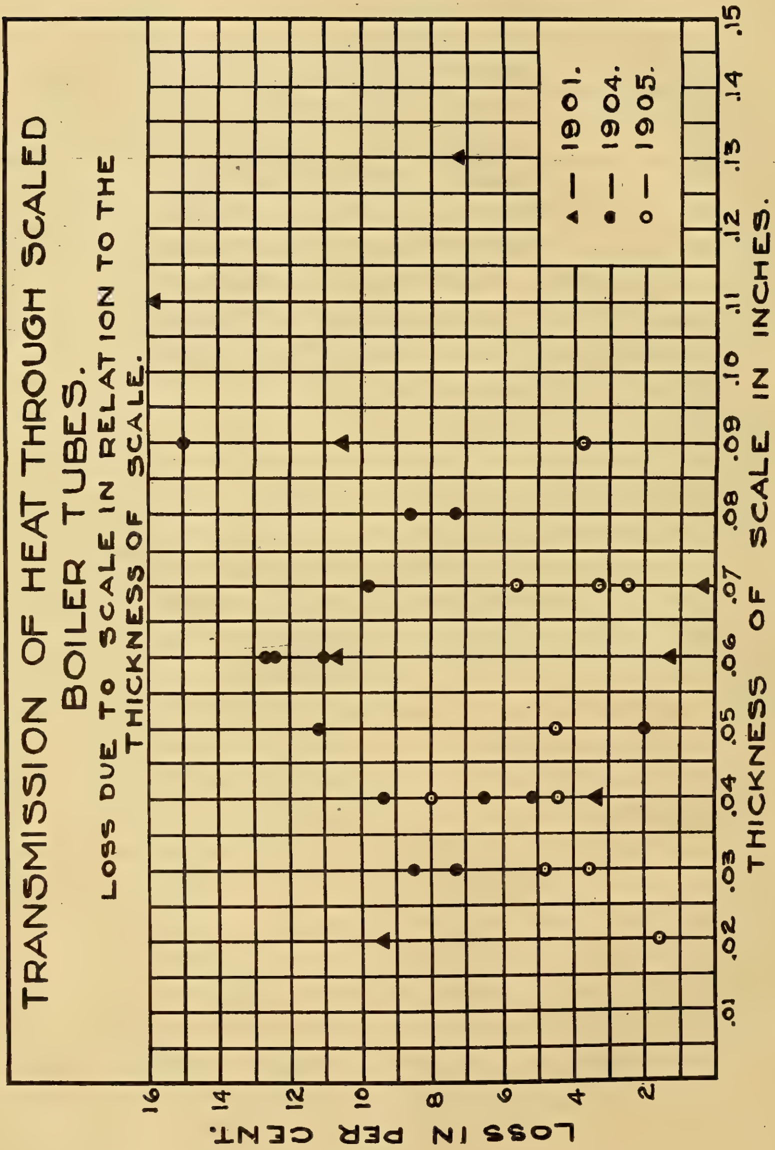
TRANSMISSION OF HEAT THROUGH SCALED BOILER TUBES.
LOSS DUE TO SCALE IN RELATION TO THE THICKNESS OF SCALE.
▲ — 1901.
● — 1904.
○ — 1905.
16
14
12
10
8
6
4
2
LOSS IN PER CENT.
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
.15
THICKNESS OF SCALE IN INCHES.

FIG. 6

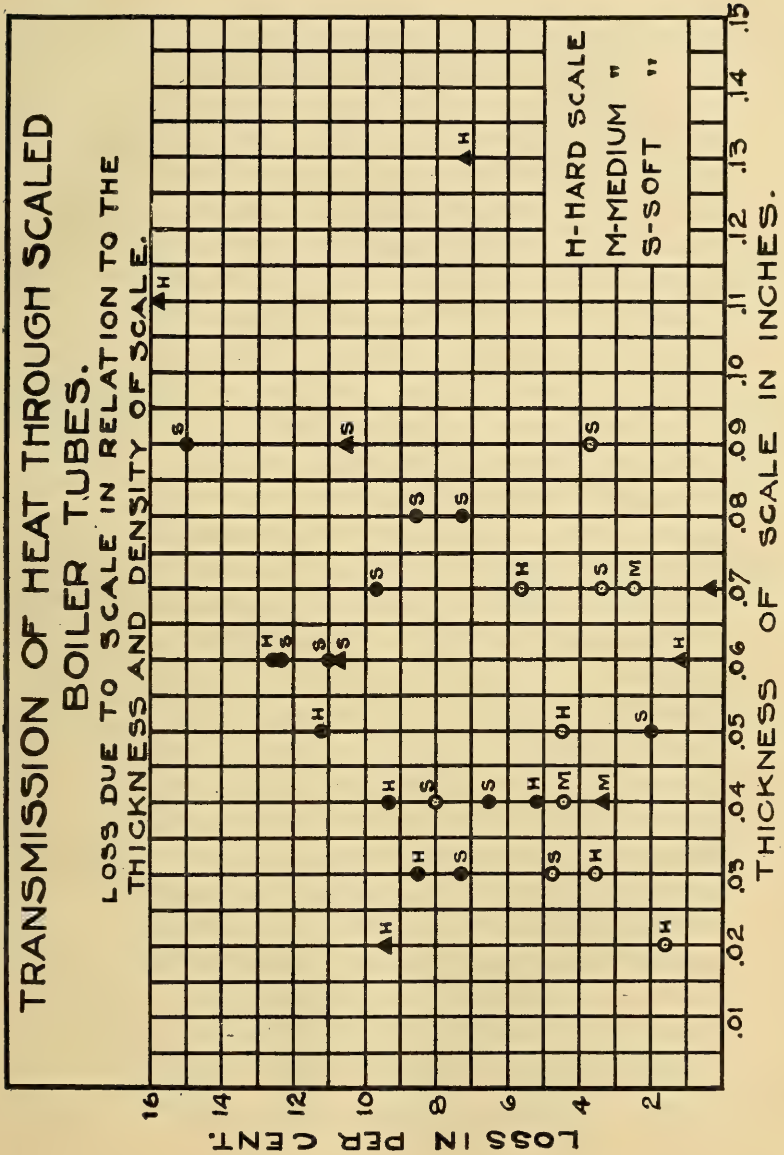
TRANSMISSION OF HEAT THROUGH SCALED BOILER TUBES.
LOSS DUE TO SCALE IN RELATION TO THE THICKNESS AND DENSITY OF SCALE.
H-HARD SCALE
M-MEDIUM "
S-SOFT "
LOSS IN PER CENT.
16
14
12
10
8
6
4
2
THICKNESS OF SCALE IN INCHES.
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
.15
H
M
S

Fig. 7

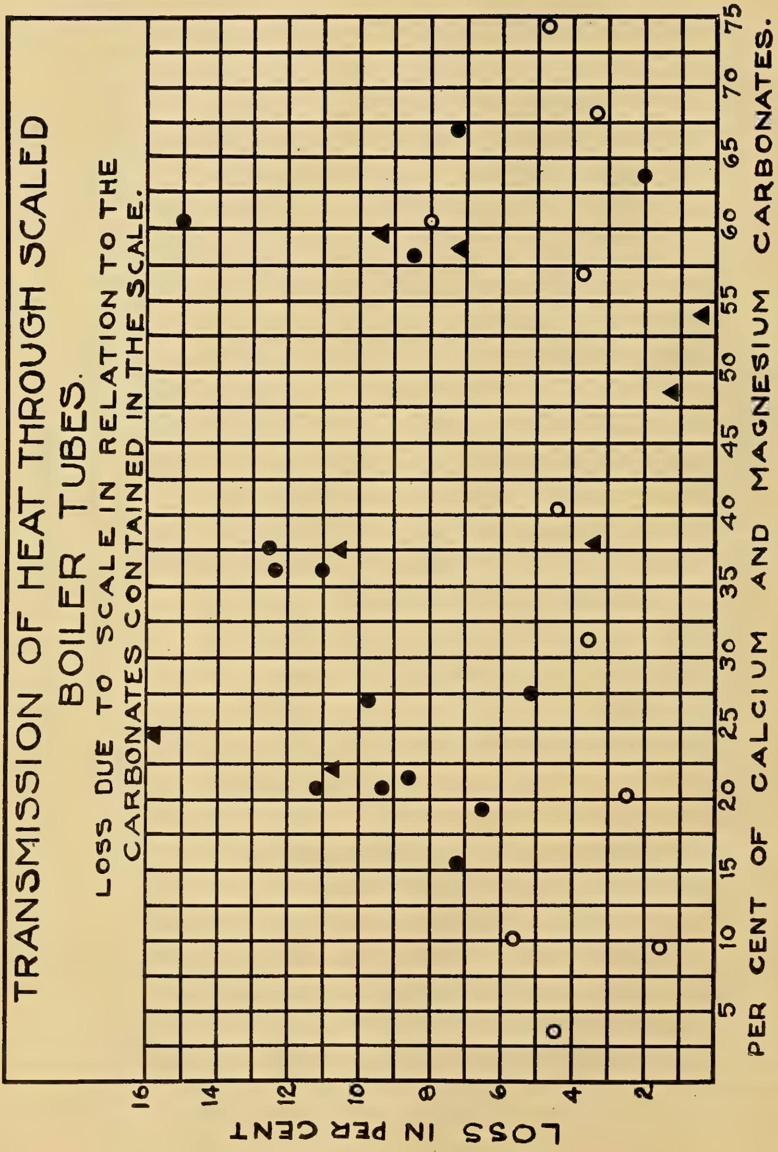
TRANSMISSION OF HEAT THROUGH SCALED BOILER TUBES.
LOSS DUE TO SCALE IN RELATION TO THE CARBONATES CONTAINED IN THE SCALE.
LOSS IN PER CENT
16
14
12
10
8
6
4
2
PER CENT OF CALCIUM AND MAGNESIUM CARBONATES.
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75

FIG. 8

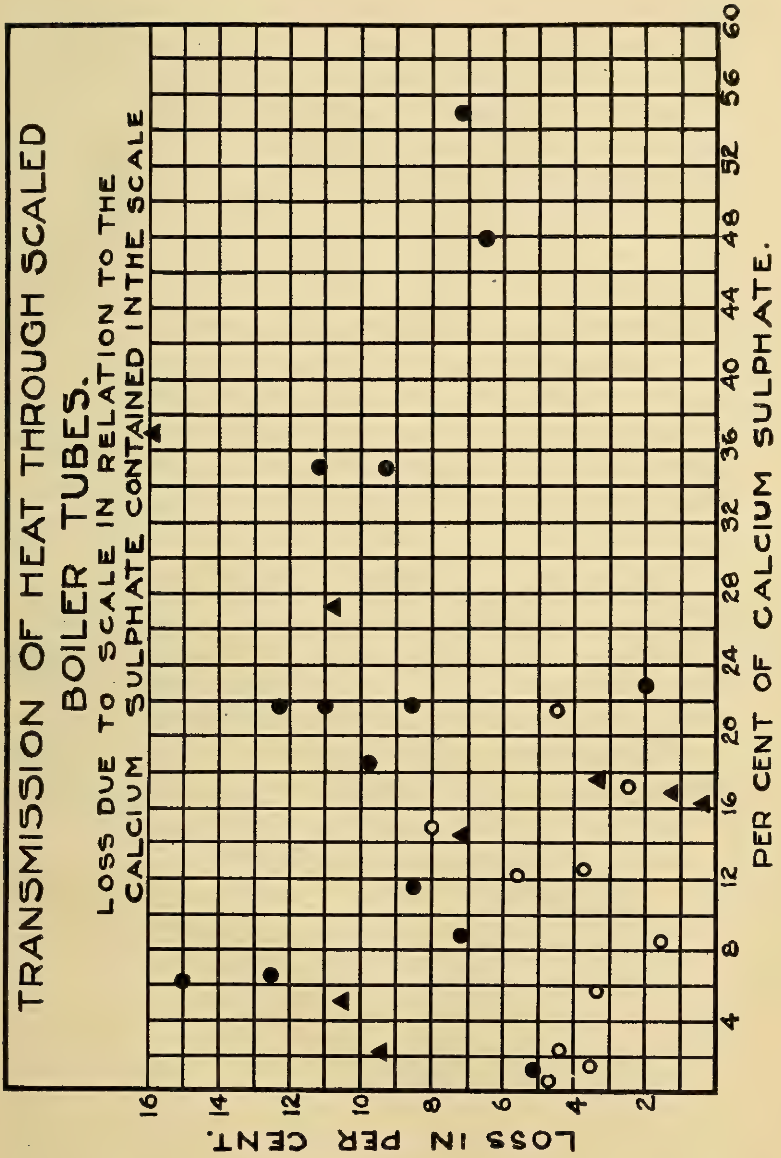
TRANSMISSION OF HEAT THROUGH SCALED BOILER TUBES.
LOSS DUE TO SCALE IN RELATION TO THE CALCIUM SULPHATE CONTAINED IN THE SCALE
LOSS IN PER CENT.
16
14
12
10
8
6
4
2
PER CENT OF CALCIUM SULPHATE.
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60

Fig. 9

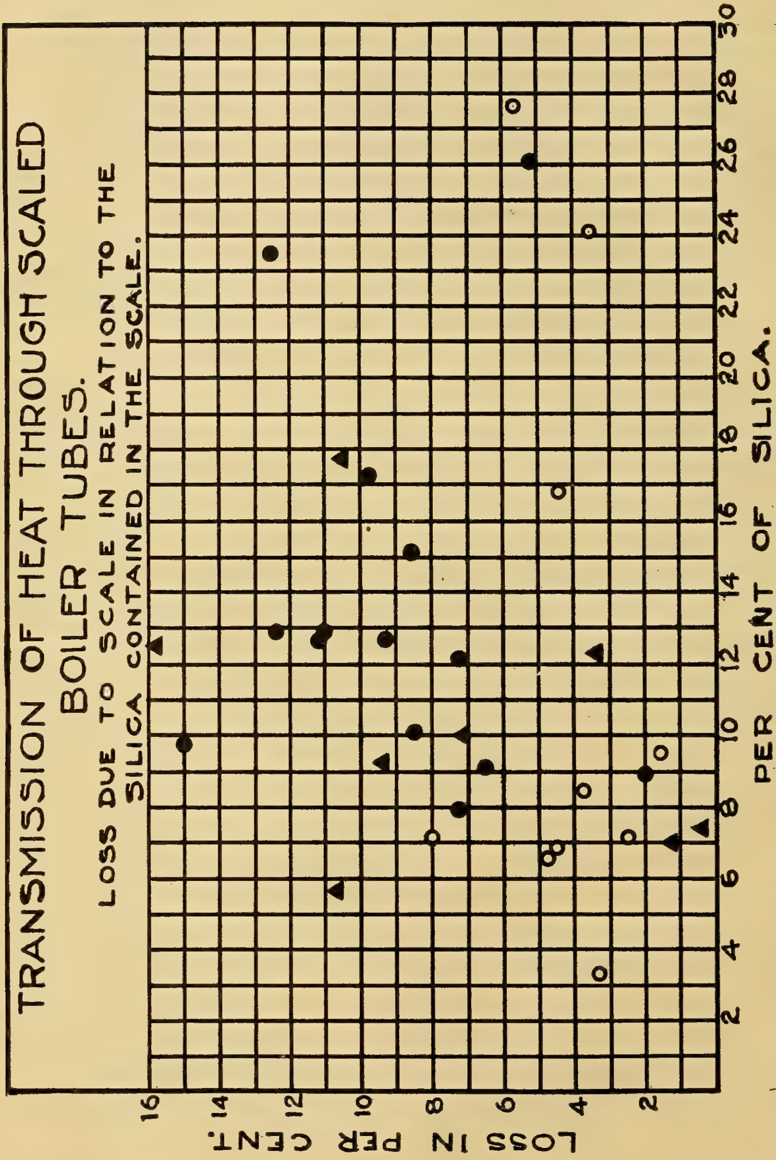
TRANSMISSION OF HEAT THROUGH SCALED BOILER TUBES.
LOSS DUE TO SCALE IN RELATION TO THE SILICA CONTAINED IN THE SCALE.
LOSS IN PER CENT.
16
14
12
10
8
6
4
2
PER CENT OF SILICA.
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

Fig. 10

In considering Fig. 6 and 7 it must be borne in mind that the tubes tested were taken from locomotives which had been in service in different parts of the country and that the scale on each tube was made up of the mineral constituents of many different water supplies. What is designated as hard scale in one case may be very different in structure—in porosity, for example—from what is designated as hard scale on another tube. Fig. 7 cannot therefore be considered as providing conclusive evidence concerning variation of conductivity with structure. The results may properly be interpreted as indicating that mechanical structure is at least as important a factor in the change in heat transmission due to scale as is the mere thickness. Such a conclusion is, of course, in accord with the facts concerning other heat insulators.

Fig. 8, 9 and 10, in which the loss in heat transmission is plotted with reference to the principal chemical constituents of the scale, do not warrant the conclusion that its chemical composition has any direct influence on its conductivity.

From the point of view of the physicist the experiments are open to objection as to method. From the engineer's viewpoint it is believed that the possible errors of the experiments do not, by any means, account for all the irregularity in the plotted results, and considering the controversy upon this subject and the comparatively meager information available, it is deemed proper to publish at this time the results as they stand in the hope that they contribute additional information which may be of interest in some quarters.

Conclusions:

In so far as generalization is warranted we may sum up the results of the tests in the following conclusions:

1. Considering scale of ordinary thickness, say of thicknesses varying up to $\frac{1}{8}$ inch, the loss in heat transmission due to scale may vary in individual cases from insignificant amounts to as much as 10 or 12 per cent.
2. The loss increases somewhat with the thickness of the scale.
3. The mechanical structure of the scale is of as much or more importance than the thickness in producing this loss.
4. Chemical composition, except in so far as it affects the structure of the scale, has no direct influence on its heat transmitting qualities.

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TESTS OF REINFORCED CONCRETE T-BEAMS

SERIES OF 1906

BY

ARTHUR N. TALBOT

BULLETIN NO. 12 OF THE UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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THIS BULLETIN is the twelfth of a series of bulletins published by the Engineering Experiment Station of the University of Illinois. The Engineering Experiment Station was established by action of the Board of Trustees December 8, 1903. It is the purpose of the Station to carry on investigations along various lines of engineering, and to study problems of importance to professional engineers and to the manufacturing, railway, mining, constructional and industrial interests of the state.

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 12. FEBRUARY 1907

TESTS OF REINFORCED CONCRETE T-BEAMS SERIES OF 1906.

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS.

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I. Introduction.

1. *Preliminary.*—The series of tests on T-beams herein described was undertaken with two objects in view,—to determine whether the width of slab within the limit used in the experiments is a controlling element in the strength of the beam, and to test the efficacy of vertical reinforcing stirrups in resisting web stresses.

When a reinforced concrete floor and its supporting beams are built as one piece, the resulting composite structure forms a system of T-shaped beams. There are differences of opinion on the action of the T-beams so formed, and also differences as to the width of flange or floor which may be considered to contribute to the strength and stiffness of the beam. T-shapes may also be used in the design of bridge girders and other structures for special conditions. The large amount of reinforcement which may be put into a T-beam without encroaching on the compressive strength of the concrete too far and the resulting high web stresses developed in the stem of the T-beam, make the T-beam an advantageous form of test piece for determining the efficacy of various forms and amounts of web reinforcement. It is felt that this feature of T-beam testing is, in itself, sufficient reason for the conduct of tests on T-beams.

2. *Scope of Tests.*—Three top widths of beam were used, equal, respectively, to two, three, and four times the width of the stem of the beam. The beams were reinforced vertically with U-shaped stirrups. One size and spacing of stirrups was used. To insure that failure would not occur by slipping of the vertical reinforcement in the concrete, the stirrups were made of deformed bars. The amount of longitudinal reinforcement was made proportional to the width of flange of the beam. Both mild steel plain round rods and high-carbon Johnson corrugated bars were used for the longitudinal reinforcement, as it was not known whether the point of elastic limit of the metal would control the amount of load or method of failure. As no data on the amount of the web stresses which may be resisted by vertical stirrups were available and as little seemed to be known on the effect of the width of flange or slab, the beams were designed to give considerable latitude in the results The series was considered as preliminary and leading to a set of

tests which should include various forms and amounts of web reinforcement.

3. *Acknowledgment.*—The investigation was made in the Laboratory of Applied Mechanics of the University of Illinois as a part of the work of the University of Illinois Engineering Experiment Station. Assistance in the tests and in the calculations was given by F. S. Hewes and C. A. Hewes, senior students in civil engineering, class of 1906, who used the results in their thesis. Immediate supervision of the work of making the beams and conducting the tests was given by D. A. Abrams, Assistant in the Engineering Experiment Station. Acknowledgment is also made to W. R. Robinson, Assistant in the Engineering Experiment Station, for aid in the preparation of this bulletin. The stone, sand and cement used in making the beams were furnished by the Joint Committee on Concrete and Reinforced Concrete.

An analysis of some of the elements of flexure of T-beams will be given. This will be followed with a description of the test pieces and method of testing, the experimental data, and a discussion of the results.

II. RESISTANCE OF T-BEAMS TO FLEXURE

4. *General.*—The analysis of the resistance of T-beams to flexure may be made to follow the general lines of analysis for rectangular beams. If the tensile strength of the concrete be not considered at sections having a maximum bending moment and if the flange or slab extends down to the neutral axis of the beam, the resisting moment may be expected to be the same as that for a rectangular beam of width equal to the width of the flange, provided, of course, that the integrity of the plane section is conserved. If the flange does not extend down to the neutral axis an examination of the effect of the omission of a part of the compression area must be made to find whether the formula for rectangular beams is still applicable. With relatively shallow flanges some modification of the formulas used in rectangular beams for determining neutral axis, tensile and compressive stresses, and resisting moment may be required. In the determination of web stresses, a slightly different treatment will be needed on account of the narrowed width of beam through the stem. The stiffness and integrity of the flange next to its junction with the stem will require investigation.

In the treatment here given, the usual assumptions of beam action noted in Bulletin No. 4 will be made. These include the assumptions that a plane section before bending will remain a plane section after bending and that tensile stresses in the concrete at the section of greatest bending moment may be neglected. The stresses developed in the flange to conserve the plane section will be considered separately. The general treatment will follow that given in Bulletin No. 4 of the University of Illinois Engineering Experiment Station, under I. RESISTANCE OF BEAMS TO FLEXURE, and equations from this source will be quoted without demonstration. The term "inclosing rectangle" will be used to denote the rectangle inclosing the flange of the T-beam and the stem down to the centroid of the reinforcing bars.

5. *Notation.*—The following notation will be used, Fig. 1:

h = thickness of flange or slab of T-beam.

b = breadth of flange or slab of T-beam.

b' = breadth of stem of T-beam.

d = distance from the compressive face to the centroid of the longitudinal reinforcement.

d' = distance from the center of the longitudinal reinforcement to center of gravity of compressive stresses.

A = area of cross section of longitudinal reinforcement.

bd = area of inclosing rectangle (rectangle inclosing flange and stem of T-beam down to center of the longitudinal reinforcement).

$p = \frac{A}{bd}$ = ratio of area of longitudinal reinforcement to area of inclosing rectangle.

o = circumference or periphery of one reinforcing bar.

m = number of reinforcing bars.

E_s = modulus of elasticity of steel.

E_c = initial modulus of elasticity of concrete in compression, a term defined in Bulletin No. 4.

$n = \frac{E_s}{E_c}$ = ratio of two moduli.

f = tensile stress per unit of area in longitudinal reinforcement.

c = compressive stress per unit of area in most remote fiber of concrete.

c' = compressive stress per unit of area which causes failure by crushing.

ε_s = deformation per unit of length in the longitudinal reinforcement.

ε_c = deformation per unit of length in most remote fiber of the concrete.

ε'_c = deformation per unit of length when crushing failure occurs; i. e., ultimate or crushing deformation.

$q = \frac{\varepsilon_c}{\varepsilon'_c}$ = ratio of deformation existing in most remote fiber to ultimate or crushing deformation.

k = ratio of distance between compression face and neutral axis to distance d.

z = distance from compression face to center of gravity of compressive stresses.

M = resisting moment at the given section.

s = horizontal tensile stress per unit of area in the concrete.

t = diagonal tensile stress per unit of area in the concrete.

u = bond stress per unit of area on the surface of the reinforcing bar.

v = vertical shearing stress and horizontal shearing stress per unit of area in the concrete.

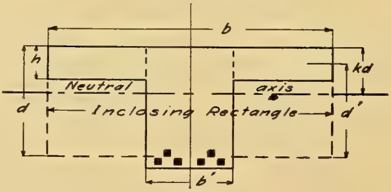

FIG. 1. T-BEAM AND INCLOSING RECTANGLE.

6. *Longitudinal, Tensile and Compressive Stresses, and Location of Neutral Axis.—Approximate Solution.*—Under the assumption that horizontal tensile stresses in the concrete are not to be considered and that a plane section before bending remains a plane section after bending, the formulas for rectangular beams may be made applicable to T-beams with a close degree of approximation by the substitution of the rectangle inclosing the flange and stem down to the centroid of the reinforcing bars (called here the inclosing rectangle) shown in Fig. 1. If the flange of the T-beam were thick enough to extend down to or below the neutral axis, the part of the concrete not included in the inclosing rectangle would, by the assumptions quoted above, not affect the resistance of the beam

to flexure. If the flange does not extend to the neutral axis, a part of the compression area assumed to exist for the inclosing rectangle is not available in the T-beam, but the portion so cut off has, by reason of its proximity to the neutral axis, a comparatively small influence upon the flexural action of the beam. The effect is so little that we may, within certain limits of proportions, substitute the inclosing rectangle for the T-section in determining the position of the neutral axis, the tensile stress in the reinforcement, the compressive stress in the most remote fiber of the concrete, and the resisting moment of the section.

For this approximation, the formulas given in Bulletin No. 4 for the proportional depth of the neutral axis may be repeated:

For a constant modulus of elasticity in the concrete (straight-line stress-deformation relation),

$$k = \sqrt{2\,pn + p^2n^2} - pn \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

When the deformation in the most remote fiber is equal to one fourth of the ultimate or crushing deformation ($q = \frac{1}{4}$), a condition approaching that found in beams of the usual percentage of reinforcement, the general equation for neutral axis reduces to,

$$k = \sqrt{\tfrac{24}{11}\,pn + \tfrac{144}{121}\,p^2n^2} - \tfrac{12}{11}pn \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Equation (8) of Bulletin No. 4 may be used for the general case and (9) when the concrete is at the limit of the compressive strength.

The diagram given on page 16 of Bulletin No. 4 will be useful in determining the position of the neutral axis if equation (11) is to be used. With 1% reinforcement, for a ratio n equal to 12, k will be .40; for a ratio n equal to 15, k will be .43.

For a beam in which the compressive stress developed is less than the ultimate strength of the concrete (and this condition covers all the usual cases of T-beams), the formula for the resisting moment of the beam may best be expressed in terms of the tensile stress in the reinforcing bars, as given in the two equations:

$$M = Af\,(d - z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$M = A\,f\,d' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

If $k = .40$, or .43, the latter equation may be written $M = .86\ A f d$, and this equation for the resisting moment of the T-beam may be used with sufficient accuracy for many purposes, even for quite a range of reinforcement. If the bending moment is known, the stress in the steel may be calculated by substituting the value of the bending moment for M in equation (13) or in the reduced form $M = .86\ A\ f\ d$. The effect of substituting the inclosing rectangle for the exact T-shape in determining the position of the neutral axis and the center of the compressive stresses is so slight that the error may be neglected and the above formulas used when the flange extends at least two thirds of the distance to the neutral axis or, say, when the thickness of the flange is at least one fourth of the depth of the beam. Even for thinner flanges the error in the use of these formulas will not be large.

Values for compressive stresses in T-beams are not of very general usefulness since the percentage of reinforcement (based on the inclosing rectangle) will generally not be large and the compressive stresses may therefore not need to be considered and may be sufficiently guarded by limiting the percentage of reinforcement. To illustrate, if in a T-beam made up of a beam and its connecting portion of the floor a width equal to four times the width of the beam itself be considered to be tributary to the T-beam and if the area of the reinforcing bars is 4% of the area inclosing the stem of the T-beam ($b'd$, of Fig. 1), this reinforcement will be only 1% of the inclosing rectangle of the assumed T-beam. Under conditions of good construction, the compressive stress developed in the concrete, would, as has been shown for rectangular beams, be well below any danger of a compression failure. This amount of steel is larger than would ordinarily be used in such construction.

If, however, it is desired to compute the compressive stress, an approximate solution may be made by equation (15) of Bulletin No 4,

$$c = \frac{2Af}{kbd}\cdot\frac{1-\frac{1}{2}q}{1-\frac{1}{3}q}. = \frac{2pf}{k}\cdot\frac{1-\frac{1}{2}q}{1-\frac{1}{3}q}\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

where b and p refer to the inclosing rectangle. For the conditions of T-beams the fraction $\frac{1-\frac{1}{2}q}{1-\frac{1}{3}q}$ will likely range between

.92 and .97, so that its use or non-use does not affect the results materially.

There is a greater proportional error in the use of equation (15) for beams in which the flange does not extend down to the neutral axis than there is in the use of equation (13), but as the purpose is only to find whether a limiting value is exceeded, the limits for depth of flange used with equation (13) may be considered allowable. This is further borne out by the fact that in floor systems a width of floor even greater than here used will generally be tributary to the T-beam, and hence the compressive stress will be lower than for an assumed ratio of 4.

7. *Web Stresses.—Bond, Shear, and Diagonal Tension.*—In T-beams the bond stresses developed are practically the same as would be found with the same steel in a rectangular beam. The shearing stresses developed and the corresponding diagonal tensile stresses are higher, since the width of stem is relatively small; and even with a moderate amount of reinforcement the resistance to web stresses may constitute the weakest element of the beam.

The bond stress developed in a T-beam when the longitudinal reinforcing rods are laid horizontally throughout the length of the beam may be determined by equation (17) of Bulletin No. 4, (p. 19),

$$u = \frac{V}{mod'} \qquad \text{...} (17)$$

where u is the bond unit-stress developed, V is the total external vertical shear, m is the number of reinforcing bars, o is the effective circumference or periphery of one bar, and d' is the distance from the center of the longitudinal reinforcement to the center of gravity of the compressive stresses. The only approximation to be made is in getting the value of d'. As in the preceding article, d' may be taken to be $d\ (1 - .34\ k)$, and k may be obtained as before suggested by the use of the method of rectangular beams with the width of the beam taken as that of the inclosing rectangle. For most conditions d' may be called $.86d$.

The reasoning for the determination of vertical and horizontal shearing stresses given on page 20 of Bulletin No. 4 is directly applicable to T-beams, and formula (18) will give the horizontal shearing unit-stress (and therefore its equal, the vertical shearing unit-stress), if we use in it the width b', the width of the stem of the T-beam.

$$v = \frac{V}{b'd'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

In equation (18) v is either the horizontal or the vertical shearing unit-stress, the two shearing unit-stresses being always equal. For most conditions d' may be taken as $.86d$, as in the last paragraph. The amount of the vertical shearing unit-stress developed in T-beams may be much higher than in ordinary rectangular beams, but it will be well below the ultimate shearing strength of the concrete.

The diagonal tensile stress in the stem of the T-beam is a function of the shearing stress and the horizontal tensile stress in the concrete. As the horizontal tensile stresses may not be well determined, it seems best for our purpose to use the horizontal and vertical shearing stresses as a means of comparison of the diagonal tensile stress developed. If part or all of the reinforcing bars are bent up at the ends, the problem is further complicated. The beams described in this bulletin were reinforced with vertical stirrups, and as the concrete itself failed to resist the diagonal tension at points well below the maximum load put on the beam, the resistance of the concrete to diagonal tension will be disregarded for these tests and the effect of the vertical stirrups studied.

Two forms of metallic web reinforcement are used to counteract the diagonal tension: (1) bending the reinforcing bars or strips sheared from them into a diagonal position, and (2) vertical stirrups carried under and around the longitudinal reinforcement and extended upward to the top of the beam or to some anchorage. The T-beams tested were reinforced with vertical stirrups, and hence this form of web reinforcement will be considered further.

The diagonal tension existing in the web may be resolved into horizontal and vertical components. Considering the longitudinal bars to be all horizontal, we may expect, when the bond resistance is sufficient, that the horizontal component will be taken by the longitudinal reinforcement. Considering that the test has passed the point where the concrete of the web resists the diagonal tension, we may count that the whole vertical component of the web stresses is taken by the stirrups. The amount of this vertical component per unit of length of beam is for the T-beams equal to the horizontal shear for the width of the stem as given by equation (18), and $vb' = \frac{V}{d'}$ will give the rate of vertical stress per unit of length of beam which will go to the stirrups. If the stirrups are 6 inches apart, the stress to be taken by the two prongs of the stirrup will be 6 vb'. This calculation is on the basis of a test loading which gives a constant shear from support to load point, as, for example, a loading at the one-third points. For a uniformly distributed load, the value vb' will be the rate of stress at a given section. To illustrate further, for a T-beam having a width of stem of 8 in., and $d' = 8.6$ in., and a load of 60 000 lb., $V = 30\,000$ lb., and $v = 437$ lb. per sq. in. With stirrups 6 inches apart the total stress on one prong of the stirrup will be 10 500 lb. These calculations do not take into account the resistance to bending of the reinforcing bars themselves. In the T-beams having also a part of the bars bent up, in order to avoid complicated calculations and to give a general comparison, the same formulas will be used herein, although the results are not an accurate measure of the stresses produced.

8. *Integrity of Flanges.*—In discussions on T-beams it is frequently stated that the thickness of the flange must be at least one half as great as the width of the stem or rib, in order that the shearing stresses at the junction of the flange and stem may not exceed those in the lower part of the stem. As the actual shearing stresses in both sections are well within the shearing strength of the concrete, there is no danger of failure by shear in either section, using the term shear in its strict sense. In the stem we have used the shearing unit-stress as a measure or method of comparison for the diagonal tensile stresses. In the flange these stresses do not need consideration. Longitudinal shearing stresses in the flange, then, will not require consideration, and the limit of depth given above is unnecessary.

Another statement sometimes made is that the compressive stress in the flange varies across the width of the flange from a maximum amount next to the stem to zero at the edge of the flange. A little consideration will show that this cannot be true. The flange must transmit stress laterally to its edges, acting in a way as a beam with a load applied longitudinally and horizontally. The width of the overhang of the flange may be considered to be the span of this cantilever beam, and the distance from the load to the support (one-third the span length in the beams tested) may be thought of as the depth. Evidently this will be a very stiff beam, and the result will be a close approach to uniformity of compressive stress at points across the width of the flange. Little variation from a plane section may therefore be expected, the change that may exist being produced more largely by other causes. Of course, there must be a limit to this assumed integrity of the cross section, but for T-beams with the ordinary amount of reinforcement the compressive stress developed is small and the variation may generally be neglected.

As usually constructed, the floor, and hence the flange of the T-beam, will have reinforcement at right angles to the stem. This will resist a breaking of the flange next to the stem and assist in giving the whole flange the curved shape which the beam takes when the load is applied.

9. *Method of Treatment.*—For the investigation herein recorded, the preceding method of analysis will be used. The T-beam will be treated as a rectangular beam of the dimensions given by the inclosing rectangle, when tensile, compressive, and bond stresses are under consideration. The width of the stem will be considered to enter into calculations for shear and for diagonal tension. The integrity of the cross section will be investigated somewhat, and the bearing of the approximation involved in the above assumptions will be looked into.

10. *Other Formulas for T-beams.*—If it is desired to take into account the exact section above the neutral axis the following formula for the proportionate depth of the neutral axis may be used, if we consider $q = 0$, or if we use a straight-line modulus of elasticity. Other values of q may be used.

$$k = \sqrt{2\,pn\left(\frac{b}{b'}\right) - \frac{h^2}{d^2}\left(1 - \frac{b}{b'}\right) + \left[pn\left(\frac{b}{b'}\right) + \frac{h}{d}\left(\frac{b}{b'} - 1\right)\right]^2} - pn\left(\frac{b}{b'}\right) - \frac{h}{d}\left(\frac{b}{b'} - 1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (10')$$

For a beam with 1% reinforcement and having a ratio of depth of flange to effective depth of steel $\left(\frac{h}{d}\right)$ of $\frac{1}{4}$, using for comparison $n = 15$, this formula gives $k = .44$, and the approximate method previously described gives $k = .42$. For $\frac{h}{d} = \frac{1}{5}$, $k = .46$. It is easily seen that the effect of this difference upon the calculated resisting moment of the beam (when based on tension in the steel) is small.

III. MATERIALS, TEST PIECES, AND METHOD OF TESTING.

11. *Materials.*—The stone was a good quality of rather hard limestone from Kankakee, Illinois, ordered screened through a 1-in. screen and over a ¼-in. screen. It contained 45% to 50% voids and weighed 85 pounds per cubic foot. In the determination of the voids of both stone and sand, the material was poured slowly into water so that the voids became filled with water and no air was caught.

The sand was of good quality from near the Wabash river at Attica, Indiana. It was fairly clean, sharp and well graded, contained 28% voids, and weighed 115 lb. per cu. ft. Table 1 gives the results of a mechanical analysis of this sand.

TABLE 1.

MECHANICAL ANALYSIS OF SAND.

Sieve No.	Per cent Passing
4	100
10	73
20	36
30	12
74	5
100	2

The cement used was furnished by the Joint Committee on Concrete and Reinforced Concrete. It was made up of a mixture of five standard American portland cements, selected and mixed by the manufacturers and was of excellent quality.

The concrete was made of the proportions of 1 of cement, 2 of sand, and 4 of stone, measured by loose volume. The mixing was done and the beams made by men skilled in concrete work. Care was taken in measuring, mixing, and tamping, to secure as uniform a concrete as possible. The mixing was done with shovels by hand; the stone having been wetted a day or so before being used. The sand and cement were first mixed dry, the stone being then added and the mass mixed until uniform in appearance. Water was added in such proportion that the tamping of the central portion of the beam really amounted to a churning action.

The steel used for the horizontal reinforcement in some of the beams was $\frac{3}{4}$-in. mild steel plain round rods, and in the rest was $\frac{3}{4}$-in. high steel Johnson corrugated bars. The stirrups were in all cases $\frac{1}{2}$-in. high steel Johnson corrugated bars. Table 2 gives physical tests of these rods, the samples tested being from the ends left after the bars were cut for the beam. The average ultimate strength of the plain round rods was 58 200 lb. per sq. in. and the yield point 38 300 lb. per sq. in. The $\frac{3}{4}$-in. Johnson bars developed an average ultimate strength of 88 400 lb. per sq. in., and a yield point of 53 800 lb. per sq. in.

TABLE 2.

Tension Tests of Steel Used in T-beams.

Average Values.

T-beam No.	Size of Bar inches	Per cent Elongation in 8 in.	Yield Point pounds	Maximum Load pounds	Yield Point lb. per sq. in.	Maximum Load lb. per sq. in.
1	$\frac{3}{4}$-in. Johnson	13.5	30 700	51 300	54 900	91 600
2	bar (new section)	15.0	30 250	48 000	53 750	86 200
3	Area=.56 sq. in.	14.5	29 700	50 000	52 700	89 000
5		15.0	30 100	48 900	53 380	86 980
Av.		14.5	30 200	49 500	53 800	88 400
4	.750 plain round	34.5	16 600	24 600	35 100	55 500
8	.752 plain round	30.8	18 100	26 600	40 700	59 900
9	.750 plain round	32.0	17 300	26 100	39 100	59 100
Av.		32.4	17 300	25 800	38 300	58 200

12. *Test Specimens.*—Table 3 shows the specimens which were made and tested. The nine T-beams were 11 ft. long and the span used was 10 ft. The following dimensions were constant for all the beams: thickness of flange, $3\frac{1}{4}$ in.; width of web, 8 in.; depth over all, 12 in. and depth from top fiber to center of steel, 10 in. The width of flange varied, three speci-

mens being made of each of the widths; 16 in., 24 in., and 32 in.

The proportion of reinforcement was from .92% to 1.10%, based on the area of the inclosing rectangle, as shown in Fig. 2; that is, it was obtained by dividing the area of the steel by the product of the breadth of the flange and the effective depth of the beam. This percentage was made as near 1.00 as the size of the reinforcing bars would permit. It will be noted that if the percentage were based on the area of the rectangle inclosing the stem, it would be very high, the amount for the beams 32 in. wide amounting to about 4%. The reinforcing rods were symmetrically arranged with respect to the axis of the beam. In some, a part of the rods was turned up beyond the one third points to within 4 in. of the top of the beam. Ten U-shaped stirrups of ½-in. high steel Johnson corrugated bars were used in each beam. These stirrups were spaced 6 in. apart on the portions of the beam between the load points and supports. They passed under the corrugated

TABLE 3.

LIST OF TEST SPECIMENS.

All Beams Have Stirrups of ½ in. Johnson Bars Placed as Shown in Fig. 2.

T-beams					Minor Specimens	
		Reinforcement				
			Area			
T-beam No.	Width of Flange inches	Kind	sq. in.	per cent	Cubes	Cylinders
1	16	3 ¾-in. Johnson bars	1.68	1.05	1_1 1_2 1_3	1
2	32	6 ¾-in. Johnson bars	3.36	1.05		2
3	24	4 ¾-in. Johnson bars	2.24	0.93	3_1 3_2 3_3	3
4	16	4 ¾-in. plain round	1.76	1.10		4
5	32	6 ½-in. Johnson bars 2 bars bent up	3.36	1.05	5_1 5_2	5
6	24	5 ¾-in. plain round 2 bars bent up	2.20	0.92		
7	16	4 ¾-in. plain round	1.76	1.10	7_1 7_2 7_3	7
8	24	5 ¾-in. plain round 2 bars bent up	2.20	0.92	8_1 8_2 8_3	8
9	32	7 ¾-in. plain round 3 bars bent up	3.08	0.97		

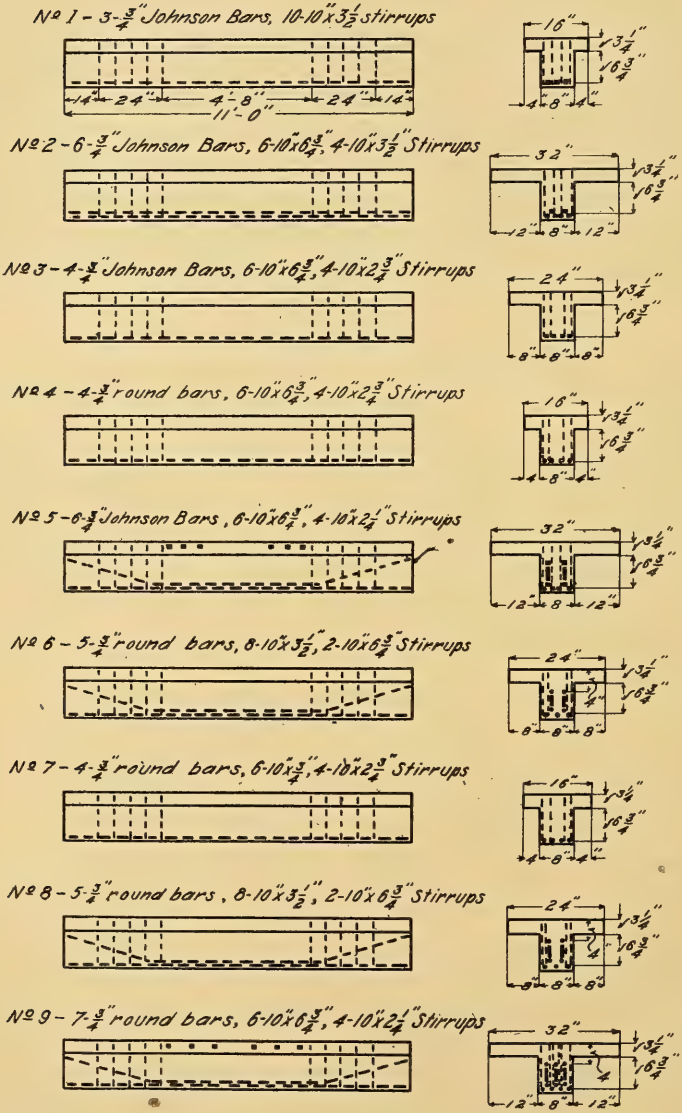
No 1 - 3-¾" Johnson Bars, 10-10"x3½" stirrups
14" 24" 4'-8" 24" 14"
11'-0"
16" 3¼" 6¾" 4" 8" 4"
No 2 - 6-¾" Johnson Bars, 6-10"x6¾", 4-10"x3½" Stirrups
32" 3¼" 6¾" 12" 8" 12"
No 3 - 4-¾" Johnson Bars, 6-10"x6¾", 4-10"x2¾" Stirrups
24" 3¼" 6¾" 8" 8" 8"
No 4 - 4-¾" round bars, 6-10"x6¾", 4-10"x2¾" Stirrups
16" 3¼" 6¾" 4" 8" 4"
No 5 - 6-¾" Johnson Bars, 6-10"x6¾", 4-10"x2¼" Stirrups
32" 3¼" 6¾" 12" 8 12"
No 6 - 5-¾" round bars, 8-10"x3½", 2-10"x6¾" Stirrups
24" 3¼" 6¾" 4" 8" 8" 8"
No 7 - 4-¾" round bars, 6-10"x¾", 4-10"x2¾" Stirrups
16" 3¼" 6¾" 4" 8" 4"
No 8 - 5-¾" round bars, 8-10"x3½", 2-10"x6¾" Stirrups
24" 3¼" 6¾" 4" 8" 8" 8"
No 9 - 7-¾" round bars, 6-10"x6¾", 4-10"x2¼" Stirrups
32" 3¼" 6¾" 4 12" 8" 12"

FIG. 2. ARRANGEMENT OF REINFORCEMENT.

reinforcing bars, alternate ones inclosing only the middle bars. In two of the 32-in. beams three ½-in. Johnson corrugated bars were placed transversely in the upper side of the flange at and near the load points. Fig. 2 shows the arrangement of the reinforcement in all of the beams. In some of the beams in which the reinforcing bars were placed in two layers, the upper bars sank down in the mortar until they were in contact with the lower layer of bars.

Fourteen 6-in. cubes were tested. The numbering is the same as that for the beams for which the batch was made. Seven 8-in. cylinders were made in a similar manner.

13. *Forms.*—Fig. 3 shows the plan of the forms used in making the T-beams. 2-in. planks were used for the sides and ends. Clamps and struts were of 2-in. x 4-in. pieces. The planks were soaked in water for some time before using, in order to prevent the absorption of water from the beams.

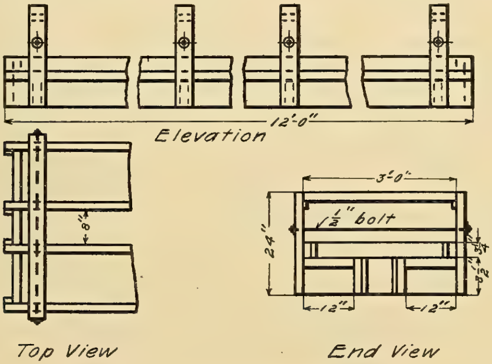

Fig. 3. Plan of Forms.

14. *Fabrication and Storage.*—The beams were made on the concrete floor of the laboratory, strips of building paper being laid on the floor to prevent the concrete from adhering to it. As already stated, the concrete was proportioned by loose volume and mixed by hand. Generally, two batches were mixed in making a beam.

After the form was set up, a layer of concrete 1-in. to 1½ in. thick was put in, the reinforcement placed, and the rest of the concrete filled in in layers about 3 in. thick. The sides and ends were spaded, the concrete tamped, then spaded again and tamped. This gave a very good surface to the test pieces.

The forms were kept on until a day or so before testing the beams, the beams being sprinkled twice daily in the meantime. The temperature of the room ranged from 60° to 70° F. during the period of storage.

15. *Method of Testing.*—Fig. 4 shows the method of loading used. The beams were tested in the 600 000-lb. Riehle' testing machine, the load being applied at the one-third points through turned steel rollers, which were 2 in. or $4\frac{1}{2}$ in. in diameter, according to the size of the beam. Each of these rested on a cast iron plate, 2 in. x 3 in. x 2 ft. 8 in., faced on both sides, which distributed the load laterally over the beam. The supports had curved bottoms to permit rocking, and plates 8 in. x $1\frac{1}{8}$ in. x 8 in. were used as bearing plates. These plates, together with the bearing plates on the top of the beam, were bedded in plaster of paris which was allowed to harden under the weight of the beam and the aparatus used in loading, before the load was applied. The movement of the head, in testing, was $\frac{1}{20}$ inch per minute. Center deflections were read on all T-beams and deformations were observed on a few. The deflections were obtained by means of a thread stretched between two points over the supports and about seven inches from the top of the beam. The frame used for holding the extensometer dials was improvised and its lack of rigidity seriously impaired its usefulness, the readings being sometimes inconsistent and therefore untrustworthy. Since at the present time tests are being made here in which reliable deformation readings are expected, diagrams from the readings taken in this series will not be reproduced.

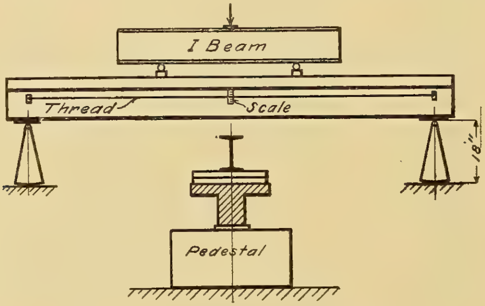

FIG. 4. METHOD OF TESTING.

The cubes, cylinders, and steel were tested on the 100 000-lb.

Riehle' machine, the speed of the head being .1 inch per minute. The cubes and cylinders were bedded in plaster of paris before testing.

IV. Experimental Data and Discussion.

16. *Cube and Cylinder Test Data.*—Table 4 gives the results of the tests of the cubes and cylinders. The breaking strength of the cylinders is somewhat below that of the cubes. The concrete showed very good quality.

17. *Deflection Diagrams.*—Fig. 7 to 15 at the end of the text give the load-deflection diagrams for the T-beams. The ordinates (vertical distances) represent the applied loads, and the abscissas (horizontal distances) the corresponding deflection at the center of the beam. It will be seen that at or near the maximum load the curve changes direction abruptly, and that for nearly all the

TABLE 4.

Results of Cube and Cylinder Tests.

Cubes			
No.	Age at Test Days	Maximum Load	
		Total Pounds	lb. per sq. in.
1_1	59	62 120	1720
1_2	59	84 690	2350
1_3	59	66 390	1840
Av.			1970
3_1	57	42 120	1170
3_2	57	49 620	1380
3_3	57	46 200	1280
Av.			1280
5_1	58	69 800	1940
5_2	58	81 500	2260
Av.			2100
7_1	56	57 600	1600
7_2	56	71 100	1980
7_3	56	68 300	1900
Av.			1830
8_1	54	68 100	1890
8_2	54	55 600	1540
8_3	54	94 500	2620
Av.			2020
Average of all cubes,			1820

Cylinders			
No.	Age at Test Days	Maximum Load	
		Total pounds	lb. per sq. in.
1	63	95 000	1890
2	63	94 200	1870
3	61	88 200	1760
4	59	67 000	1330
5	58	60 000	1190
7	56	73 000	1450
8	54	88 000	1750
Av.			1610

beams the load does not fall off materially until a considerable deflection has been obtained. The stress-deformation curves found (not reproduced) show an abrupt change on the tension side of the beam.

18. *Phenomena of Tests.*—In Table 5 (p. 25) are given the maximum loads which the beams sustained. Fig. 5 and 6 give sketches showing position of cracks as they appeared after the maximum load and also the shape of the flange in Beam No. 2 after it broke off. The general phenomena of the tests of the T-beams were quite similar to those attending the tests of rectangular beams

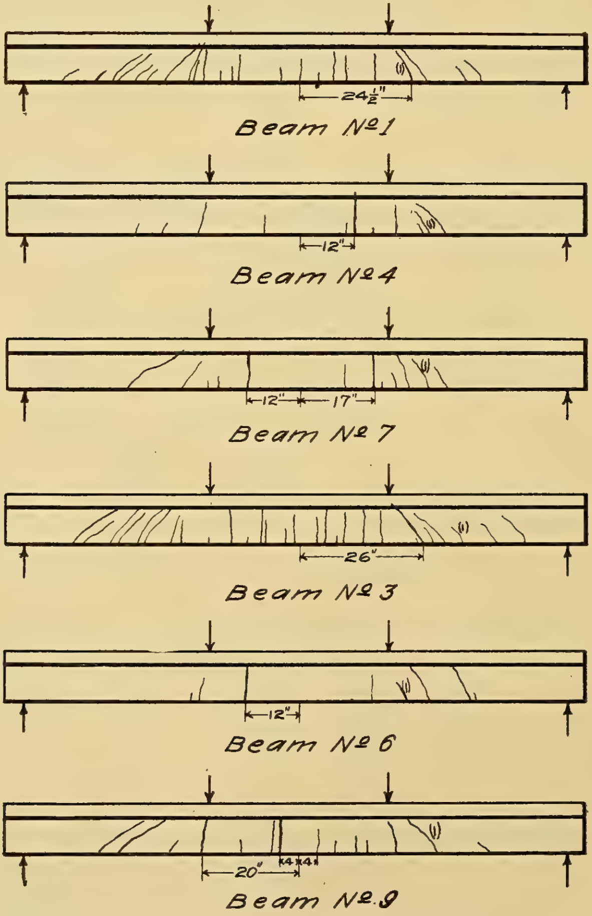

FIG. 5. SKETCH OF BEAMS AFTER FAILURE.

having a similar percentage of reinforcement. When the tension in the steel reached 13 000 to 20 000 lb. per sq. in., minute vertical cracks became visible in the concrete on the lower part of the faces of the stem of the T-beam at points in the middle third of the span length. These grew more distinct as the load was increased. When the load became sufficient to strain the steel to its yield point, these cracks opened up and finally became quite large and extended up into and along the flange of the

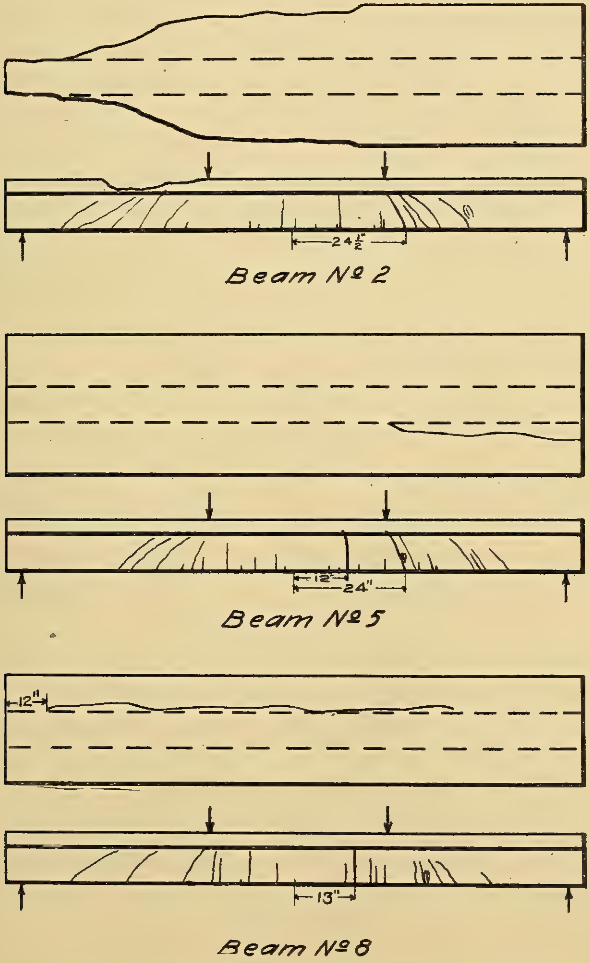

FIG. 6. SKETCH OF BEAMS AFTER FAILURE.

beam. The evidences of failure by tension in the steel were apparent in every beam, as much so in those reinforced with corrugated bars as in those having plain rods.

At loads somewhat above the amounts at which it would be expected that failures by diagonal tension in the concrete would occur in beams not having metallic web reinforcement, minute diagonal cracks appeared on the face of the stem in the outer thirds of the span length. These cracks had the direction and appearance of cracks attending failure by diagonal tension. Their consideration will be taken up under "21. *Web Stresses*," (p. 26).

In general, the flanges gave no sign of failure. In one (Beam No. 2) the flange split off after the maximum load was reached, as shown in Fig. 6. In this beam there was no lateral reinforcement in the flange. It seems evident that the tearing off occurred by reason of the warping of the flange as bending took place, and as this effect was not found in the other two beams of 32-in. width, the presence of the lateral rods in the beams was probably advantageous. Such rods would, of course, be present in ordinary floor construction, and uneven bending would be resisted by this lateral reinforcement.

In the representation of diagonal and vertical cracks after the maximum load had been reached, shown in Fig. 5 and 6, the tension crack which caused failure is indicated by a heavy line. The diagonal crack first to appear is numbered 1. These diagonal cracks generally intersected the plane of the reinforcing bars at their intersection with the stirrups. The description of the action of the beams which follows is given only in sufficient detail to indicate their general behavior, though individual peculiarities of beams are also noted.

Beam No. 1.—Two tension cracks at points at the level of the reinforcing bars between the load points appeared at an applied load of 16 000 lb. Before 20 000 lb. was reached eight more tension cracks appeared in the middle third of the span length. At 20 000 lb. the first diagonal crack appeared. At 26 000 lb. it had extended to within 2 inches of the flange. With further loading a large number of diagonal cracks developed. After reaching a load of 42 000 lb., (deflection 0.5 in.), the load fell off to 41 000 lb., as the machine speed did not keep up with the deflection of the beam under the given load. The load then slowly rose to 44 500 lb., the deflection increasing to 1.34 in. During this period the tension cracks opened up quite rapidly, while the diagonal cracks remained about the same, one diagonal crack becoming $\frac{1}{32}$ in. wide. Finally, at a deflection of 1.6 in., and an applied load of 43 800 lb., the concrete crushed at one of the load points.

Beam No. 2.—The failure of this beam was peculiar in that after it had passed the maximum load and had shown all the evidences of failure by tension of the steel, instead of holding about the same load and deflecting considerably more as the machine was run down, the final breaking was by the flanges splitting off as shown in Fig. 6. This was the only one of the three beams with 32-in. flanges which did not have lateral reinforcing rods in the flange at and near the load points. The first tension crack appeared at a point 10 in. west of the center at a load of 26 000 lb. and others appeared at 28 000 lb.

The first diagonal crack was observed 19 in. west of the west load point at 36 000 lb., and reached half-way to the flange. A number of other diagonal cracks appeared as the load was increased. At 56 000 lb. one diagonal crack had reached the flange and small horizontal cracks formed at the junction of the flange and stem. The beam carried the load well and reached a maximum of 78 300 lb. applied load. The load gradually fell off as the deflection was increased, and finally the flange on both sides from the east end to the west load point broke off suddenly, as shown in Fig. 6.

Beam No. 3.—In beam No. 3 it was noticed that each diagonal crack passed through the intersection of the stirrups and the longitudinal reinforcement. At the maximum load one of the tension cracks extended well into the flange.

Beam No. 4.—In Beam No. 4, some time after the maximum load of 29 900 lb. had been reached, the principal tension crack extended to the flange and then extended along the junction of the stem and flange, but no consequences were traceable to this. As in the other cases, the failure was by tension in the steel, followed finally by compression in the concrete.

Beam No. 5.—At an applied load of 25 000 lb., two tension cracks and one diagonal crack appeared, others following later. At 74 000 lb. the deflection curve makes an abrupt change of direction, the deflection at this load being .54 in. The tension cracks increased in size and failure by tension in steel was evident. With increased deflection the load ran up slowly, reaching a maximum of 80 000 lb. at a deflection of 1.4 in., when one roller rolled off its bearing and one pedestal fell forward, allowing the beam to drop and break. At the maximum load there were evidences of crushing in the concrete.

Beam No. 6.—In Beam No. 6, after the maximum load of 37 000 lb. was reached the principal tension crack extended into the flange and finally extended horizontally along the junction of flange and stem. The test was then discontinued.

Beam No. 7.—In Beam No. 7, the maximum load was maintained from a deflection of .3 in. to one of 1.9 in. before final failure.

Beam No. 8.—In Beam No. 8, the maximum load was maintained from a deflection of .3 in. to one of 2.6 in. before final failure. The stress-deformation diagram gave a compressive unit deformation of .0005 at the point where the steel yielded.

Beam No. 9.—The stress-deformation curve and the deflection curve show failure by tension in the steel. The position of cracks is shown in Fig. 5.

19. *Tension in Steel.*—As already noted, the abrupt turn in the deflection diagrams (Fig. 7 to 15, following the text), taken in connection with the action of the concrete on the compressive side, indicates that the beams failed in every case through the steel becoming stressed beyond the yield point. The stress-deformation diagrams of the four beams on which observations of longitudinal deformations were made, also give marked evidence of failure by tension in the steel. In Table 5 are given the calculated stresses in the reinforcement at the maximum load, based on the formula $M = .86\ A\ f\ d$. The weight of the beam and of the loading apparatus was included in the calculations. The average stress in the reinforcement in the beams reinforced with plain mild steel rods at the maximum load is 39 800 lb. per sq. in. The average stress in the corrugated bars at the maximum load is 58 700 lb. per sq. in. It will be recalled that the average yield point for the coupons taken from the ends of the reinforcing bars was 38 300 lb. per sq. in. for the mild steel plain rods and 53 800 lb. per sq. in. for the corrugated bars. The calculated stresses at the maximum load are therefore somewhat above the average yield point of the metal. It may also be noted that

there is no marked difference in the results obtained for beams of different width of flange. The tests and calculations go to show that equation (13) may be used for calculating the resisting moment, and it is believed that this formula may properly be used at least up to a reinforcement equal to 1% of the inclosing rectangle.

TABLE 5.

Results of Tests of T-beams.

$M = .86\ A\ f\ d.$

All failed by tension in steel.

Beam No.	Flange Width inches	Longitudinal Reinforcement		Maximum Load pounds			Bending Moment M lb. -in.	Stress in Steel f lb. per sq. in.
		Per cent	Kind	Applied	With Beam and Apparatus	Per 8 inches of Width		
1	16	1.05	3 ¾-in. Johnson bars	44 500	46 700	23 350	923 000	64 300
4	16	1.10	4 ¾-in. plain round	29 900	32 410	16 200	631 000	41 500
7	16	1.10	4 ¾-in. plain round	27 300	30 100	15 050	579 000	38 100
3	24	0.93	4 ¾-in. Johnson bars	53 500	55 700	18 570	1 107 500	57 500
6	24	0.92	5 ¾-in. plain round 2 bars bent up	36 800	39 300	13 100	773 500	40 700
8	24	0.92	5 ¾-in. plain round 2 bars bent up.	37 300	40 100	13 370	783 500	41 200
2	32	1.05	6 ¾-in. Johnson bars	78 300	80 500	20 120	1 608 000	55 700
5	32	1.05	6 ¾-in. Johnson bars 2 bars bent up. (6 transverse bars in flange)	80 800	83 300	20 820	1 658 000	57 400
9	32	0.97	7 ¾-in. plain round 3 bars bent up. (6 transverse bars in flange)	48 100	50 900	12 720	1 004 000	37 600

20. *Compression in Concrete.*—The stress-deformation diagrams obtained in four beams show clearly that the full compressive strength of the concrete was not developed when the yield point of the beam was reached, the amount of shortening being about .0005. The fact that the final deflection of the beam generally was several times as much as that at the yield point of

the beam shows that with the stretch of the steel beyond the yield point the neutral axis must have risen very much before final or ultimate failure by crushing of the concrete occurred, and this corroborates the preceding statement. The results bear out the assertion that beams with 1% reinforcement will have only a part of the compressive strength of the concrete developed, even when steel of 54 000 lb. per sq. in. yield point is used. As in T-beams having a width of flange equal to four times the thickness of the stem a 1% reinforcement (4% of the stem area) is very large, it does not seem that the compressive strength of the concrete in the upper fiber may be expected to be a controlling element in floor construction of this kind.

21. *Web Stresses.*—Rectangular beams which do not have metallic web reinforcement and in which the longitudinal bars are laid horizontally may be expected (for concrete of this quality) to fail by diagonal tension when the calculated vertical shearing unit stresses reach, say, 120 lb. per sq. in., provided, of course, the amount of reinforcement and relation of depth to length of span are not such that failure by tension or compression will occur before this amount of stress is developed. This is on the assumption that the method of calculation given on page 10 is applicable to T-beams so reinforced. For the T-beams tested, no matter what the width of flange, the load which would produce failure by diagonal tension in the concrete calculated in this way would be 17 000 lb. As all beams failed by tension in the steel and as the loads ran as high as 80 000 lb. it is apparent that the metallic web reinforcement was efficient and adequate.

The conditions of testing were not favorable for determining the time of appearance of the first diagonal cracks (the minute cracks which appear in the outer third of the span length and which in beams without metallic web reinforcement presage early and sudden failure), and in some cases these may have appeared before their presence was noted. However, the load at which their existence was observed is given in Table 6. In Beam No. 7 the load for the first visible crack, 18 000 lb., gives by equation (18) a value of the vertical shearing unit stress of 147 lb. per sq. in., not much above that at which a beam without metallic web reinforcement would fail. The general average time of observation of the first visible diagonal crack corresponded to a value of about 180 lb. per sq. in. This higher value of v may be an indication that the stiffness of the stirrups holds back the growth of cracks of the size at which they become visible or even that the stirrups act with the concrete in taking these secondary stresses up to the load where the cracks become visible, or else that the method of calculation gives too high results.

After the diagonal cracks have appeared, it must be considered that the secondary tensile stresses are taken mainly or wholly by the metallic reinforcement. In Table 6 are given the values of the vertical shearing unit-stresses for the maximum loads, calculated by equation (18). As in no case was there a failure by diagonal tension, we are most interested in the beams which carried the highest loads and hence developed the greatest shearing unit-stresses. Beam No. 2 developed 585 lb. per sq. in. and beam No. 5, 605 lb. per sq. in. These results are several times as great as the values obtained with rectangular beams without metallic reinforcement which failed by diagonal tension of the concrete.

TABLE 6.

Web Stresses.

Beam No.	Flange Width inches	At First Diagonal Crack			At Maximum Load	
		Load in Pounds		v lb. per sq. in.	Bond, u lb. per sq. in.	Shear, v lb. per sq. in.
		Applied	Including Beam and Apparatus			
1	16	20 000	22 200	161	302	340
4	16	29 900	32 100	234	200	236
7	16	18 000	20 200	147	186	219
3	24	28 000	30 500	222	270	405
6	24	20 000	22 500	164	194	286
8	24	27 000	29 500	214	198	292
2	32	36 000	38 800	282	260	585
5	32	35 000	37 800	275	269	605
9	32	32 000	34 800	253	180	370

The stresses in the stirrups may be calculated tentatively by the method given under "7. *Web Stresses*", (p. 9). For a load of 80 000 lb., $V = 40000$ lb.; $v = 580$. For the width of stem of 8 in. and stirrups 6 in. apart the tension in each prong of the stirrups, calculated in this way, will be 13 900 lb., equivalent to 55 500 lb. per sq. in. The bond developed is also very high.

In the foregoing discussion the horizontal and vertical shearing stresses have been used as a means of comparing resistances to diagonal stresses. Viewed by themselves, they are also interesting. 600 lb. per sq. in. is evidently well below the actual shearing resistance of concrete, but it is considerably above the values given as the ultimate shearing strength of concrete by those who hold that shearing strength is but little more than the tensile strength of concrete.

22. *Beam Deflection.*—Considering the beams all as rectangular beams having the full section of the inclosing rectangle and as having reinforcement equal to 1% of this rectangle, the deflection in beams of different width of flange should be the same at loads proportional to the flange width. That is, a load of 10 000 lb. on a rectangular beam 8 in. wide, one of 20 000 lb. on a T-beam with flange 16 in. wide, one of 30 000 lb. on a T-beam with 24-in. flange, and one of 40 000 lb. on a T-beam with 32-in. flange, all having 1% reinforcement, should, on this assumption, give the same deflection. However, the T-beams do not have the full compressive area just above the neutral axis. As the concrete on the tension side of the neutral axis also adds to the stiffness of the beam, especially in the part in the outer thirds of the span length, the narrow width of the stem will detract from the stiffness of the beam. It may be expected, then, that the deflection of the T-beams will increase somewhat as the width of flange increases, when compared on the basis of loads proportional to the width of flange or on the basis of a load which gives the same stress in the steel. In Table 7 are given the deflections of the beams for loads which give a stress of 35 000 lb. per sq. in. in the steel, as calculated by the formula, $M = .86\ A\ f\ d$. The results average .27 in. for the 16-in. flange, .27 in. for the 24-in. flange, and .32 in. for the 32-in. flange. The decrease in stiffness for the wide flange, while apparent, is not very great.

23. *Position of Neutral Axis.*—Little can be said here concerning the position of the neutral axis except that the data and analysis go to confirm the statement that for ratios of width of flange to width of stem not exceeding those used in the experiments, the position given by considering the T-beam as a rectangular beam of the full section of the inclosing rectangle is

sufficiently close for purposes of ordinary calculation. The stress-deformation diagrams for the beams on which observations were made, so far as these results may be used, also corroborate this view. As the compressive stresses in T-beams will generally be low, the error in any use of the assumed inclosing rectangle likely to be made will be small enough to be neglected, especially if we limit the use to the condition that the flange shall extend two-thirds of the distance to the neutral axis.

TABLE 7.

DEFLECTIONS FOR A STRESS OF 35 000 LB. PER SQ. IN. IN THE STEEL.

$M = .86\ A\ f\ d.$

Beam No.	Flange Width inches	Per cent Reinforcement	Applied Load pounds	Deflection inches
1	16	1.05	25 200	0.24
4	16	1.10	26 500	0.25
7	16	1.10	26 500	0.31
Av.				0.27
3	24	0.93	33 600	0.25
6	24	0.92	33 200	0.27
8	24	0.92	33 200	0.28
Av.				0.27
2	32	1.05	50 500	0.31
5	32	1.05	50 500	0.31
9	32	0.97	46 700	0.34
Av.				0.32

24. *Applicability of Results.*—The tests are not numerous enough or sufficiently diversified to show that the results are generally applicable to beam construction. It seems clear, however, from the general behavior of these beams that for calculations on strength the T-section may be considered to be the equivalent of rectangular beams of the size of the inclosing rectangle for widths of flange equal at least to four times the width of stem. It seems probable that this relation may be applicable to even greater widths of flange. However, the actual value of this limit cannot be of great practical importance, since a greater width would not materially change the value of the calculated resisting moment (considering it based upon tensile strength of steel and moment arm measured to center of compressive stress), and since the amount of steel will at any rate

be limited by the space in the stem and by practical considerations in placing and bedding it to an amount which will keep the maximum compressive stress within a reasonable limit.

The method of calculating the tension in the stirrups is used tentatively and is not regarded as a final method for use. It is probable that experiments will show that a different method of calculation should be adopted. The efficacy of stirrups of this kind is well brought out, but at the same time, it should be noted that the size, spacing, and bonding of the web reinforcement are of larger capacity than is usually given. The fact that all the beams, and especially the wide beams, escaped failure by diagonal tension is of especial importance.

Nothing in the observed phenomena of the tests indicates that there was an appreciable distortion from a plane cross section even in the flanges of the beam. The tearing of the flange in one beam was evidently due to unequal bending and seems not to be traceable to shear or to variations from a plane cross section.

25. *Summary.*—The following summary of the discussion is given:

1. Beams of flange width of two, three, and four times the width of stem or web and reinforced in each case with steel equal to 1% of the inclosing rectangle exhibited in a common way the characteristics of rectangular beams, and the critical failure in every case came through the longitudinal reinforcement becoming stressed beyond its yield point.

2. The full compressive strength of the concrete at the most remote fiber was not developed at the yield point of the beam, even in the beams which were reinforced with steel of 54000 lb. per sq. in. yield point.

3. The beams with the wide flanges were deflected somewhat more than the narrower beams, as may be expected from the lack of full width of concrete on both the compression and the tension sides of the neutral axis, the deficiency affecting the stiffness of the beam but having little effect upon its strength at points of maximum bending moment.

4. The vertical stirrups used proved to be very effective web reinforcement. The diagonal tension cracks appeared at or above the loads at which failure by diagonal tension may be expected in beams without web reinforcement. A high resistance

to diagonal tensile stresses was developed, as measured by the calculated maximum vertical shearing unit-stress, which in one beam was 605 lb. per sq. in. Since no beam failed by diagonal tension, the limit of strength of the web reinforcement was not determined.

5. The observed phenomena of the tests give no indication of distortion from a plane cross section and there was no indication that the thin flange was an element of weakness. The tearing of the flange of one beam after the maximum load was reached was due to causes which would not exist in a floor system as usually constructed. It seems clear that the limit of useful width of flange was not reached in the beams tested.

6. The maximum strength of T-beams to resist horizontal tension and compression (flange stresses) may well be calculated by using the ordinary methods and formulas in use for rectangular beams and considering the inclosing rectangle of the T-beam to be the equivalent rectangular beam. This approximation is at least applicable for reinforcement not exceeding 1% of the inclosing rectangle. It gives little error when the thickness of flange is at least one quarter of the depth of beam and the width of flange not more than four times the width of stem, and may be used for an even greater range without great error. The inclusion of a greater width of flange than four times the width of stem would not materially change the calculated strength of the beam, since the amount of steel which may be put into the stem is usually limited by considerations which of themselves will hold the compressive stresses within proper limits, and since the moment arm of the horizontal couple will not change much with an increase in width of flange. The web stresses, which here are very important, will differ from those found for rectangular beams, and for T-beams the actual width of stem must be used in the calculations for vertical shear and diagonal tension.

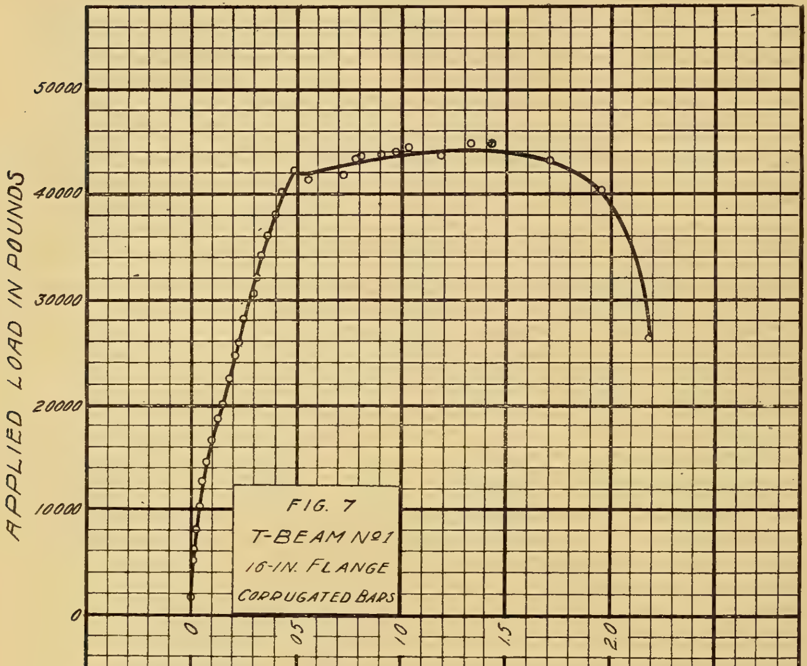

FIG. 7
T-BEAM No. 1
16-IN. FLANGE
CORRUGATED BARS

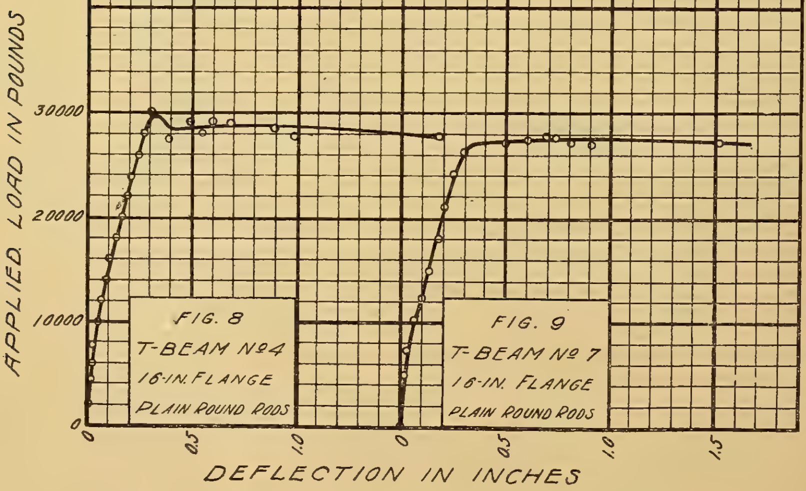

FIG. 8
T-BEAM No. 4
16-IN. FLANGE
PLAIN ROUND RODS

FIG. 9
T-BEAM No. 7
16-IN. FLANGE
PLAIN ROUND RODS

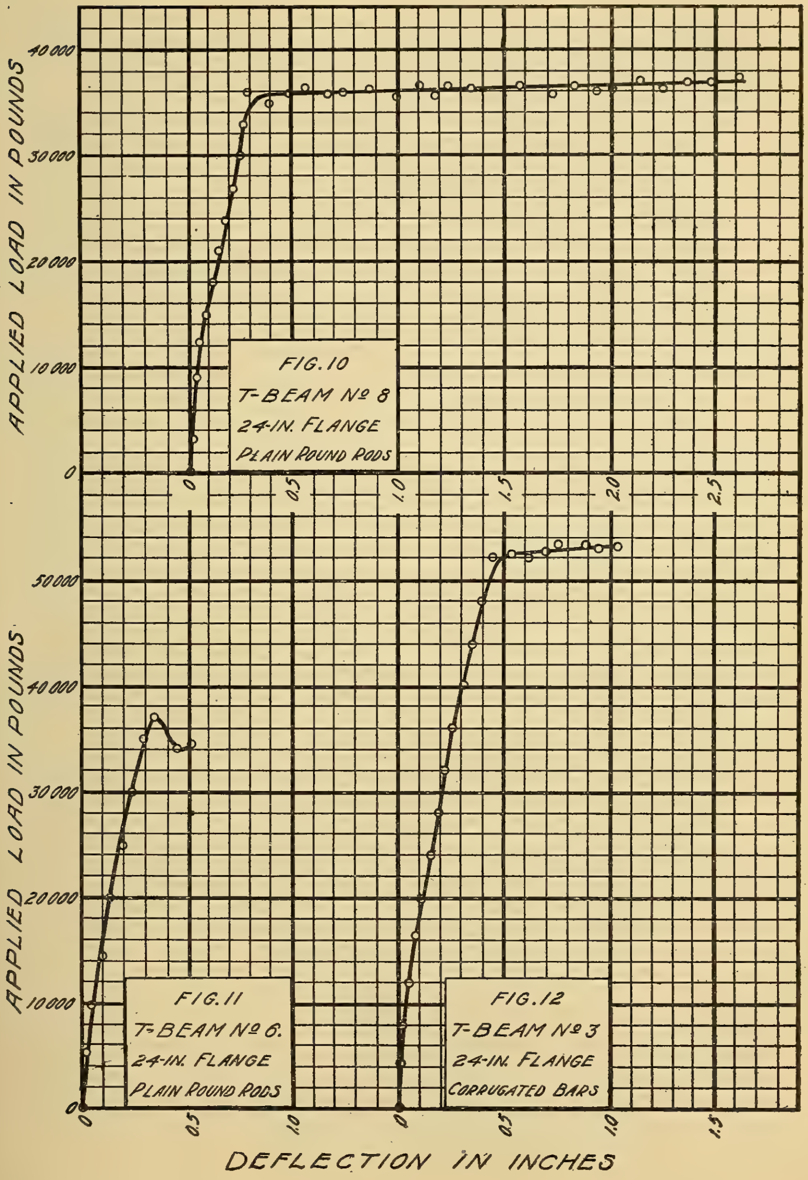

FIG. 10
T-BEAM No 8
24-IN. FLANGE
PLAIN ROUND RODS

FIG. 11
T-BEAM No 6.
24-IN. FLANGE
PLAIN ROUND RODS

FIG. 12
T-BEAM No 3
24-IN. FLANGE
CORRUGATED BARS

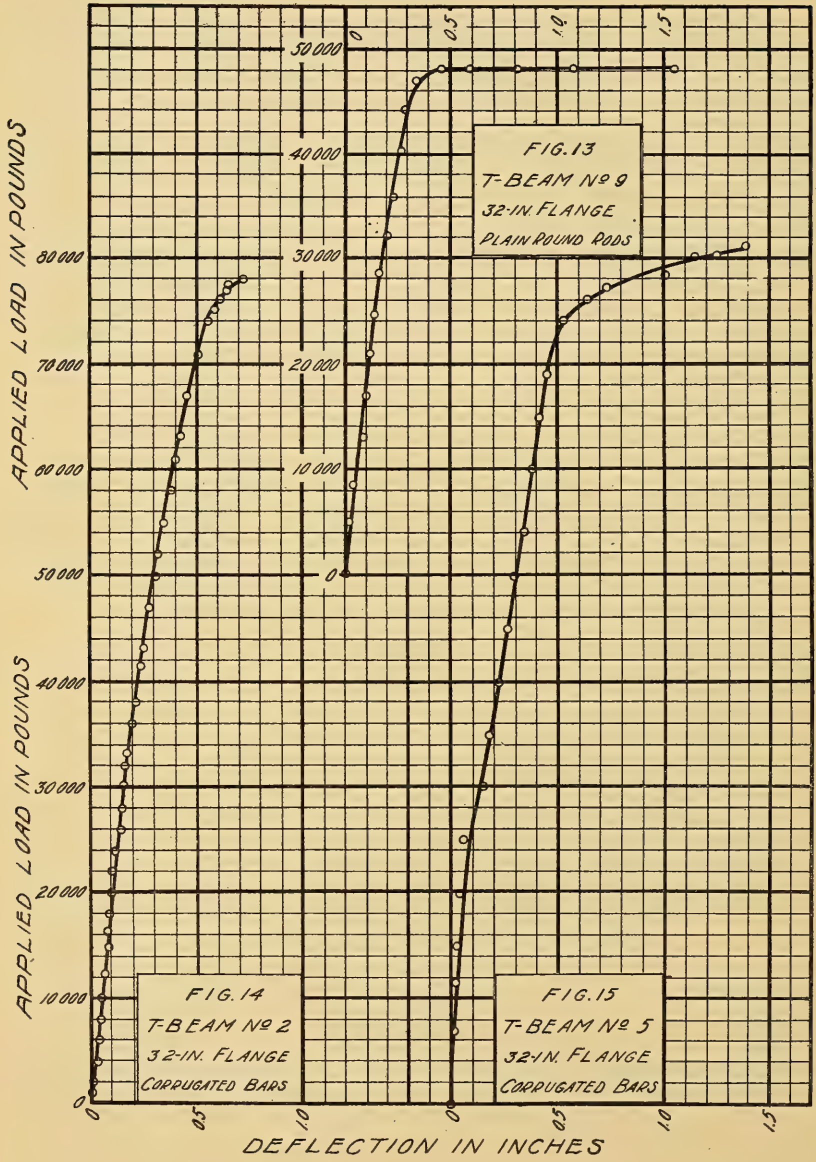

FIG. 13
T-BEAM No 9
32-IN. FLANGE
PLAIN ROUND RODS

FIG. 14
T-BEAM No 2
32-IN. FLANGE
CORRUGATED BARS

FIG. 15
T-BEAM No 5
32-IN. FLANGE
CORRUGATED BARS

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BULLETIN NO. 13

AN EXTENSION OF THE DEWEY DECIMAL SYSTEM OF CLASSIFICATION APPLIED TO ARCHITECTURE AND BUILDING

By N. CLIFFORD RICKER

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 13 NOVEMBER, 1906

AN EXTENSION OF THE DEWEY DECIMAL SYSTEM OF CLASSIFICATION APPLIED TO ARCHITECTURE AND BUILDING

by

N. CLIFFORD RICKER, D. ARCH., PROFESSOR OF ARCHITECTURE

INTRODUCTION

The Dewey Decimal Classification was invented and introduced by Dr. Melvil Dewey, also the originator of the schools for the training of librarians and their assistants. Several successive and much extended editions of his book have appeared, and the system has been found so convenient for practical uses that it is now more commonly employed in America and Europe than are any competing systems of arrangement.

All science, arts, literature, and tangible objects to be classified are distributed into ten classes or centuries:

0 to 100	General Works
100 to 200	Philosophy
200 to 300	Religion
300 to 400	Sociology
400 to 500	Philology
500 to 600	Natural Science
600 to 700	Useful Arts
700 to 800	Fine Arts
800 to 900	Literature
900 to 1000	History

Each one of these primary classes or centuries is next divided into ten secondary divisions or decades, like that for the Fine Arts:

700	Fine Arts
710	Landscape Architecture
720	Architecture
730	Sculpture
740	Drawing
750	Painting
760	Engraving
770	Photography
780	Music
790	Amusements

Each secondary class or decade is further divided into ten subdivisions or units, for example, like Architecture:

720	Architecture
721	Construction
722	Ancient
723	Mediæval
724	Modern
725	Public
726	Religious
727	Educational
728	Residence
729	Design

In this manner are obtained 1000 unit divisions, which form a system usually found sufficient for a small library, a collection of general memoranda, or a series of notes and clippings of moderate extent. But a specialist requires a much more extended subdivision of perhaps only a few of these units comprising the topics in which he is chiefly interested. This introduces a very useful property of the decimal system, namely, that it may be almost infinitely extended in any part by the subdivision of certain units without affecting others or requiring any rearrangement of the general system.

First subdivide into tenths of the unit, like Modern Architecture, for example:

724.	Modern Architecture
.1	Renaissance

.2	Grecian revival
.3	Gothic revival
.4	Tudor revival
.5	Queen Anne
.6	Neo Grec
.7	Swiss revival
.8	Romanesque revival
.9	Other modern styles

If one is particularly interested in the comparative study of the various types of the Renaissance style found in different countries, it will be convenient to subdivide 724.1 into hundredths, for example:

724.1	Renaissance
.11	Scotland
.12	England
.13	Germany
.14	France
.15	Italy
.16	Spain
.17	Russia
.18	Scandinavia
.19	Minor countries

A collection of notes and memoranda or photographs of Italian Renaissance buildings may then be so large, that it may profitably be subdivided into thousandths of the unit, as follows:

724.15	Renaissance in Italy
.151	Cinquecento
.152	High Renaissance
.153	Decadence
.154	Barocco
.155	Rococo

This would evidently arrange the materials most conveniently for a thorough study of the Italian Renaissance style in the order of its historical development. Therefore it is sufficiently evident that the decimal system of classification is capable of unlimited extension in any of its parts.

To apply the system to a collection of books, memoranda, or other materials, each particular item is simply numbered as indicated by the decimal classification. The collection is then arranged in numerical sequence, which places the objects classified in the same order. Persons not accustomed to the use of the decimal system of classification may at first find some difficulty in finding the proper number for a particular object or article. Therefore an alphabetically arranged index of the more important topics and their numbers has been added for their convenience.

It is believed that this extension of the Dewey Decimal Classification will be found useful and convenient by architects, builders, engineers and all other persons practically or theoretically concerned with Architecture and Building. Most of it has been in use for many years in the department of Architecture of this University for classifying extensive collections of lantern slides, photographs, a card index to architectural periodicals, and other materials for instruction. As meriting careful attention, the following points are suggested:

1. The suggested classification of all materials relating to each one of the architectural styles together, in order to bring them into the most compact and convenient arrangement for the student of the History of Architecture.

2. The careful separation of 690, Building Materials and Trades; 721, Architectural Construction; and 729, Architectural Forms and Design, making it now easy to assign any topic to its proper place.

TABLE
OF
CLASS NUMBERS

---

690 Building Materials Trades

See 721 for Architectural Construction.
See 729 for Architectural Forms or Design.

.1	**Theories of construction**
.11	Systems of construction
.2	**Compends**
.21	Manuals
.22	Handbooks
.23	Recipes, collections of
.3	**Dictionaries**
.31	Cyclopedias
.4	**Essays**
.41	Lectures
.42	Discussions
.5	**Periodicals**
.51	Daily
.52	Weekly
.53	Monthly
.54	Quarterly
.55	Annual
.6	**Societies; Proceedings**
.61	Trade unions
.62	Exhibitions
.621	Materials
.622	Methods
.623	Construction

690.7	**Education and study**
.71	Training of workmen
.72	Apprenticeship
.73	Tools and their uses
	(See special trade for special tools.)
.74	Shop practice
.75	Trade schools
.76	Manual training
.8	**Museums**
.81	Collections
.82	Patents
.83	Inventions
.84	Machines for manufacturing
.841	Wood
.842	Stone
.843	Steel and iron
.844	Bricks
.845	Tiles
.846	Cement and lime
.847	Concrete
.848	Asphalt
.849	
.9	**History of Building Materials**
.91	Ancient
.92	Mediæval
.93	Renaissance
.94	Modern
.95	History of building construction
.96	Ancient
.97	Mediæval
.98	Renaissance
.99	Modern
691	**Materials Processes Preservatives**
	See 620.1 for Strength of Materials. See 693 to 699 for Uses of Prepared Materials.
.1	**Woods**
.11	Hard conifers
.111	Pine, longleaf
.112	Pine, hard

691.113	Pine, yellow
.114	Pine, Norway
.115	Pine, pitch
.116	Tamarack; hackmatack
.117	
.118	Yew
.119	
.12	Soft conifers
.121	Pine, white
.122	Pine, shortleaf
.123	Cedar, white
.124	Cedar, red
.125	Hemlock
.126	Spruce
.127	Fir
.128	Cypress
.129	
.13	Hard leaf woods
.131	Oak
.132	Beech
.133	Sycamore
.134	Maple, sugar
.135	Ash
.136	Hickory
.137	Walnut, black
.138	Locust
.139	
.14	Soft leaf woods
.141	Poplar
.142	Gum
.143	Birch
.144	Maple, white, red
.145	Basswood; linden
.146	Elm
.147	Catalpa
.148	Butternut
.149	
.15	Defects of woods
.151	Sapwood
.152	Shakes; cracks
.153	Spots; streaks
.154	Knots
.155	Decay
.156	Stains
.157	Pitch
.158	Shrinkage
.159	

691.16	Injuries to woods
.161	Wet rot
.162	Dry rot
.163	Ants
.164	Borers
.165	Fungus
.169	
.17	Preservation of woods
.171	Painting
.172	Oiling
.173	Creosoting
.174	Zincking
.175	By corrosive sublimate
.176	By crude kerosene
.177	By fireproof paint
.178	By sulphate copper
.179	
.2	**Stone Material Protection**
.211	Limestone
.212	Marble
.213	Gypsum
.214	Bedford; Oolite
.221	Granite
.222	Syenite
.223	Porphyry
.231	Sandstone, ordinary
.232	Sandstone, brown
.233	Sandstone, portage
.24	Slate
.251	Serpentine
.252	Soapstone
.253	Talc
.261	Trap
.262	Basalt
.271	Tufa
.272	Travertine
.273	Peperino
.28	Other building stones
.29	Preservation
.291	By painting
.292	By oiling
.293	By paraffine
.294	By silicate of soda
.295	By glue and tannin
.296	By cement wash
.299	

691.3	**Stone, artificial Concrete**
.31	Beton coignet
.32	Ransome
.33	Hollow block
.34	Selenitic
.35	Lime concrete
.36	Cement concrete
.39	Aggregate
.4	**Bricks Tiles Ceramic products**
.41	Bricks, ordinary
.42	Bricks, pressed
.43	Bricks, moulded
.44	Bricks, glazed or enameled
.45	Bricks, self-colored
.46	Tiles
.461	Roofing
.1	Spanish
.2	Ludovici
.3	Celadon
.4	Ornamented
.9	
.462	Flooring
.1	Self-colored
.2	Inlaid
.3	Embossed
.4	Glazed
.9	
.463	Wall tiles
.1	Self-colored
.2	Inlaid
.3	Embossed
.4	Glazed
.5	Picture
.6	Paience panels
.7	Mouldings
.8	Painted
.9	
.47	Hollow structural tiles
.471	Floor
.1	Side arch
.2	End arch
.3	Mixed arch
.9	

691.472	Roof and ceiling tiles
.1	Book tiles
.2	Ceiling plates
.9	
.473	Wall tiles
.1	Wall linings
.2	Bond courses
.3	Partition
.48	Terra cotta
.481	Manufacture
.482	Face blocks
.483	Bands and mouldings
.484	Columns and pilasters
.485	Cornices
.486	Ornamental
.49	Sewer tiles
.491	Salt-glazed
.492	Drain tiles
.493	Fittings
.499	
.5	**Lime Cement Plaster**
.51	Lime, ordinary
.52	Lime, hydraulic
.53	Lime, selenitic
.54	Cement, natural
.55	Cement, portland
.56	Plaster of paris
.57	Keene's cement
.58	Hard plaster
.59	
.6	**Glass**
.61	Window
.62	Plate, rough Rolled
.63	Plate, polished
.64	Luxfer prism, etc.
.65	Sidewalk lights
.66	Moulded
.67	Cathedral
.68	Stained
.69	

691.7	**Iron Steel Anti-rust processes**
.71	Cast iron
.72	Malleable cast iron
.73	Wrought iron
.74	Steel, blister or tool
.75	Steel, crucible
.76	Steel, bessemer
.77	Steel, open hearth
.79	Protection of iron and steel
.791	Painting
.792	Tinning
.793	Zincking (galvanizing)
.794	Electroplating
.795	Bower-Barff process
.796	Cement coating
.797	
.798	
.799	
.8	**Other metals**
.81	Copper
.82	Brass
.83	Zinc
.84	Lead
.85	Aluminum
.86	Bronze
.87	Tin
.88	
.89	
.9	**Other materials**
.91	Mineral wool
.921	Hair
.922	Jute fiber
.923	Hemp fiber
.93	Paper
.931	Sheathing
.932	Cabot's quilt
.933	Slating
.934	Roofing
.94	Fabrics woven
.941	Duck

691.942	Burlap
.943	Carpets
.944	Rugs
.945	Linings
.95	Asbestos
.951	Pipe coverings
.952	Hot air insulation
.953	Fire protection
.961	Asphalt, Trinidad
.962	Asphalt, rock
.963	Pitch
.964	Resin
.965	Coal tar
.966	Tar
.967	Damp-proof felt
.969	
.97	Felt, roofing
.98	Linoleum
.99	

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.11	Plan, location
.12	Plan, foundation
.13	Plans, floor
.14	Plan, roof
.15	Elevations
.151	Front
.152	Side
.153	Rear
.154	Court
.159	
.16	Sections
.161	Longitudinal
.162	Cross
.169	
.19	Other general drawings
.2	**Detail drawings**
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.211	Stone-cutting
.212	Stone-setting
.213	Marble

692.214	Tiles
.215	Slate
.216	Brickwork
.217	Bricks, ornamental
.218	Terra cotta
.219	
.22	Woodwork
.221	Carpentry
.222	Joinery
.223	Cabinet work
.224	Furniture, special
.229	
.23	Metal work
.231	Cast iron, structural
.232	Cast iron, ornamental
.233	Steel, structural
.234	Steel, ornamental
.235	Copper
.236	Tin
.237	Zinc
.238	Brass
.239	
.24	Plumbing
.241	Water supply
.242	Sanitary fixtures
.243	Drainage
.249	
.25	Heating and ventilating
.251	Furnaces
.252	Steam
.253	Hot water
.254	Gas
.255	Electric
.256	Combination systems
.257	
.258	Ventilation, natural
.259	Ventilation, forced
.26	Plastering
.261	Plain
.262	Ornamental
.263	Stucco work
.269	
.27	Roofing
.271	Gutters
.272	Leaders

692.28	Glass work
.281	Stained
.282	Beveled
.289	
.29	Other drawings
.3	**Specifications**
.31	General conditions
.32	Excavation and grading
.33	Masonry
.331	Stonework
.332	Brickwork
.333	Terra cotta
.334	Concrete, plain
.335	Concrete, reinforced
.34	Carpentry
.341	Carpentry proper
.342	Joinery
.343	Cabinet work
.35	Metal work
.351	Cast iron
.352	Wrought iron
.353	Steel
.354	Copper
.355	Tin
.356	Ornamental
.357	Bronze
.359	
.36	Plastering
.37	Painting and glazing
.371	Painting
.372	Glazing
.373	Finishing
.38	Plumbing
.39	Other specifications
.391	Lighting
.392	Heating
.393	Ventilation
.399	
.4	**Contracts Bids Advertisements**
.41	Advertisements
.42	Bids
.43	Agreements, informal

692.44	Contracts, signed
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.52	Volume
.53	Area
.54	Units
.55	Quantities, priced
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.57	Classification of data
.58	
.59	
.6	**Superintendence**
.61	Occasional
.62	Architect's stated
.63	Constant
.64	Clerk of works
.65	Special
.69	
.7	**Supervision of accounts**
.71	Bookkeeping, system of
.72	Approval of bills
.73	Oversight of payments
.74	Statements to owner
.75	Reserved payments
.76	Extras and omissions
.77	Settlement, final
.79	
.8	**Professional services Fees Commissions**
.81	Services, ordinary
.82	Services, extra
.83	Services, special

692.84	Rates of commission
.85	Account of payments
.86	Expenses, extra
.87	Bye commissions
.89	
.9	**Building laws Liabilities of architects, owners, and contractors**
.91	General or state laws
.92	City ordinances
.93	Town or village ordinances
.94	Other laws
.95	Liabilities of architects
.96	Liabilities of owners
.97	Liabilities of contractors
.98	Lien laws
.99	
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.1	**Stone construction**
	See 515.8 for Stereotomy See 736 for Stone Carving
.11	Bonds
.12	Stonecutting
.13	Stonesetting
.14	Stone tools
.19	
.2	**Brick construction**
.21	Bonds
.22	Adobes
.23	Cut or rubbed
.24	Ornamental
.25	Polychrome
.26	Tools
.29	
.3	**Terra cotta construction**
.31	Setting
.32	Anchoring
.33	Backing

693.4	**Hollow tile and porous terra cotta**
.41	Walls
.42	Floors
.43	Roofs
.44	Partitions
.45	Linings
.49	
.5	**Concrete and beton construction** See 693.3 for Reinforced Concrete
.51	Massive
.52	Layers
.53	Hollow block
.54	Sidewalks
.55	Ornamental
.56	Surface finish
.59	
.6	**Plastering**
.61	External
.62	Internal
.63	Ornamental
.64	Scagliola
.69	
.7	**Reinforced concrete**
.71	Systems of construction
.711	Ordinary round rods
.712	Ransome
.713	Unit
.714	Hennebique
.715	Roebling
.716	Thacher
.719	
.72	Forms and centers
.73	Testing and inspection
.74	Data from experiments
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.81	Systems
.811	Hollow tiles
.812	Porous terra cotta
.813	Brickwork
.814	Concrete
.815	Wood and sheet metal
.816	Mortar
.819	
.82	Walls and partitions
.83	Floors
.84	Roofs
.85	Columns
.86	Girders
.87	Trusses
.88	Vaults
.89	
.9	
694	Carpentry Joinery Cabinet-Work Stairbuilding
.1	Wood construction in general
.2	Framing and joints
.3	Strengthened beams
.4	Posts and columns
.5	Paneled and lattice construction
.6	Joinery in general
.7	Cabinet work
.8	Stairbuilding
.9	
695	Roofing
.1	Shingle
.2	Slate
.3	Tile

695.4	**Metal**
.41	Tin
.42	Copper
.43	Lead
.44	Zinc
.5	**Iron or steel**
.6	**Asphalt**
.7	**Felt and gravel**
.71	Asbestos
.72	Paper
.8	**Fabrics**
.81	Duck
.82	Canvas
.9	
696	**Plumbing Gas and Steam Fitting**
.1	**Plumbing**
.11	Piping
.12	Fixtures
.13	Tools
.19	
.2	**Gas Fitting**
.21	Piping
.22	Fixtures
.23	Tools
.29	
.3	**Steam Fitting**
.31	Piping
.32	Fittings
.33	Tools
.4	**Rivets and riveted joints**
.41	Tools
.5	**Screws and screw joints**
.51	Tools

696.6	**Rust or calked joints**
.61	Rust joints
.62	Calked joints
7.	**Anchors Bond irons**
.71	Bond irons
.72	Anchors, wall
.73	Anchors, angle
.74	Anchors, through
.75	Anchors for gutters and leaders
.76	Anchors for masonry
.79	
.8	**Other branches**
.9	**Plumbing laws and ordinances**
.91	General or state
.92	City ordinances
.93	Town or village ordinances
697	**Heating and Ventilation**
.1	**Fireplaces**
.11	Ordinary
.12	Ventilating
.2	**Stoves**
.3	**Furnaces, hot air**
.31	Furnaces
.32	Pipes, cold air
.33	Pipes, hot air
.34	Registers
.35	Regulators
.4	**Hot water**
.41	High pressure
.42	Low pressure
.43	Conservatory
.44	Heaters with steam supply
.45	Boilers
.46	Pipes
.47	Regulators

697.5	**Steam**
.51	High pressure
.52	Low pressure
.53	Exhaust
.54	Vacuum
.55	Boilers
.56	Pipes
.57	Regulators
.58	Air valves
.59	
.6	**Gas**
.61	Illuminating
.62	Fuel
.63	Natural
.64	Acetylene
.7	**Electric and other**
.8	**Smoke flues and chimneys**
.81	Built in walls
.82	Isolated
.9	**Ventilation Ducts Fans**
.91	Natural
.92	Plenum
.93	Exhaust
.94	Fresh air ducts
.95	Foul air ducts
.96	Treatment of air
.961	Filtration
.962	Dampening
.963	Cooling
.964	Testing
.97	Regulation of air supply
.98	Fans
.981	Side fans
.982	Central fans
.99	

698	**Painting Glazing Finishing Paperhanging**
.1	**Painting Oil**
.11	Lead
.12	Zinc
.13	Iron
.14	Mixed paints
.15	Oil
.16	Turpentine
.17	Dryers
.18	Tools
.19	
.2	**Distemper and Fresco**
.21	Distemper
.22	Milk wash
.23	Cement wash
.24	Other washes
.25	Fresco, real
.26	Fresco, other methods
.27	Stencils
.28	Tools
.29	
.3	**Varnishing Polishing**
.31	Varnish
.311	Oil
.312	Spirit
.319	
.32	Shellac
.33	French polish
.34	Wax
.35	Oiling
.36	Other finishes
.37	Polishing
.38	Tools
.39	

698.4	**Other modes of protection**	
.41	Asphalt	
.42	Tar	
.43	Graphite	
.49		
.5	**Glazing**	
.51	Puttying, ordinary	
.52	Puttying, hard	
.53	Cutting	
.54	Setting	
.55	Leading	
.56	Tools	
.59		
.6	**Paperhanging**	
.61	Paper	
.62	Paste	
.63	Hanging	
.64	Tools	
.69		
.7	**Textile Hangings**	**Tapestry**
.71	Burlap	
.72	Chintz	
.73	Tapestry, real	
.74	Tapestry, painted	
.75	Tools	
.79		
.8	**Relief work**	
.81	Embossed paper	
.82	Lincrusta	
.83	Stamped leather	
.84	Tools	
.89		
.9	**Other branches**	
699	**Car and Shipbuilding**	

700 Fine Arts

701	Theories	Esthetics	
702	Compends	Manuals	Outlines
703	Dictionaries	Cyclopedias	
704	Essays	Lectures	Discussions
705	Periodicals		
.1	Daily		
.2	Weekly		
.3	Monthly		
.4	Quarterly		
.5	Annual		
.6	Occasional		
.9			
706.1	Societies		
.11	Painters		
.12	Sculptors		
.13	Illustrators		
.14	Transactions		
.15	Proceedings		
.16	Reports		
.19			
707.1	Education		
.11	Study		
.12	Instruction		
.13	Practice		
708	Art Galleries and Museums		
.11	Scotland		
.15	Ireland		
.2	England		
.31	Germany		
.36	Austria		

708.4	**France**
.5	**Italy**
.61	Spain
.69	Portugal
.7	**N. America**
.71	Canada
.72	Mexico
.73	United States
.8	**S. America**
.9	**Australia**
.992	Holland
.993	Belgium
.994	Switzerland
709	**History of Art in General**
.3	**Ancient**
.311	China
.312	Japan
.313	Korea
.32	Egypt
.331	Phœnicia
.332	Philistine
.333	Judea
.334	Carthage
.335	Cyprus
.336.1	Pelasgian
.2	Etruria
.34	India
.351	Chaldea
.352	Assyria
.355	Persia
.357	Sassania
.36	Celtic
.37	Roman
.38	Greece
.39	Minor countries
.4	**Modern Europe**
.411	Scotland
.415	Ireland
.42	England

709.43	Germany
.431	Northern
.434	Southern
.436	Austria
.44	France
.442	Normandy
.445	Auvergne
.446	Poitou
.449	Provence
.45	Italy
.452	Lombardy
.453	Venice
.455	Tuscany
.457	Naples
.458	Sicily
.461	Spain
.469	Portugal
.47	Russia
.48	Scandinavia
.481	Norway
.485	Sweden
.489	Denmark
.49	Minor countries
.492	Holland
.493	Belgium
.494	Switzerland
.495	Modern Greece
.496	Turkey in Europe
.499	
.5	**Asia**
.51	China
.52	Japan
.54	India
.55	Persia
.59	Farther India
.6	**Africa**
.61	Northern
.62	Egypt
.63	Abyssinia
.64	Morocco
.65	Algeria
.69	

709.7	**North America**
.71	Canada
.72	Mexico
.728	Central America
.729	West Indies
.73	United States
.8	**South America**
.81	Brazil
.82	Argentina
.83	Chili
.84	Bolivia
.85	Peru
.87	Venezuela
.89	
.9	**Oceanica**

710 Landscape Architecture

711	Parks
712	Private Grounds
713.1	Walks
.2	Roads
.3	Drives
714.1	Lakes
.2	Streams
.3	Fountains
715.1	Trees
.2	Shrubs
.3	Hedges
.4	Vines
716.1	Plants
.2	Flowers
.3	Conservatories

716.4	Window gardens
.5	Ferneries
717.1	Arbors
.2	Summer Houses
.3	Seats
.4	Outlooks
.5	Pergolas
.6	Garden walls
.7	Niches
.8	Statues Vases
.9	
718.1	Monuments
.2	Tombs
.3	Mausoleums
719.	Cemeteries
.1	Gates
.2	Walls
.3	Walks
.4	Drives
.5	Vaults

720 Architecture

.11	Theories, general
.12	Esthetics
.13	Architectonics
.21	Compends
.22	Manuals
.23	Handbooks
.31	Dictionaries
.32	Encyclopedias

720.33	Glossaries
.34	Dictionaries of foreign terms
.41	Essays, general
.42	Lectures
.43	Articles
.44	Discussions
.5	**Periodicals**
.51	Daily
.52	Weekly
.53	Monthly
.54	Quarterly
.55	Annual
.56	Occasional
.6	**Societies**
.61	School
.62	Architects' local
.63	Draftsmen's
.64	National
.65	State
.71	Education
.72	Study
.73	Training, professional
.74	Schools, architectural
.741	Scotland
.742	England
.743	Germany
.5	Austria
.744	France
.745	Italy
.746	Spain Portugal
.747	Russia
.1	Canada
.2	Mexico
.3	United States
.8	South America
.748.1	Norway
.5	Sweden
.9	Denmark
.749	

720.8	**Collections**
.81	Materials
.82	Fixtures
.83	Fittings
.84	Drawings
.85	Models
.86	Photographs
.89	
.9	**History of architecture, general**
	Classify under 722, 723 or 724, if possible.
.94	Europe
.941	Scotland
.5	Ireland
.942	England
.943	Germany
.1	Prussia
.2	Central Germany
.3	Bavaria
.4	South Germany
.5	Northwest Germany
.6	Austria
.7	Bohemia
.8	Poland
.9	Hungary
.944	France
.1	Brittany, etc.
.2	Normandy
.3	Isle de France
.4	Burgundy
.5	Auvergne
.6	Poitou
.7	Gascony
.8	Languedoc
.9	Provence
.945	Italy
.1	Piedmont
.2	Lombardy
.3	Venice
.5	Tuscany
.7	Naples
.8	Sicily
.9	Sardinia
.946	Spain
.9	Portugal

720.947	Russia
.1	Norway
.5	Sweden
.9	Denmark
.949	Minor countries
.2	Holland
.3	Belgium
.4	Switzerland
.5	Modern Greece
.7	Servia
.8	Roumania
.9	
.95	Asia
.96	Africa
.962	Egypt
.97	North America
.971	Canada
.972	Mexico
.973	United States
.98	South America
.981	Brazil
.982	Argentina
.983	Chili
.984	Bolivia
.985	Peru
.986	Equador
.987	Venezuela
.989	Paraguay
.99	Oceanica
.993	New Zealand
.994	Australia
.999	

721 Architectural Construction

See 729 for Forms and Design.
See 690, etc., for Materials, Trades, etc.

.1	**Foundations**
.11	Dimension stone
.12	Rubble
.13	Concrete
.131	Plain
.132	Reinforced by bars
.133	Reinforced by rails
.134	Reinforced by beams
.14	Sand or gravel

721.15	Gratings, wooden
.16	Piles
.161	Wood
.162	Sand
.163	Steel
.164	Concrete, plain
.165	Concrete, reinforced
.17	Wells, sunken
.18	Tubes, sunken
.181	Caissons, wood
.182	Caissons, steel
.19	
.191	Piers, stone
.192	Piers, brick
.193	Piers, concrete
.194	Piers, concrete-steel
.195	Piers, steel-framed
.2	**Walls**
.211	Wood
.212	Half-timber
.22	Stone
.231	Concrete
.232	Hollow block
.233	Concrete-steel
.234	Pise (Tamped earth)
.241	Brick, solid
.242	Brick, hollow
.25	Bases, belts, etc.
.251	Base Plinths
.252	Water tables
.253	Belt courses
.254	String courses
.261	Colonnades
.1	Stone
.2	Brick
.3	Terra cotta
.4	Steel
.5	Iron
.6	Concrete
.7	Concrete-steel
.8	Wood
.262	Arcades
.1	Stone
.2	Brick

721.262.3	Terra cotta
.4	Steel
.5	Iron
.6	Concrete
.7	Concrete-steel
.8	Wood
.9	
.263.1	Arcade, blind
.2	Arcade band
.3	Arched frieze
.27	Cornices
.271	Stone
.272	Brick
.273	Terra cotta
.274	Steel
.275	Iron
.276	Concrete
.277	Concrete-steel
.278	Wood
.279	
.28	Pediments
.281	Stone
.282	Brick
.283	Terra cotta
.284	Steel
.285	Iron
.286	Concrete
.287	Concrete-steel
.288	Wood
.289	
.29	Gables
.291	Stone
.292	Brick
.293	Terra Cotta
.294	Steel
.295	Iron
.296	Concrete
.297	Concrete-steel
.298	Wood
.299	
.3	**Piers Columns**
.31	Wood
.311	Exposed
.312	Protected
.32	Stone

721.33	Concrete
.331	Solid
.332	Block
.333	Concrete-steel
.34	Brick
.341	Brick, cut
.342	Brick, moulded
.343	Terra cotta
.35	Metal
.351	Cast iron
.352	Wrought iron
.353	Steel
.1	Rolled Solid
.2	Riveted
.3	Latticed
.36	Fireproof Protected
.361	By mortar
.362	By hollow tiles
.363	By porous terra cotta
.364	By brickwork
.365	By concrete
.39	

.4 Arched construction

See 624.5 for Bridges.
See 729.3 for the different forms.
See 690 for Materials.

.41	Stone
.411	Dressed
.412	Rubble
.42	Brick
.43	Terra cotta
.44	Steel
.45	Cast iron
.46	Concrete
.461	Massive
.462	Hollow block
.463	Solid block
.464	Concrete-steel
.47	Wood
.48	Plaster Staff
.49	

721.5 Roofs

See 695 for Roof Coverings.
See 729.35 for Forms of Roofs.

.51 Wood
.511 Sheathing
.512 Rafters
.513 Purlins
.514 Ceiling
.515 Truss Wood
.1 Types
.2 Loads
.3 Stress diagrams
.4 Dimensioning
.5 Connections
.6 Weight
.7 Cost
.8 Economy
.9

.52 Masonry

.53 Glass
.531 Glass
.532 Rafters
.533 Purlins
.534 Ceiling

See 721.515 or 721 545 for Truss.

.54 Metal
.541 Sheathing
.1 Wood
.2 Corrugated steel
.3 Lining for drip
.542 Rafters
.543 Purlins
.544 Ceiling
.545 Truss, steel or iron
.1 Types
.2 Loads
.3 Stress diagrams
.4 Dimensioning
.5 Connections
.6 Weight
.7 Cost
.8 Economy
.9

.55 Windows Openings in roofs
.551 Luthern

721.552	Dormer
.553	Skylights
.554	Scuttles
.555	Gables
.56	Roof crestings, etc.
.561	Crestings
.562	Balustrades
.563	Reliefs
.564	Statuary
.565	Cornice
.566	Bands
.57	Spires
.571	Stone
.572	Brick
.573	Terra cotta
.574	Steel
.575	Iron
.576	Concrete
.577	Concrete-steel
.578	Wood
.579	
.58	Gablets
.59	
.6	**Floors**
.61	Wood
.611	Flooring
.612	Deafening
.613	Sheathing
.614	Joists
.615	Beams
.616	Girders
.619	
.62	Stone
.621	Slabs
.622	Slabs and beams
.623	Paneled
.624	Coffered
.625	Marble
.63	Brick Tiles
.631	Brick arched
.632	Hollow tile, arched
.633	Hollow tile, flat
.634	Guastevino, arched
.64	Metal
.641	Cast Iron

721.642	Steel
.649	
.65	**Composite**
.651	Stone and cast-iron beams
.652	Stone and steel beams
.653	Hollow tile and steel beams
.654	Porous tile and steel beams
.655	Concrete and steel beams
.656	Concrete-steel
.657	Concrete and Roebling wire Arches
.658	Concrete and expanded metal
.659	
.66	**Parquetry floors**
.67	**Mosaic**
.671	Marble, plain
.672	Marble, patterns
.673	Marble, geometrical
.674	Marble, ornamental
.675	Marble and cement
.676	Inlaid concrete
.679	
.68	**Tiles**
.681	Inlaid
.682	Self-colored
.683	Glazed
.684	Embossed
.685	Tesserae
.69	
.691	Cement
.692	Artificial stone
.693	Xylolith
.694	Linoleum
.699	
.7	**Ceilings**
	See 721.4 for Construction of Vaulted Ceilings See 729.6 for Forms of Ceilings.
.71	**Wood**
.711	Matched and beaded
.712	Panels planted on
.713	Beam
.714	Mill-floor system
.715	Paneled
.716	Coffered
.719	

721.72	Stone
.721	Flat
.722	Beam
.723	Paneled
.724	Coffered
.73	Brick or tile and steel
.731	Flat, plain
.732	Flat, enameled
.733	Flat, ornamented
.734	Paneled
.735	Coffered
.74	Metal
.741	Cast iron with steel beams
.742	Steel trough plates and beams
.743	Steel buckled plates and beams
.744	Steel panels and beams
.745	Steel sheets on wood sheathing
.746	Steel stamped on wood ceiling
.75	Concrete Plaster
.751	Concrete, plain
.752	Concrete-steel
.753	Concrete-steel beams
.754	Concrete-steel paneled
.755	Plaster, plain
.756	Plaster, ornamental
.757	Plaster, paneled
.758	Plaster, painted
.759	
.79	Other ceilings
.8	**Doors Enclosures Windows**
.81	Doors, wood
.811	Single
.812	Double
.813	Sliding
.814	Folding
.815	Concealed or secret
.816	Glazed
.82	Doors, metal
.821	Single
.822	Double
.823	Sliding
.824	Concealed
.825	Fire-proof
.826	Sheet metal on wood

721.827	Wire-glazed
.828	Vault
.829	
.84	Windows, external
.841	Sliding
.842	Casement
.843	Fixed
.844	Wire-glazed
.845	Fireproof
.846	Leaded
.847	Transoms
.85	Windows, internal
.851	Sliding
.852	Hinged
.853	Fixed
.854	Wire-glazed
.855	Fireproof
.856	Leaded
.857	Transoms
.86	Enclosures of doors and windows
.861	Architrave
.862	Cap, horizontal
.863	Pediment on consoles
.864	Pediment on pilasters
.865	Pediment on engaged columns
.866	Pediment on free columns
.867	Lintel only
.868	Sill and stool
.869	
.87	Shutters Blinds Screens Grilles
.871	Shutters, wood
.872	Shutters, steel
.873	Blinds, ordinary
.874	Blinds, Venetian
.875	Screens, fly
.876	Grilles, plain
.877	Grilles, ornamental
.88	Fastenings Locks
.881	Shutter fastenings
.882	Blind fastenings
.883	Door locks
.1	Rebated
.2	Mortise
.3	Rim
.4	Dead
.5	Latch

721.883.6	Sliding door
.7	Bolts
.8	Espagnolette bolts
.9	
.89	Other fixtures
.891	Hinges, blind
.892	Hinges, door
.1	Fast
.2	Loose joint
.3	Pin
.4	Spring
.5	Self-closing
.893	Door closers
.894	Sash fasts
.895	Blind fasts
.899	
.9	**Iron and composite structures** See 620.1 for Strength of Materials. Classify here only that which cannot be placed elsewhere, under 721 etc.
.91	Cast-iron structures
.92	Wrought-iron structures
.93	Steel structures
.94	Composite structures
.95	Steel and wood
.96	Steel and stone
.97	Steel and ceramic
.971	Steel and brick
.972	Steel and tile
.973	Steel and terra cotta
.98	Steel and glass
.99	Wood and glass

722, 723, 724 History of Architecture

Classify modern American buildings of importance in the History of Architecture under 724; generally all other American buildings under 725 to 728 inclusive.

Modern foreign buildings are usually placed under 724, unless of special importance as examples of the class or purpose, when they are to be treated like American buildings.

Classification of Historical Data

The following subdivision is recommended for the convenient arrangement of all data particularly referring to the History of Architecture. It is readily applicable to each subordinate or general division under 722, 723 and 724, being simply annexed to the style numbers.

.001	General
.0011	Country
.0012	Climate
.0013	History
.0014	Religion
.0015	Government
.0016	Social conditions
.0017	Character of style
.0018	Derivation of style
.0019	Influence of style
.002	Materials
.0021	Wood
.0022	Stone
.0023	Concrete
.0024	Bricks Tiles
.0025	Mortar Cement
.0026	Glass
.0027	Iron Steel
.0028	
.0029	
.003	Construction
.0031	System employed
.0032	Arch Vault Dome
.0033	Foundations
.0034	Floors
.0035	Walls Supports
.0036	Ceilings
.0037	Roofs Spires
.0038	Doors Windows
.0039	
.004	Design
.0041	Facades
.0042	Sections
.0043	Ceilings, treatment of
.0044	Roofs, forms of
.0045	Bases of buildings
.0046	Colonnades

.0047	Arcades
.0048	Cornices Belts Entablatures
.0049	Proportions, system of
.005	Decoration
.0051	Mouldings
.0052	Statues
.0053	Sculptures
.0054	Painting
.0055	Mosaics
.0056	Gilding
.0057	Furniture
.0058	Fabrics
.0059	Pottery
.006	Sanitation
.0061	Water supply
.0062	Drainage
.0063	Sewage disposal
.0064	Plumbing
.0065	Lighting
.0066	Heating
.0067	Ventilation
.0068	Burial
.0069	
.007	Buildings, kinds of
.0071	Religious
.0072	Mortuary
.0073	Memorial
.0074	Military
.0075	Residence
.0076	Public
.0077	Amusement
.0078	Engineering
.0079	
.008	Examples described
.0081	Religious
.0082	Mortuary
.0083	Memorial
.0084	Military
.0085	Residence
.0086	Public
.0087	Amusement
.0088	Engineering
.0089	
.009	Biographies
.0091	Architects
.0092	Sculptors

.0093	Painters
.0094	Art-workers
.0095	Connoisseurs
.0096	Builders
.0097	Engineers
.0098	Rulers
.0099	
722.	**Ancient or primitive architecture**
.0	**Prehistoric**
.011	Scotland
.015	Ireland
.019	Wales
.02	England
.031	Germany
.036	Austria
.04	France
.051	Italy
.052	Sardinia
.053	Sicily
.061	Spain
.069	Portugal
.07	Russia
.071	Canada
.072	Mexico
.073	United States
.1	Indian
.2	Mound-builders
.3	Pueblo
.4	Cave-dwellers
.074	S. American
.075	Peru
.08	Scandinavia
.081	Norway
.085	Sweden
.089	Denmark
.09	
.091	Egypt
.092	Holland
.093	Belgium
.094	Switzerland
.095	Greece
.096	Turkey
.097	Russia
.098	N. Africa
.099	

722.11	China
.12	Japan
.13	Korea
.14	Philippine
.2	**Egypt**
.21	Nubia
.22	Abyssinia
.3	**Phoenician, Jewish, etc.**
.31	Phoenicia
.32	Philistia
.33	Judea
.34	Carthage
.35	Cyprus
.4	**India, East**
.41	Buddhist
.42	Jaina
.43	Himalayan
.44	Dravidian
.45	Chalukyan
.46	Indo-Aryan
.47	Burmah
.48	Siam
.49	
.5	**Western Asia**
.51	Babylonia
.52	Assyria
.53	Persia, ancient
.54	Sassania
.61	Pelasgian
.611	Greece
.612	Asia Minor
.62	Etruria
.7	**Roman**
.71	Rome
.72	England
.73	Germany Austria

722.74	France
.75	Italy, except Rome
.76	Spain Portugal
.77	Asia
.78	Africa
.79	
.8	**Grecian**
.81	Athens
.82	Peninsula
.83	Mainland, except Athens
.84	Archipelago
.85	Asia Minor
.86	Italy
.87	Sicily
.88	Africa
.89	
.9	**Other ancient styles**
723.	**Mediaeval Christian Mohammedan**
.1	**Early Christian**
.11	Syria
.12	England Saxon
.13	Germany Austria
.14	France
.15	Italy
.151	Rome
.152	Ravenna
.16	Coptic in Egypt
.17	N. Africa
.18	Scandinavia
.19	
.2	**Byzantine**
.21	Byzantine proper
.22	Armenia
.23	Russia
.24	Bulgaria
.25	Greece
.29	

723.3	**Mohammedan**
.311	Arabia
.312	Syria
.32	Egypt
.33	Persia
.34	Turkey
.35	India
.36	Spain Moorish
.37	North Africa
.38	
.39	
.4	**Romanesque**
.411	Scotland
.415	Ireland
.42	England
.431	Germany
.436	Austria
.44	France
.441	Anjou
.442	Normandy
.445	Auvergne
.446	Poitou
.449	Provence
.45	Italy
.451	Piedmont
.452	Lombardy
.453	Venice
.455	Tuscany
.457	Naples
.458	Sicily
.459	
.461	Spain
.469	Portugal
.48	Scandinavia
.481	Norway
.485	Sweden
.489	Denmark
.49	
.492	Holland
.493	Belgium
.494	Switzerland
.499	

723.5	**Gothic**
.511	Scotland
.515	Ireland
.52	England
.521	Early English
.522	Decorated
.523	Perpendicular
.531	Germany
.536	Austria
.537	France
.55	Italy
.552	Lombardy
.553	Venice
.555	Tuscany
.558	Sicily
.561	Spain
.569	Portugal
.58	Scandinavia
.581	Norway
.585	Sweden
.589	Denmark
.59	Minor countries
.592	Holland
.593	Belgium
.594	Switzerland
.599	
724.	**Modern**
.1	**Renaissance**
.111	Scotland
.115	Ireland
.12	England
.121	Elizabethan
.122	Jacobean
.123	17th Century
.124	18th Century
.131	Germany
.136	Austria
.14	France
.141	Francis I
.142	Henry IV
.143	Louis XIV
.144	Louis XVI
.145	Empire

724.15	Italy
.151	Cinquecento
.152	High Renaissance
.153	Decadence
.154	Barocco
.161	Spain
.169	Portugal
.17	Russia
.171	Canada
.172	Mexico
.173	United States
.1	Old colonial
.2	Spanish colonial
.178	South Americ
.1	Brazil
.2	Argentina
.3	Chili
.4	Bolivia
.5	Peru
.6	Ecuador
.7	Venezuela
.9	Paraguay
.18	Scandinavia
.181	Norway
.185	Sweden
.189	Denmark
.19	Minor countries
.192	Holland
.193	Belgium
.194	Switzerland
.199	
.2	**Classical revival Grecian**
.211	Scotland
.212	Ireland
.22	England
.231	Germany
.236	Austria
.24	France
.25	Italy
.261	Spain
.269	Portugal
.27	Russia
.271	Canada
.272	Mexico

724.273	United States
.278	South America
.281	Norway
.285	Sweden
.289	Denmark
.292	Holland
.293	Belgium
.294	Switzerland
.299	
.3	**Gothic revival**
.311	Scotland
.315	Ireland
.32	England
.331	Germany
.336	Austria
.34	France
.35	Italy
.361	Spain
.369	Portugal
.37	Russia
.371	Canada
.373	United States
.381	Norway
.385	Sweden
.389	Denmark
.392	Holland
.393	Belgium
.394	Switzerland
.4	**Tudor Gothic revival**
.411	Scotland
.412	Ireland
.42	England
.471	Canada
.473	United States
.49	
.5	**Queen Anne revival**
.511	Scotland
.512	Ireland
.52	England
.571	Canada
.573	United States
.59	

724.6	**Neo Grec**
.62	England
.631	Germany
.636	Austria
.64	France
.65	Italy
.661	Spain
.669	Portugal
.671	Canada
.673	United States
.68	Scandinavia
.69	
.7	**Half-timber** **Swiss**
.711	Scotland
.712	Ireland
.72	England
.731	Germany
.736	Austria
.74	France
.75	Italy
.761	Spain
.769	Portugal
.77	Russia
.771	Canada
.772	Mexico
.773	United States
.781	Norway
.785	Sweden
.789	Denmark
.792	Holland
.793	Belgium
.794	Switzerland
.799	
.8	**Romanesque revival**
.811	Scotland
.812	Ireland
.82	England
.831	Germany
.836	Austria
.84	France
.85	Italy
.861	Spain
.869	Portugal

724.871	Canada
.873	United States
.881	Norway
.885	Sweden
.889	Denmark
.892	Holland
.893	Belgium
.894	Switzerland
.899	
9	**Other recent styles**
.911	Scotland
.915	Ireland
.92	England
.931	Germany
.936	Austria
.94	France
.95	Italy
.961	Spain
.969	Portugal
.97	Russia
.971	Canada
.972	Mexico
.973	United States
.978	South America
.1	Brazil
.2	Argentina
.3	Chili
.4	Bolivia
.5	Peru
.6	Ecuador
.7	Venezuela
.8	Paraguay
.9	
.981	Norway
.985	Sweden
.989	Denmark
.992	Holland
.993	Belgium
.994	Switzerland
.999	
725	**Public Buildings**
.1	**Administration Government**
.111	National capitols
.112	State capitols
.113	Provincial capitols

725.114	Houses of parliament
.115	Provincial buildings
.12	Ministries
.121	State
.122	Finance
.123	War
.124	Navy
.125	Foreign affairs
.126	Interior
.127	Education
.128	Commerce
.129	
.13	City buildings
.131	City halls
.132	Town halls
.133	Guild halls (public)
.134	Office buildings
.14	Custom houses, etc.
.141	Custom houses
.142	Customs warehouses
.143	Bonded storehouses
.144	Excise offices
.15	Court houses Record offices
.151	Supreme court houses
.152	Appeal court houses
.153	Court houses
.154	Justice courts
.155	Record office buildings
.156	Archive buildings
.159	
.16	Postal buildings
.161	National buildings
.162	City post offices
.163	Village post offices
.164	Railway postal cars
.165	City postal cars
.17	Official residences Palaces of rulers
.171	National
.172	State
.173	City
.18	Barracks Police buildings
.181	National barracks
.182	State barracks
.183	Armories
.184	National police buildings

725.185	State police buildings
.186	City police buildings
.19	Fire buildings
.191	Fire administration
.192	Engine houses
.193	Fire patrol
.194	Fire alarm stations
.2	**Business and commercial buildings**
.21	Stores
.211	Wholesale
.212	Department
.213	Retail city
.214	Retail village
.215	Warehouses, wholesale
.216	Warehouses, retail
.22	Mixed store, office and apartment buildings
.221	Stores and offices
.222	Stores and flats
.223	Offices and flats
.224	
.23	Office buildings
.231	Office only
.232	Telegraph and office
.233	Insurance and office
.234	Hall and office
.24	Bank buildings
.241	Banks only
.242	Bank and office
.243	Savings banks
.244	Savings bank and office
.245	Safe deposit
.25	Exchanges Boards of trade
.251	Stock exchanges
.252	Provision exchanges
.253	Cotton exchanges
.254	Lumber exchanges
.255	Oil exchanges
.259	
.26	Markets
.261	City
.262	Provisions
.263	Commission
.264	Retail

725.27	Cattle markets
.271	Stock yards
.272	Cattle
.273	Horses Mules
.274	Sheep Goats
.275	Hogs
.276	Fowls
.279	
.28	Abattoirs
.281	Public
.282	Private
.283	Packing houses
.284	Storehouses
.29	Other commercial buildings
.3	**Transportation and storage**
.31	Railway passenger stations
.311	Country
.312	City through
.313	City terminal
.314	Union
.315	
.316	Electric passenger
.317	Street-car
.318	Elevated
.319	Underground
.32	Railway freight houses
.321	Terminal
.322	Local
.323	Express
.33	Railway shops
.331	Metal
.332	Wood
.333	Painting
.334	Round houses
.335	Car barns
.336	Water tanks
.337	Storehouses
.338	Tool houses
.339	
.34	Dock buildings
.341	Passenger
.342	Freight
.343	Wharf boats
.344	Wharf storehouses

725.35	Warehouses
.351	Merchandise
.352	Cold storage
.353	Ice plants
.354	Storage of furniture, etc.
.36	Grain elevators
.361	Brick
.362	Tiles
.363	Wood
.364	Concrete-steel
.365	Steel
.37	
.371	Coal
.372	Ores
.373	Cement Lime Plaster
.374	Malt
.375	Sand
.39	
.4	**Manufactories**
.41	Textile
.411	Wool
.412	Cotton
.413	Silk
.414	Linen
.415	Hemp
.416	Jute
.419	
.42	Beer Alcohol, etc.
.421	Breweries
.422	Distilleries
.423	Wood alcohol
.424	Spirits turpentine
.425	Malteries
.43	Iron and steel
.431	Smelters
.432	Foundries
.433	Rolling mills
.434	Machine shops
.435	Pattern shops
.436	Nail and screw works
.437	Wire and fence works
.438	Ornamental
.439	

725.44	Wood
.441	Saw mills
.442	Planing mills
.443	Cabinet mills
.444	Furniture works
.445	Agricultural works
.446	Specialty works
.449	
.45	Carriage and car shops
.451	Carriages
.452	Wagons
.453	Automobiles
.454	Bicycles
.455	Car shops (contract)
.456	Locomotives (contract)
.46	Paper mills
.461	Paper
.462	Wall paper
.463	Straw board
.469	
.47	Milling
.471	Flour
.472	Meal
.473	Feed
.48	Ceramic Glass Works
.481	Bricks, ordinary
.482	Bricks, pressed and moulded
.483	Terra cotta
.484	Tiles, roofing
.485	Tiles, floor
.486	Potteries
.487	Glass, window Bottles
.488	Plate
.489	Stained
.49	
.5	**Hospitals Asylums**
.51	General
.511	Sick
.512	Surgical
.513	Eye
.514	Ear
.515	Lying-in
.516	Incurables
.517	Consumption
.518	Contagious
.519	

725.52	Insane
.521	National
.522	State
.523	City
.524	Incurables
.525	Private
.53	Feeble-minded
.531	Idiots
.532	Defectives
.533	Defective diseased
.54	Blind Deaf
.541	Blind
.542	Deaf
.543	Schools
.544	Shops
.545	Colleges
.55	Almshouses
.551	National
.552	State
.553	County
.554	City
.555	Town
.556	Endowed
.557	Subscription
.558	Society
.559	
.56	Homes for aged
.561	Public
.562	Society
.563	Endowed
.564	Subscription
.565	Private
.57	Homes for children, orphans
.571	National
.572	State
.573	City
.574	Church
.575	Society
.576	Endowed
.58	Foundlings
.581	City
.582	Church
.583	Society
.584	Private
.59	Homes for soldiers and seamen
.591	National

725.592	State
.593	Society
.594	Private
.595	Widows
.596	Orphans
.6	**Prisons Reformatories**
.61	Penitentiaries
.611	National
.612	State
.613	City
.62	Jails
.621	County
.622	City
.623	Village
.624	Police cells
.63	Reformatories for adults
.631	Houses of correction
.632	Houses of detention
.633	Work houses
.634	Houses for women
.64	Reformatories for young
.641	Boys
.642	Girls
.643	Parental schools
.644	Truant schools
.65	Asylums for inebriates
.651	Washingtonian
.652	Hospitals
.653	Drug victims
.654	Incurables
.7	**Refreshments Baths Parks**
.71	Cafes Restaurants
.711	Buffets
.712	Dairies
.713	Cafes
.714	Restaurants
.715	Restaurant gardens
.72	Saloons Sample rooms
.73	Baths
.731	Ordinary
.732	Medicated
.733	Turkish
.734	Russian
.735	Shower

725.736	Rain
.737	Vapor
.739	
.74	Swimming baths
.741	Public
.742	Society
.743	Private
.75	Buildings for watering places
.751	Spring houses
.752	Casinos
.753	Bowling alleys
.754	Tennis courts
.755	Porticos
.76	Buildings for parks
.761	Shelters
.762	Music pavilions
.763	Kiosks
.764	Animals
.765	Plants
.766	Conservatories
.767	Refreshments
.768	Toilet
.769	
.8	**Recreation**
.811	Music halls
.812	Concert halls
.813	Orchestra halls
.821	Theatres
.822	Marionette theatres
.823	Opera houses
.824	Vaudevilles
.831	Lecture halls
.832	Recital halls
.841	Bowling alleys
.842	Billiard halls
.843	Card halls
.851	Gymnasiums
.852	Turn halls
.853	Drill halls for boys
.86	Rinks
.861	Skating
.862	Roller skating
.863	Bicycle
.864	Running

725.87	Boat houses
.871	Club
.872	Private
.873	Public
.881	Riding halls
.882	Riding schools
.883	Bicycle halls
.89	Shooting galleries
.891	Public
.892	Military
.893	Club
.894	Society
.895	Private
.9	**Other public buildings**
.91	Exhibition buildings
.911	International
.912	National
.913	State
.914	County
.915	City
.916	Memorial
.917	Art
.918	Antiquities
.919	
.92	Halls for temporary purposes
.921	Shooting contests
.922	Musical contests
.923	Religious meetings
.924	Chautauqua assemblies
.925	Political meetings
.926	Convention halls
.93	Workingmen's clubs, etc.
.931	Clubs
.932	Institutes
.933	Unions
726	**Ecclesiastical and religious buildings**
.1	**Temples**
.11	Egyptian
.12	Assyrian
.13	Jewish
.14	Etruscan
.15	Grecian

726.16	Roman
.17	Chinese
.18	Japanese
.19	
.2	**Mosques**
.21	Courtyard
.22	Byzantine
.23	Indian
.3	**Synagogues**
.4	**Chapels S. S. buildings**
.41	University
.42	College
.43	Asylum
.44	Memorial
.45	Cemetery
.46	Private
.47	Sunday school buildings
.49	
.5	**Churches**
.51	Roman Catholic
.52	Protestant Episcopal
.53	Greek
.54	Methodist Episcopal
.55	Presbyterian Congregational
.56	Baptist Christian
.57	Friends
.58	Christian Science
.59	
.6	**Cathedrals**
.61	Roman Catholic
.62	Protestant Episcopal
.63	Greek
.7	**Monasteries**
.71	Abbeys
.72	Monasteries

726.73	Priories
.74	Convents
.75	Houses for clergy
.76	Paulist fathers
.8	**Mortuary buildings**
.81	Chapels, cemetery
.82	Chapels, memorial
.83	Vaults, public
.84	Vaults, family
.85	Tombs, memorial
.86	Tombs, family
.87	Tombs, society
.88	Tombs, private
.89	
.9	**Y. M. C. A. buildings**
.91	Y.M.C.A. houses
.92	Y.M.C.A. hotels
.93	Y.W.C.A. houses
.94	Y.W.C.A. hotels
727	**Educational and scientific**
.1	**Schools**
.11	Public
.12	Private
.13	Defectives
.14	Preparatory
.15	Military for boys
.16	Orphans
.17	Manual training
.18	Trades
.19	
.2	**Academies Seminaries**
.21	Academies
.22	Seminaries (not professional)
.23	Boarding schools, boys'
.24	Boarding schools, girls'

727.3	**Colleges Universities**
.31	National
.32	State
.33	City
.34	Graduate
.35	Sectarian
.36	Scientific
.4	**Professional schools**
.41	Theology
.42	Medicine
.43	Law
.44	Normal
.441	National
.442	State
.443	City
.444	Private
.449	
.45	Engineering
.451	Architecture
.452	Architectural engineering
.453	Civil
.454	Electrical
.455	Mechanical
.456	Railway
.457	Sanitary
.458	Gas
.459	
.46	Music
.461	Voice
.462	Piano
.463	Organ
.464	Violin
.465	Minor instruments
.47	Art
.471	Elementary
.472	Painting
.473	Sculpture
.474	Illustration
.475	Institutes
.476	Academies
.481	Chemistry
.482	Agricultural
.49	
.491	Dairy

727.5	**Laboratories**
.51	Agriculture
.52	Horticulture
.53	Chemistry
.54	Physics
.55	Engineering
.551	Materials
.552	Hydraulic
.553	Steam
.554	Gas
.555	Fuel
.556	Electricity
.557	Mechanics
.558	Machines
.559	
.56	
.57	Zoölogical gardens
.58	Botanic gardens
.59	Aquariums
.6	**Museums**
.61	Ethnology
.62	Zoölogy
.63	Botany
.64	Industry
.65	History
.66	War
.67	Art
.68	Private
.69	
.7	**Art galleries**
.71	Painting
.72	Sculpture
.73	Engravings
.74	Art industries
.751	Medals
.752	Coins
.753	Postal
.76	
.771	Studios, painters'

727.772	Studios, sculptors'
.773	
.78	Private
.79	
.8	**Libraries**
.81	National
.82	State
.83	City
.84	Town
.851	College
.852	University
.853	Society
.86	Professional
.861	Theology
.862	Medicine
.863	Law
.864	Normal
.865	Engineering
.866	Music
.869	
.87	Art
.88	History
.89	
.9	**Learned societies' buildings**
.91	Art
.911	Painting
.912	Sculpture
.913	Art industries
.914	Engraving
.915	Medals and coins
.92	Science
.921	Physics
.922	Chemistry
.923	Natural science
.1	Biology
.2	Zoölogy
.3	Botany
.4	Entomology
.5	Geology
.6	Microscopy
.7	Ethnology
.9	

727.93	Engineering
.931	Architecture
.932	Architectural
.933	Civil
.934	Electrical
.935	Mechanical
.936	Railway
.937	Sanitary
.939	
.94	Education
.941	National
.942	State
.943	County
.944	City
.945	Local
.95	Religion
.951	Ministerial
.952	Missionary
.953	Benevolent
.96	Medicine
.961	National
.962	State
.963	County
.964	City
.965	Local
.969	
.99	
728	**Residences**
.1	**Tenements**
.11	City, for poor
.12	City, for workers
.13	City, for clerks, etc.
.14	Country
.15	Factory
.17	Society
.19	
.2	**Apartment buildings**
.21	Flats, small
.22	Flats, medium
.23	Flats, large
.24	Bachelor's

728.25	Women's
.26	Family
.27	Double houses
.28	Rooming houses
.29	
.3	**City houses**
.31	Small, wood
.32	Small, brick
.33	Small, stone
.34	Medium, brick
.35	Medium, stone
.36	Mansions, inside
.37	Mansions, corner
.38	Mansions, detached
.39	
.4	**Buildings for clubs and societies**
.41	Clubs, dining
.42	Clubs, meeting
.43	Masonic
.44	Odd Fellows
.45	Knights of Pythias
.46	Elks
.47	G.A.R.
.48	Insurance, fraternal
.49	
.5	**Hotels**
.51	Country inns
.52	Village inns
.53	City, rooming
.54	City, European
.55	City, American
.56	City, largest
.57	Suburban
.58	Summer
.59	

728.6	**Village and country homes**
.61	Small
.62	Brick
.63	Stone
.64	Concrete
.65	Plastered
.66	Masonry and wood
.67	Farm houses
.68	Cottages for laborers
.69	
.7	**Summer homes**
.71	Tents
.72	Portable
.73	Wood
.74	Brick
.75	Stone
.76	Mixed
.77	Concrete
.78	Metal
.79	
.8	**Country seats**
.81	Castles
.82	Chateaux
.83	Manor houses
.84	Villas
.89	
.9	**Out buildings**
.91	Gate lodges
.92	Cottages for helpers
.921	Servants
.922	Laborers
.923	Gardeners
.924	Grooms
.925	Coachmen
.926	Keepers
.927	Foresters
.929	

728.93	Kitchens, laundries, etc.
.931	Kitchens
.932	Laundries
.933	Dairies, house
.934	Smoke houses
.935	Store houses
.936	Granaries
.937	Commissaries
.938	
.94	Stables, carriage houses, etc.
.941	Stables, village
.942	Stables, city
.943	Stables, largest
.944	Kennels
.945	Carriage houses
.949	
.95	Barns, granaries, etc.
.951	Barns, small
.952	Barns, village
.953	Barns, farm
.954	Barns, store
.955	Granaries, small
.956	Granaries, large
.957	Corn cribs
.959	
.96	Dairies
.961	Dairies, large
.962	Dairy stables
.963	Dairy stores
.97	Refrigeration accommodations
.971	Ice houses
.972	Ice plants
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.339	Arcades
.34	Vaults and domes
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BULLETIN NO. 14

TESTS OF REINFORCED CONCRETE BEAMS

SERIES OF 1906

BY

ARTHUR N. TALBOT

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 14 AUGUST 1907

TESTS OF REINFORCED CONCRETE BEAMS
SERIES OF 1906.

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY
ENGINEERING AND IN CHARGE OF THEORETICAL
AND APPLIED MECHANICS.

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I. INTRODUCTION.

1. *Preliminary.*—The tests on reinforced concrete in the Laboratory of Applied Mechanics of the University of Illinois were continued during the college year of 1905-6. The results on shear and bond were reported in Bulletin No. 8 of the University of Illinois Engineering Experiment Station; those on columns, in Bulletin No. 10; those on T-beams, in Bulletin No. 12. The tests on rectangular beams will be described in this bulletin. The analytical theory of reinforced concrete beams was quite fully treated in Bulletin No. 4, and the methods and nomenclature used in this bulletin will follow those there given.

2. *Scope of Tests.*—The discussion given in Bulletin No. 4 suggested many topics for investigation. Several of these were taken up and considerable information obtained, though it was not feasible to make the investigations complete enough to be fully conclusive. Effect of quality of concrete, effect of method of loading, effect of repetitive loading, and diagonal tension failures were among the topics considered. The beams were of the standard width and depth adopted by the Joint Committee on Concrete and Reinforced Concrete, but in the investigation of resistance to diagonal tension failure the length was varied and shortened to ensure this form of failure. Plain round rods of mild steel were generally used for reinforcement, but a deformed bar was used in some of the beams. The concrete was varied in quality both in the richness of the mixture and in the conditions of fabrication. These tests have since been followed with others bearing on some of the same topics.

3. *Acknowledgment.*—These tests were part of the work of the University of Illinois Engineering Experiment Station. The tests on effect of quality of concrete and effect of method of loading were made in co-operation with the Joint Committee on Concrete and Reinforced Concrete through the sub-committee on tests, of which Mr. Richard L. Humphrey is chairman. The work of testing the beams was done principally as thesis work. The students conducted the tests in a careful and skillful manner, and showed considerable discrimination in making observations and in drawing conclusions. The following members of the class of 1906 in Civil Engineering were connected with the work:

E. W. Sanford, Effect of Quality of Concrete.

H. R. Armeling, Comparison of Methods of Loading.
C. E. Andrew and J. L. Bannon, Effect of Repetition of Load.
T. E. Phipps and R. H. Whipple, A Study of Diagonal Tension Failures.

The work was under the direct supervision of D. A. Abrams, Assistant in the Engineering Experiment Station, and to him and to W. R. Robinson, Assistant in the Engineering Experiment Station, acknowledgment is made for aid in the interpretation of results and in the preparation of this bulletin.

II. MATERIALS, TEST PIECES, AND METHOD OF TESTING.

4. *Materials.*—Materials for the tests on "Effect of Method of Loading" and "Effect of Quality of Concrete" were furnished by the Joint Committee on Concrete and Reinforced Concrete through Mr. Richard L. Humphrey, chairman of the Committee on Tests. Materials for the tests on "Effect of Repetition of Load" and "Diagonal Tension Failure" were furnished by the Engineering Experiment Station. The terms "Joint Committee tests" and "Experiment Station tests" will be used to designate this difference of work and materials.

Stone.— The stone for the Joint Committee tests was a good quality of limestone from Kankakee, Illinois, ordered screened through a 1-in. screen and over a $\frac{1}{4}$-in. screen. It contained from 45% to 50% voids and weighed 85 lb. per cu. ft. loose. The stone for the Experiment Station tests was also a Kankakee limestone somewhat softer than the other, screened as above, and contained 50% to 54% voids. It was somewhat finer than the Joint Committee stone. In the determination of the voids in both stone and sand, the material was poured slowly into the water so that the voids became filled with water and no air was caught.

Sand.—The sand used was the same for all tests. It came from near the Wabash river at Attica, Indiana. It was of good quality, well graded, and fairly clean. It weighed 115 lb. per cu. ft. loose, and contained 28% voids. Table 1 gives the result of a mechanical analysis of this sand.

Cement.—The cement furnished by the Joint Committee was made up of a mixture of five standard American portland cements, selected and mixed by the manufacturers, and was of excellent quality. The cement furnished by the Experiment Station was Chicago AA portland, purchased in the open market of a local

TABLE 1.

MECHANICAL ANALYSIS OF SAND.

Sieve No.	Size of Mesh inches	Per cent passing
4	.208	100
10	.073	73
20	.034	36
50	.011	12
74	.0078	5
100	.0045	2

TABLE 2.

TENSILE STRENGTH OF CHICAGO AA PORTLAND CEMENT.

Ref. No.	Ultimate Strength, lb. per sq. in.			
	Age 7 days		Age 60 days	
	Neat	1–3	Neat	1–3
1	634	283	890	443
2	717	281	916	440
3	732	275	840	422
4	687	217	942	365
5	580	206	872	352
6	731	189	885	....
Av.	680	242	891	404

dealer. The tensile strength of this last cement, as determined from briquettes made by standard methods, is given in Table 2.

Concrete.—Men accustomed to making concrete mixed the materials and made the test beams. Care was taken in measuring, mixing, and tamping, to secure as uniform a concrete as possible. All materials were measured by loose volume. The mixing was done with shovels by hand. The sand and cement were first mixed dry. The stone was then added and the mass mixed until uniform in appearance. Water was added in such proportion as to give a slightly wet concrete.

Steel.—The longitudinal reinforcement consisted generally of ½-in. or ¾-in. mild-steel plain round rods. In a few beams ½-in. high-steel Johnson corrugated bars were used. The results of tensile tests of the steel used are given in Table 3. The plain round bars had an average yield point of 40 500 lb. per sq. in. and an ultimate strength of 60 000 lb. per sq. in. The corrugated bars developed an average yield point of 57 300 lb. per sq. in. with an ultimate strength of 87 400 lb. per sq. in. In general, the yield points of the various bars in one beam varied less than 3% from one another.

TABLE 3.

Tension Tests of Steel Used in Beams.

Values given are in general the average of results of tests of specimens cut from four different rods.

Specimens Taken from Beams No.	Nominal Size inches	Diameter inches	Per cent Elongation in 8 inches	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.
2	¾	.750	31.0	34000	56300
13	¾	.747	33.0	38800	56900
14	¾	.750	29.7	38700	58000
15	½	.501	29.3	41400	61700
16	½	.500	28.5	37700	58100
17	½	.500	28.0	36700	57900
18	½	.497	29.0	40300	60200
20	½	.502	28.0	44400	62500
21	½	.502	31.2	41800	58000
23	¾	.750	31.5	35000	56900
26	½	.500	29.0	37200	58600
27	¾	.748	31.2	39400	57100
28	½	.503	29.1	42700	60800
30	½	.503	29.1	41000	61100
31	½	.504	28.5	41800	60900
34	¾	.749	32.5	38800	56000
35	½	.502	29.5	40800	58400
36	½	.500	28.0	36300	58100
38	¾	.750	29.0	35800	54700
39	¾	.749	32.5	38800	56200
40	½	.502	28.1	42800	62200
42	½	.502	29.9	42500	61800
43	½	.501	28.4	42400	62100
44	½	.503	28.7	42100	61800
45	½	.503	29.6	42000	61900
50	½	.503	29.5	42800	61700

TABLE 3—*Concluded.*

Specimens Taken from Beams No.	Nominal Size inches	Diameter inches	Per cent Elongation in 8 inches	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.
51	½	.502	28.5	41800	60900
52	½	.501	27.9	44200	63100
53	½	.504	29.2	42500	62800
54	½	.505	28.0	42100	61200
55	½	.503	27.8	42000	62300
56	⅝	.622	30.7	37800	60600
58	¾	.753	29.5	40300	62900
59	¾	.753	26.0	41000	65300
60	¾	.750	28.5	39200	63000
61	¾	.752	30.5	39000	59100
62	¾	.750	32.3	40600	58100
63	¾	.748	30.3	42350	62600
64	¾	.748	33.5	37800	54400
65	¾	.747	29.5	41700	61600
68	¾	.754	29.0	44200	61900
69	¾	.747	29.5	42200	62700
70	¾	.752	29.5	44500	60700
71	¾	.749	32.2	40100	59200
72	¾	.748	31.5	39900	58900
73	¾	.750	31.2	40700	56900
74	½	.499	27.6	43900	60100
66*	½ in. (net section = 0.25 sq. in.)		16.1	58800	91100
67*			17.4	57300	87300

* Corrugated bars.

5. *Test Beams.*—In all of the tests herein discussed the cross-section of the beams was 8 in. x 11 in., the center of the steel being placed 10 in. below the top surface except in some cases where the ends of the bars were bent up. In the tests on "Diagonal Tension Failure" the test span varied from 6 ft. to 12 ft. In all the other series of tests a 12-ft. span was used. Unless otherwise specified the reinforcing bars were straight and were placed horizontally throughout the beam. In the beams marked "Bars bent up" the bars were bent up at a point about 3 in. outside the load points and passed diagonally either in a straight line or in a slightly curved line to a point within 2 in. or 3 in. of the top of the beam near its ends. In several beams stirrups of ½-in. plain

round bars were used. These stirrups were placed 12 in. apart longitudinally throughout the outer thirds of the span length, beginning at the load points. They were U-shaped and passed under all the reinforcing bars and extended nearly to the top of the beam. The stirrups were left very close to the sides of the beam. For general data on all beams see Table 5.

TABLE 4.

COMPRESSION TESTS OF CUBES AND CYLINDERS.

Concrete as in Beam No.	Kind of Concrete	Compressive Strength lb. per sq. in.	
		Cubes	Cylinders
59	1–2–4	2030	1700
61	1–2–4	3500	....
69	1–2–4	1510	....
Av.	....	2350	1700
51	1–3–5½	....	1470
58	1–3–5½	1580	1060
60	1–3–5½	1690	1060
65	1–3–5½	2360	....
68	1–3–5½	1850	885
70*	1–3–5½	2250	1100
72	1–3–5½	1770	540
73*	1–3–5½	....	770
Av.	....	1920	980
5†	1–3–6	1390	
6†	1–3–6	1420	
7†	1–3–6	1050	
12†	1–3–6	1080	
14	1–3–6	1600	
18	1–3–6	1380	
19	1–3–6	1170	
Av.	....	1300	
71	1–5–10	1230	

* Poorly mixed.
† Poorly made.

TABLE 5.

General Data on Beams.

Beam No.	Kind of Concrete	Reinforcement		Span, ft.	Age at Test days	Classification	
		Per cent	Amount and Disposition			Table No.	Kind
3	1–3–6	0.98	4 ½-in. round.	12	72	10	E. S.
4	1–3–6	0.98	4 ½-in. round.	12	74	12	E. S.
5	1–3–6	0.98	4 ½-in. round.	12	71	10	E. S.
6	1–3–6	0.98	4 ½-in. round.	10	71	10	E. S.
7	1–3–6	0.98	4 ½-in. round. Bars bent up.	10	71	11	E. S.
8	1–3–6	2.21	4 ¾-in. round.	10	76	10	E. S.
9	1–3–6	2.21	4 ¾-in. round.	10	76	10	E. S.
10	1–3–6	0.98	4 ½-in. round.	12	73	12	E. S.
11	1–3–5½	0.98	4 ½-in. round.	12	73	10	E. S.
12	1–3–6	2.21	4 ¾-in. round.	10	70	12	E. S.
13	1–3–6	2.21	4 ¾-in. round. Bars curved up.	10	69	11	E. S.
14	1–3–6	2.21	4 ¾-in. round. Bars curved up.	10	68	11	E. S.
15	1–3–6	0.98	4 ½-in. round.	8	69	10	E. S.
16	1–3–6	0.98	4 ½-in. round.	8	69	10	E. S.
17	1–3–6	0.98	4 ½-in. round. Bars bent up.	8	70	11	E. S.
18	1–3–6	0.98	4 ½-in. round.	6	71	10	E. S.
19	1–3–6	0.98	4 ½-in. round.	6	67	10	E. S.
20	1–3–6	0.98	4 ½-in. round. Bars bent up.	6	67	11	E. S.
21	1–3–6	0.98	4 ½-in. round. Bars bent up.	6	68	11	E. S.
22	1–3–6	0.98	4 ½-in. round. 2 bars bent up.	6	70	11	E. S.
23	1–3–5½	2.21	4 ¾-in. round.	10	67	10	E. S.
25	1–3–5½	0.98	4 ½-in. round.	6	70	10	E. S.
26	1–3–5½	0.98	4 ½-in. round. Bars curved up.	8	68	12	E. S.
27	1–3–5½	2.21	4 ¾-in. round.	8	68	10	E. S.
28	1–2–4	0.98	4 ½-in. round.	12	70	8, 9, 12	E. S.
29	1–3–5½	0.98	4 ½-in. round.	12	70	8, 9, 10	E. S.
30	1–4–7½	0.98	4 ½-in. round.	12	71	8, 9, 10	E. S.
31	1–2–4	0.98	4 ½-in. round.	12	78	8, 9, 12	E. S.
32	1–3–5½	0.98	4 ½-in. round.	12	79	8, 9, 12	E. S.
33	1–3–5½	0.98	4 ½-in. round.	6	73	10	E. S.
34	1–3–5½	2.21	4 ¾-in. round.	8	67	10	E. S.
35	1–4–7½	0.98	4 ½-in. round.	12	75	8, 9, 12	E. S.
36	1–3–5½	0.98	4 ½-in. round. Bars bent up.	10	61	11	E. S.
37	1–3–5½	2.21	4 ¾-in. round.	8	70	10	E. S.
38	1–3–5½	2.21	4 ¾-in. round. Bars bent up.	8	64	11	E. S.
39	1–3–5½	2.21	4 ¾-in. round. Bars bent up.	8	64	11	E. S.
40	1–3–5½	0.98	4 ½-in. round.	12	62	7	J. C.
42	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
43	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
44	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
45	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
46	1–3–5½	0.98	4 ½-in. round.	6	85	10	E. S.
47	1–2–4	0.98	4 ½-in. round.	6	88	10	E. S.
48	1–2–4	0.98	4 ½-in. round.	6	84	10	E. S.
49	1–3–5½	0.98	4 ½-in. round.	6	82	10	E. S.
50	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
51	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.

TABLE 5.—*Concluded*

GENERAL DATA ON BEAMS.

Beam No.	Kind of Concrete	Reinforcement		Span, ft.	Age at Test days	Classification	
		Per cent	Amount and Disposition			Table No.	Kind
52	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
53	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
54	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
55	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
56	1–3–5½	0.98	4 ½-in. round.	12	60	7	J. C.
57	1–2–4	1.15	3 ⅝-in. round,	12	76	6	J. C.
58	1–3–5½	1.10	2 ¾-in. round.	12	61	6	J. C.
59	1–2–4	1.10	2 ¾-in. round.	12	62	6	J. C.
60	1–3–5½	1.10	2 ¾-in. round.	12	61	6	J. C.
61	1–2–4	1.10	2 ¾-in. round.	12	61	6	J. C.
62	1–5–10	1.10	2 ¾-in. round.	12	62	6, 10	J. C.
63	1–5–10	1.10	2 ¾-in. round.	12	61	6, 10	J. C.
64	1–3–5½	1.10	2 ¾-in. round.	12	61	6	J. C.
65	1–3–5½	1.10	2 ¾-in. round.	12	60	6	J. C.
66	1–5–10	1.25	4 ½-in. corrugated bars.	12	59	6, 10	J. C.
67	1–5–10	1.25	4 ½-in. corrugated bars.	12	60	6, 10	J. C.
68	1–3–5½	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	60	6, 10	J. C.
69	1–2–4	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	61	6	J. C.
70	1–3–5½	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	60	6	J. C.
71	1–5–10	1.65	3 ¾-in. round. 10 ½-in. stirrups.	12	60	6, 10	J. C.
72	1–3–5½	1.10	2 ¾-in. round. 10 ½-in. stirrups.	12	60	6	J. C.
73	1–3–5½	1.10	2 ¾-in. round. 10 ½-in. stirrups.	12	60	6	J. C.
74	1–3–5½	0.98	4 ½-in. round.	10	62	12	E. S.

NOTE:—E. S. and J. C. refer to Engineering Experiment Station and Joint Committee respectively. For explanation of terms, see "4. Materials," p. 3.

6. *Making of the Beams.*—The beams were made directly on the concrete floor of the laboratory, a strip of building paper being laid beneath the forms. Several proportions of concrete were used, varying from 1-2-4 to 1-5-10 by loose volume. A number of the first beams were made of 1-3-6 mixture, and the later ones of 1-3-5½, as it was thought that the voids in the stone were not properly filled in the 1-3-6 mixture. The forms, which were of the ordinary wooden knock-down type, were removed four days after making the beams and the beams were not moved in any way for 14 days. Generally the stone for the concrete was dampened and the concrete well mixed and wet enough to secure proper hardening. The making of the beams

was skillfully done. In Beams No. 3 to 13 inclusive, however, through some oversight the stone (a porous material), was not dampened, insufficient water was used in mixing, the making and tamping were not properly done, and the concrete was allowed to become too dry. The beams so made proved to be of inferior concrete and are referred to as poorly made concrete. The low results obtained are of interest in showing the effect of improper methods even if enough cement is used.

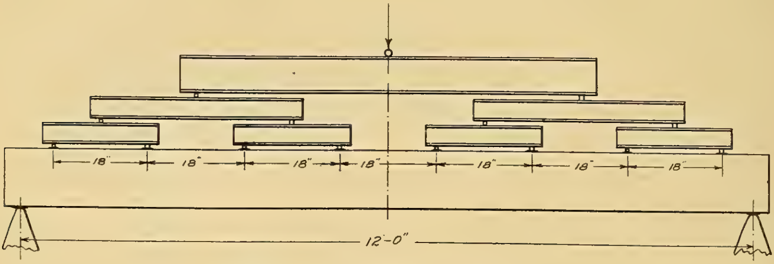


FIG. 1. 8-POINT LOADING.

7. *Minor Test Pieces.*—Tests were made on 6-in. cubes and on 8-in. cylinders 16 in. high taken from concrete used in some of the beams. The results of these tests are given in Table 4. The values given for the cubes are the averages of three test specimens. For the cylinders, a single test specimen was used.

8. *Storage.*—The beams were stored in a room the temperature of which was from 60° F. to 70° F. They were tested at the age of about 60 days.

9. *Method of Testing.*—The usual method of testing was by loads applied at the one-third points as described in Bulletin No. 4, page 34. The beams were all tested in the 200 000-lb. Olsen testing machine, and in all cases except the tests on "Effect of Method of Loading," were loaded at the one-third points. The method used for loading at eight points is shown in Fig. 1. The supports of the beam allowed longitudinal movement, the bottom of the rocker being an arc of 12-in. radius, and the top, on which cast-iron blocks rested, having a radius of 1½ in. Turned steel rollers, 2 in. in diameter, were used for applying the load at the third points. The blocks at the supports and load points were bedded in plaster of paris which was allowed to harden under the weight of the beam and the apparatus used in loading before the

load was applied. The cubes, cylinders, and steel were tested in the 100 000-lb. Riehlé and 200 000-lb. Olsen testing machines.

Center deflections were read on all the beams. Deformations of the upper fiber and steel were measured by means of four extensometers. The methods of measuring deflections and deformations were fully described in Bulletin No. 4.

TABLE 6.

EFFECT OF QUALITY OF CONCRETE.

Span 12 ft. Loaded at one-third points.

Beam No.	Maximum Load, lb.	k	Per cent Reinforcement	Stress in Steel lb. per sq. in.	Vertical Shearing Stress $v = \frac{V}{bd'}$ lb. per sq. in.	Manner of Failure
1–2–4 Concrete.						
57	11730	.43	1.15	39800	96	Tension.
59	12000	.53	1.10	44000	97	Tension.
61	10960	.47	1.10	39700	90	Tension.
69*	16000	.57	1.65	39000	129	Tension.
1–3–5½ Concrete.						
58	9860	.52	1.10	36900	82	Tension.
60	10360	.57	1.10	39400	85	Tension.
64	9850	.55	1.10	37300	82	Tension.
65	10000	.54	1.10	37600	83	Tension.
68*	14220	.57	1.65	35000	115	Diagonal Tension. [crete]
70*	13710	.69	1.65	35800	112	Compression (Poor con-
72*	10270	.47	1.10	37400	85	Tension.
73*	9900	.52	1.10	37000	82	Tension.
1–5–10 Concrete.						
62	8850	.60	1.10	34600	74	Diagonal Tension.
63	7360	.48	1.10	28100	63	Diagonal Tension.
66†	8000	.63	1.25	29400	69	Diagonal Tension.
67†	7850	.57	1.25	27100	68	Diagonal Tension.
71*	7600	.59	1.65	20200	67	Diagonal Tension.

* Stirrups.

† Reinforced with corrugated bars.

III. EXPERIMENTAL DATA AND DISCUSSION.

10. *Explanation of Tables 6 to 12.*—In Tables 6 to 12, the position of the neutral axis was obtained by the method used in Bulletin No. 4. In calculating the per cent of reinforcement the area of the beam above the center of the reinforcing bars is used. The columns headed "Maximum Applied Load" do not include the weight of the beam loading apparatus, but these weights were considered in calculating the stress in the steel. In determining the amount of the vertical shear, the weight of the beam and loading apparatus was considered, 6 lb. per sq. in. being added to the unit-stress for a 6-ft. beam, 7 for an 8-ft. beam, 8 for a 10-ft. beam, and 10 for a 12-ft. beam. In obtaining the vertical shear, the formula $v = \frac{V}{.86bd}$ or $.0145V$ was used for 1% beams and $v = \frac{V}{.81bd}$ or $.0154V$ for 2.2% beams.

11. *Effect of Quality of Concrete.*—In this series, tests were made on beams made of three kinds or grades of concrete,—1-2-4, 1-3-5½, and 1-5-10 mixtures. The purpose of this series was to determine the effect of quality of concrete upon the strength of the beam and upon the manner of failure. The beams were planned to give a variety of manners of failure,—tension in the steel, compression in the concrete, and diagonal tension in the concrete. Table 6 gives the results of this series. The calculations were made as described under "10. Explanation of Tables". No attempt is made to calculate the diagonal tensile stresses developed, but the ability to resist diagonal tension will be compared by means of the vertical shearing stresses developed. The load was applied at the one-third points of the beams. From the tables it will be noted that the manner of failure for beams of these proportions depends upon the richness of the concrete. Counting the effect of the weight of the beam and loading apparatus, it is seen that all the beams made with 1-2-4 concrete failed by tension in the steel at calculated stresses somewhat above the elastic limit of the steel. Beam No. 57, (Fig. 4), is typical of the appearance of the beams after failure. Beam No. 69, having 1.65% reinforcement, has a load-compression diagram (Fig.12) which indicates that the stress in the concrete at the maximum load was well within the limits of its ultimate compressive strength. The vertical shearing stress developed in this

beam was 124 lb. per sq. in. and there was no indication of approaching failure by diagonal tension.

Of the test beams of 1-3-5½ concrete all but two failed by tension in the steel. The calculated stresses in the steel, when allowance is made for the weight of the beam, are slightly above the elastic limit of the steel. The two not failing by tension in the steel, when compared by the stresses computed either from the bending moment or the observed deformations, gave stresses in the steel not much lower than the average of the remainder of the series. Both of these beams, therefore, had hardly reached the load which would have been followed by failure by tension in the steel. Beam No. 68 failed by diagonal tension at a calculated vertical shearing stress of 115 lb. per sq. in. No. 70 failed by compression of the concrete. The compression diagram (Fig. 13) shows that the concrete in this beam lacked stiffness, the amount of deformation being more than for the average beam but not much more than that of No. 68, its companion beam, as is seen from the load-deformation diagrams. And yet this beam of 1-3-5½ concrete and 1.65% reinforcement carried a load nearly to the elastic limit of the steel reinforcement.

All the beams made with 1-5-10 concrete failed by diagonal tension at loads which show a rather narrow range regardless of the amount or method of reinforcement. The vertical shearing unit-stress developed averaged 68 lb. per sq. in. As the beams failed by diagonal tension at loads much smaller than those at which failure by compression in the concrete may be expected, there is nothing in these tests upon which to base the limit of the concrete or the amount of reinforcement at which the compressive strength of the concrete and the tensile strength of the steel may be considered to be balanced.

A comparison of beams having 1.1% reinforcement which failed by tension in the steel shows that the 1-2-4 beams carried greater loads than the 1-3-5½ beams, the additional load amounting to 10% or 15%. This increase of load probably is due to the fact that the greater strength of the richer concrete allows the steel to be stretched a greater distance beyond the elastic limit before developing the full compressive strength of the concrete and also that the moment arm of the couple formed by the compressive stresses is somewhat greater with the richer concrete. The added strength of the richer concrete in preventing failure by diagonal tension is apparent. This feature of the series will be further discussed under "14. Diagonal Tension Failures". As will be shown afterward, the arrangement of stirrups used was not well planned and their presence seemed not to add to the strength of the beams, although in Beams No. 68 and 71 it might have been expected that well designed stirrups would prevent failure by diagonal tension.

12. *Effect of Method of Loading.*—It was the purpose of this series to determine the effect of the method of loading upon the resisting moment developed in the beam. With this in view, the beams were so proportioned that failure by tension in the steel was expected in all cases. In Bulletin No. 4, page 54, a discussion of this topic is given. It was there stated that beams loaded at the middle have been found to develop a higher moment of resistance than is to be expected if the distribution of stresses is as assumed in the ordinary theory of flexure. Six methods of loading were used in the tests of this series: (1) load applied at center of the span only; (2) load applied at two points 1½ feet apart; (3) load applied at two points 3 feet apart; (4) load applied at the one-third points; (5) load applied at two points 7½ feet apart; (6) load applied at eight points (to approximate a uniform load). The appliances used for the loading at eight points have been described under "9. Method of Testing".

Table 7 gives the results of these tests, together with the calculated stresses in the steel. All beams failed by tension in the steel, as was clearly shown by the load-deformation diagrams. If the effect of the weight of the beam and loading apparatus is included, it will be seen that the calculated stresses in the steel all lie above the elastic limit. A comparison of the resisting moments of the beams may be made by comparing the calculated stresses in the steel, given in Table 7, since the tests of the steel used in these beams show that there was little variation in the yield point of the test pieces. It will be noted that the highest stress developed was in beams having the loading at the middle, and that when the two loads were close to the middle the results were not much lower. For the other methods of loading the variation in stress developed was not large, no greater than may be expected with the difference in the materials and fabrication in such beams, though the method in which the load was applied at eight points gave a somewhat higher resisting moment. These

TABLE 7.

EFFECT OF METHOD OF LOADING

Reinforcement .98%. Concrete 1-3-5½. Span 12 ft. All failed by tension in the steel.

Beam No.	Method of Loading	Maximum Applied Load lb.	k	Stress in Steel lb. per sq. in.
50	Center.	7400	.45	45200
52	Center.	7650	.38	45400
53	2 points 1½ ft. apart.	7900	.45	42500
54	2 points 1½ ft. apart.	8250	.42	43500
55	2 points 3 ft. apart.	8900	.44	40700
56	2 points 3 ft. apart.	9610	.51	45000
40	One-third points.	10000	.46	41000
42	One-third points.	9420	.47	39000
45	2 points 7½ ft. apart.	17300	.50	40100
51	2 points 7½ ft. apart.	18000	.45	40700
43	8 points 18 in. apart.	14000	.44	42100
44	8 points 18 in. apart.	15000	.41	44300

tests go to show the general applicability of the ordinary beam theory to simple beams without end restraint or horizontal restraint for any of the usual methods of loading, with the exception of center loading, provided, of course, that the proportions of the beam are such that the method of failure is by tension in the steel. It will be seen that the beams loaded at the middle give about 10% greater resistance than the more usual methods of loading. This excess is not so great as has been found in beams having a high percentage of reinforcement. With high reinforcement the resulting moment developed is considerably greater than for loading at the one-third points. Evidently under such conditions loading at the middle gives a distribution of stresses at sections near the center of the beam which is different from that assumed in the ordinary theory of flexure.

The load-deformation curves have the same general characteristics in all of the beams. The position of the neutral axis, as determined by the method used, is nearly the same for the several methods of loading, the variation being as little as may be expected in tests of this character and no characteristic difference being noticeable in any method of loading. Of course, with the load applied in the middle the deformations were taken over so great a gauged length that a discrepancy in the distribution of stress at a section at the middle had little effect on the values found.

13. *Effect of Repetitive Loading.*—Tests were made on six beams to determine the effect of repeatedly applying and releasing the load on the beam, from 24 to 30 applications of a single load being made. Three mixtures of concrete were used, thus permitting a study of the effect of quality of concrete. All beams were reinforced with 1% of steel. The load applied was 5000 lb.

TABLE 8.

EFFECT OF REPETITIVE LOADING.

Span 12 ft. Loaded at one-third points.

Beam No.	Mixture	Age days	Per cent Reinforcement	Repeated Load lb.	No. of Applications	Failed at lb.	Manner of Failure
28	1-2-4	70	0.98	6000	26	9900	Tension.
31	1-2-4	78	0.98	6000	25	9400	Tension.
29	1-3-5½	71	0.98	5000	30	10000	Diagonal tension.
32	1-3-5½	79	0.98	6000	24	9800	Tension.
30	1-4-7½	71	0.98	5000	30	5900	Diagonal tension.
35	1-4-7½	75	0.98	5000	30	7500	Compression.

TABLE 9.

DEFLECTIONS UNDER REPETITIVE LOADING.

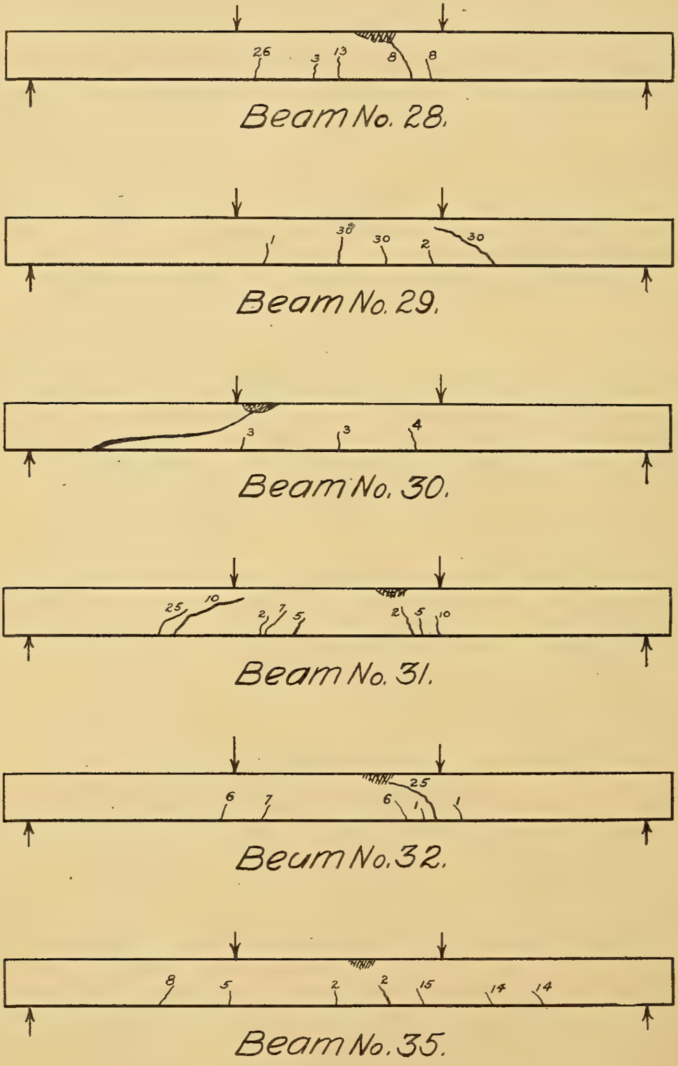
Beam No.	Mixture	Center Deflection in inches for 5000-lb. Load.						
		Application						
		1st	5th	10th	15th	20th	25th	30th
28	1-2-4	0.25	0.33	0.34	0.36	0.37	0.38	
31	1-2-4	0.25	0.33	0.35	0.35	0.36	0.37	
29	1-3-5½	0.15	0.25	0.25	0.25	0.25	0.26	0.25
32	1-3-5½	0.24	0.31	0.33	0.34	0.34		
30	1-4-7½	0.40	0.49	0.54	0.57	0.60	0.62	0.65
35	1-4-7½	0.31	0.38	0.40	0.41	0.41	0.41	0.42

in three beams, and 6000 lb. in the other three. These loads were from 50 % to 85 % of the maximum load carried by the beam when the load was finally increased to the point of failure. The deflection at the midpoint of the span was measured, as were the deformations at the top and bottom for a gauged length along the middle of the beam, though the latter measurements were not entirely satisfactory. Table 8 and Table 9 give results of these tests.

The following notes show the principal features of the tests of the several beams.

Beam No. 28. The beam was made of 1-2-4 concrete. The general behavior of this beam during the repetition of the 6000-lb. load is representative of the action of the other beams. Hair cracks appeared on the tension side of the beam in the middle third of the length, one or more of them usually on the first application of the load and others at subsequent applications, as shown in Fig. 2. In this beam hair cracks appeared at the third, eighth, thirteenth, and twenty-sixth applications. In each case the cracks closed upon the removal of the load. Upon increasing the load after the twenty-sixth application of 6000 lb., the cracks opened still further. Finally the crack marked 8 opened much more rapidly, the beam failed by tension in the steel, and this was followed by the load dropping off and the concrete finally crushing at the top at a load much less than the maximum. It may be noted that the deflection of the beam increased with the repetition of the 6000-lb. load, the amount at the twenty-fifth application being 50% more than at the first. The position of the cracks is shown in Fig. 2, the numbers indicating the applications at which the cracks were noted.

Beam No. 31. The beam was of 1-2-4 concrete. Hair cracks appeared at the second, fifth, seventh, tenth, and twenty-fifth applications of the load of 6000 lb. On increasing the load after the twenty-fifth application, the cracks opened, and at 8000 lb. crack No. 2 (Fig. 2) rapidly widened. At a load of 9400 lb., the steel passed its yield point and as usual this was finally followed with the crushing of concrete at the top of the beam. The increase in the deflection of the beam at the twenty-fifth application over that at the initial application of the load was 50%. It should be noted that the diagonal crack marked 10 formed at the 10th application of the load, but, although it widened when the load was increased beyond 6000 lb., it did not cause failure. The vertical shearing unit-stress, as calculated by equation 18 (See page 20, Bulletin No. 4), was 53 lb. per sq. in. for a load of 6000 lb., and 78 lb. per sq. in. at failure.



Beam No. 28.

Beam No. 29.

Beam No. 30.

Beam No. 31.

Beum No. 32.

Beam No. 35.

FIG. 2. SKETCHES SHOWING BEAMS AFTER FAILURE UNDER REPETITIVE LOADING.

Beam No. 29. The beam was made of 1-3-5½ concrete. A load of 5000 lb. was applied 30 times. Hair cracks appeared as shown in the sketch in Fig. 2. The deflection for the load of 5000 lb. increased 67% during the repetitions. At a load of 9000 lb. a diagonal crack appeared one foot outside the load point, and the beam finally failed by diagonal tension along this crack. The vertical shearing unit-stress at this load was 76 lb. per sq. in., and at the maximum load 83 lb. per sq. in.

Beam No. 32. The beam was made of 1-3-5½ concrete. A load of 6000 lb. was applied 24 times. Two hair cracks appeared at the first application, and several more at later applications of the load, but no additional effect was observed. As the load was finally increased to an amount near 9400 lb., a crack near the load point appeared, and failure by tension in the steel at this point followed at a maximum load of 9800 lb. The repetition of the load gave a greatly increased deformation in the upper fiber (See Fig. 14) but when the load was increased beyond 6000 lb., the upward direction of the deformation curve for the upper fiber indicates that the concrete had not lost its strength or elasticity. The changes in the deflection are shown in Fig. 10.

Beam No. 30. The beam was made of 1-4-7½ concrete. A load of 5000 lb. was applied 30 times. Hair cracks appeared on the third and fourth applications. On increasing the load a diagonal crack appeared outside the load point, and the beam failed by diagonal tension at a maximum load of 5900 lb., followed by a stripping of the bars for some distance beyond. The vertical shearing unit-stress for this load was 53 lb. per sq. in. The exposed ends of the bars showed a slip in the concrete after the maximum load was reached. Fig. 8 gives the changes in deflection.

Beam No. 35. The beam was made of 1-4-7½ concrete. Thirty applications of the load of 5000 lb. were made. Fine hair cracks appeared at the bottom over the the middle third during the repetition, and a few outside of the load points. The beam finally failed by compression in the upper face of the beam at a load of 7500 lb. It seems possible that the strength of the concrete may have been affected by the repetition of stress, although it is more likely that the test is an example of the effect of poor concrete.

These tests throw light upon the phenomena of repetitive loading and show the need of further investigation in this direction, but they are not at all conclusive. The manner of failure in general is the same as may be expected with beams of the same reinforcement and same quality of concrete loaded progressively to final failure. Whether the maximum load carried in the case of the repetitive loading is less than would have been the case with progressive loading is not known. There are some indications that the maximum load was less than it would have been without repetition.

The increase in the deflection of the beams with repetition of the load is quite apparent. Much of this increase is due to the increased amount of shortening of the concrete in the compression side of the beam with repetition. A part is due to the breaking of the concrete in tension and the transferring of the tensile stress once taken by the concrete to the steel itself. This accounts for part of the set in the deflection curve upon the release of the load. Part of the set must be due to the concrete in the lower fiber not meshing, so to speak, when the load is released after numerous fine cracks have appeared. For this reason some tension remains in the steel reinforcement after the load is taken off. It seems evident that upon the removal of the load the beam does not regain its original shape and a section which was plane before bending will not be plane upon release of the load. The plastic nature of the concrete on the compression side gives a set, and the concrete on the tension side is unable to return to its original position; the two act together to cause the fibers not to return to the original plane section. These several causes operate together to produce the permanent deflection or set.

The load applied in the cases of the leaner concretes was 67% to 85% of the maximum load which the beam finally held. In the case of the better concrete, the repeated load was 50% to 60% of the maximum load. The effect of the quality of the concrete is seen in the manner of failure.

This topic is one of such importance that it merits fuller investigation. The few tests which have been made indicate that the deflection and the deformations increase with repetition. It seems quite probable that the breaking load under a number of repetitions will be smaller than under a single load. It seems to be true also that the amount of reinforcement for which the elastic tensile strength of the steel used for reinforcement may be considered to balance the compressive strength in the concrete of the beam (which the writer calls "the balanced reinforcement") should be taken at a lower percentage in beams subjected to a repetition of load than is found necessary in the case of beams tested by means of a gradually applied load.

The term "balanced reinforcement" referred to above is a convenient term for general use. It should be taken to mean that amount of reinforcement for which the allowable stress in the steel and the allowable stress in the concrete both exist at the same time. The factors of safety for the two materials will not be the same. The determination of the balanced reinforcement for given conditions of materials, fabrication, and use is a matter involving calculation and experimentation, but in any event the judgment of the designer must enter into the choice of the amount.

14. *Diagonal Tension Failures.*—As shown in Bulletin No. 4 (pages 20, 21, and 26), certain secondary stresses or web stresses exist in the concrete of a reinforced concrete beam in addition to the horizontal or longitudinal tensile and compressive stresses which are always considered in the analysis. Strictly speaking, the shearing stresses developed under ordinary conditions are relatively light, and the actual shearing strength of concrete is considerably greater than the shearing stress which exists in ordinary beams at the time of failure. It is quite common, however, to use the term "shearing failure" as a name for a class of failures in the web of a beam, but it must not be understood from this use of terms that the failure necessarily involves actual failure by shear. Generally speaking, such failures are due to the inability of the concrete to resist the tensile stresses developed in the web in a diagonal direction, and the term "diagonal tension failure" is a much more appropriate name for this form of failure. It is a principle of mechanics that where shearing stresses exist tensile and compressive stresses are set up at an angle with the direction of the shearing stresses. If longitudinal tensile stress also exists in the concrete, the diagonal tensile stress induced by the combination of these with the shear is even higher than that due to shear alone. If v represents the horizontal and vertical unit-stress at any point in the web of a beam and s the horizontal tensile unit-stress existing in the concrete at the same

point, then, as shown in Bulletin No. 4, the formula for the maximum diagonal tensile unit-stress is

$$t = \tfrac{1}{2}s + \sqrt{\tfrac{1}{4}s^2 + v^2} \quad \ldots\ldots\ldots\ldots(19)$$

If there is no longitudinal tension in the concrete, this formula reduces to

$$t = v \quad \ldots\ldots\ldots\ldots(20)$$

and maximum diagonal tension makes an angle of 45° with the horizontal and is equal in intensity to the vertical shearing stress.

It is evident then that the amount of this diagonal tension is dependent upon both the shearing stress and the longitudinal tensile stress in the concrete at the point considered. The amount of longitudinal tension is not easy to determine and hence the actual amount of the diagonal tensile stress is uncertain. The best method for ordinary computation seems to be to compute the vertical shearing unit-stress and make all calculations upon the basis of this value. The value of the vertical shearing unit-stress, where the longitudinal reinforcement is straight (not bent up or inclined), may be computed from the formula given in Bulletin No. 4,

$$v = \frac{V}{bd'}, \quad \ldots\ldots\ldots\ldots(18)$$

where V is the total external vertical shear at the section considered, b is the breadth of the beam, d' is the distance from the center of the steel to the center of the compressive stresses. For beams with 1% reinforcement d' is about 0.86 d, d being the distance from the center of the steel to the upper face of the beam.

The value of v thus calculated for beams which fail by diagonal tension ranges from one-half to one-third of the tensile strength of the concrete. Diagonal tension failures are frequently characterized by sudden failures without much warning, as is the case in the failure of plain concrete beams. A variation from this gives a slower failure, part of the shear being carried through the reinforcing bars, and the ultimate failure involving the splitting and stripping of the bars from the beam above. When the reinforcing bars are bent up or inclined toward the ends of the beams the distribution of the vertical shear is different from that just outlined and the analysis is more complex. However, for purposes of comparison, the use of equation (18) is advantageous, and the values of v given in the tables for beams with bars inclined are calculated by this formula, using d as 10 in., though the amounts as calculated do not represent the actual vertical shear.

Forty test beams were made with a view of studying diagonal tension failure. To make a sufficient variety of conditions the concrete was varied from a fairly rich mixture to a very lean concrete. The first of the 1-3-6 concrete beams numbered up to 13 were very poorly made, as described under "6. Making of the Beams", and the general appearance of the concrete and its action during the tests go to show that the concrete was of a very inferior quality. The beams were made with the same depth, and their length was varied to give a variable relation between depth and span.

Table 10 gives the results of beams having the reinforcing bars horizontal and in which failure occurred by diagonal tension. The beams are grouped according to the quality of the concrete.

TABLE 10.

DIAGONAL TENSION FAILURES.

BARS HORIZONTAL.

All beams loaded at one-third points.

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v=\frac{V}{bd'}$	Manner of Failure
			1–5–10 Concrete.			
62	12	1.10	.60	8850	74	Diagonal tension.
63	12	1.10	.48	7360	63	Diagonal tension.
66	12	1.25	.71	8000	69	Diagonal tension.
67	12	1.25	.57	7850	68	Diagonal tension.
71	12	1.65	.59	7600	67	Diagonal tension.
Av.	......	....	.....	....	68	
			1–4–7½ Concrete.			
30	12	0.98	.71	5900	53	Diagonal tension.

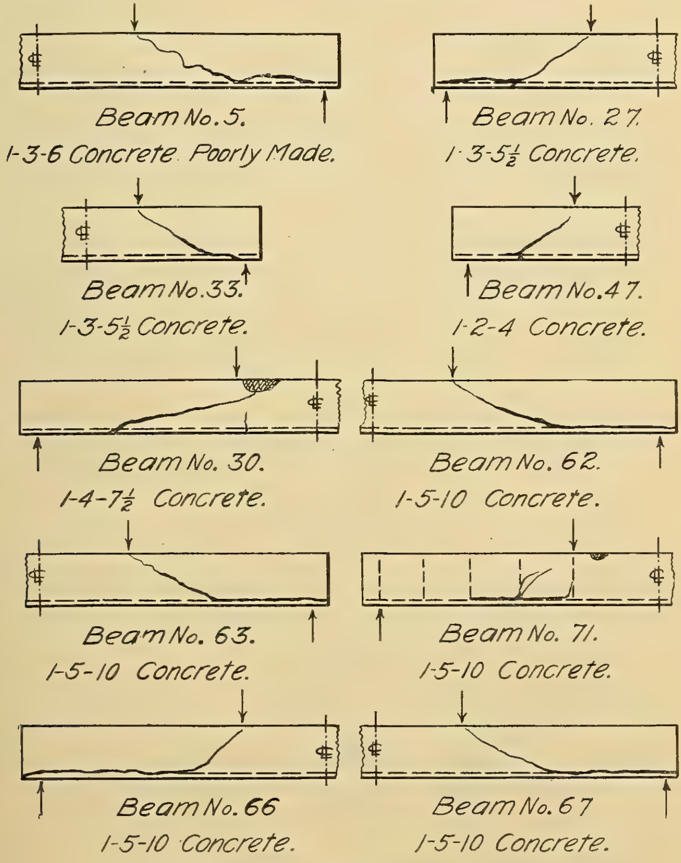
TABLE 10—*Concluded*

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v = \frac{V}{bd'}$	Manner of Failure
			1–3–6 Concrete.			
3	12	0.98	.48	7380	64	Diagonal tension.
5	12	0.98	.44	6200	55	Diagonal tension.
6	10	0.98	.47	7050	59	Diagonal tension.
8	10	2.21	.56	8470	73	Diagonal tension.
9	10	2.21	.64	9150	79	Diagonal tension.
15	8	0.98	.52	13000	107	Diagonal tension.
16	8	0.98	.41	12420	97	Diagonal tension.
18	6	0.98	.40	13000	101	Diagonal tension.
19	6	0.98	.54	12760	99	Diagonal tension.
Av.	......	....	.....	....	81	
			1–3–5½ Concrete.			
11	12	0.98	.52	10000	83	Diagonal tension.
29	12	0.98	.55	10000	83	Diagonal tension.
68	12	1.65	.57	14220	115	Diagonal tension.
23	10	2.21	.61	13830	115	Diagonal tension.
27	8	2.21	.67	11000	92	Diagonal tension.
34	8	2.21	.76	11230	94	Diagonal tension.
37	8	2.21	.70	15140	114	Diagonal tension.
25	6	0.98	.56	15430	118	Diagonal tension.
33	6	0.98	.57	15670	120	Diagonal tension.
46	6	0.98	.34	13700	106	Diagonal tension.
49	6	0.98	.49	19000	144	Diagonal tension.
Av.	......	....	......	....	108	
			1–2–4 Concrete.			
47	6	0.98	.46	18800	142	Diagonal tension.
48	6	0.98	.38	17940	136	Diagonal tension.
Av.	......	....	......	....	139	

The value of v, calculated by equation (18), offers a means of comparison of the resistance of the concrete to failure by diagonal tension. The effect of lean concrete and of poorly made concrete is quite evident. The results are instructive. The low values for the poor concrete may be helpful as a warning against assuming high web stresses for beams in which the concrete may not be well made.

The range of the values of the vertical shearing unit-stress v found in these tests with beams having the reinforcing rods in a horizontal position may be summarized as follows, the results being obtained with a single application of the load on beams about 60 days old:

1-2-4 concrete	136 to 142 lb. per sq. in.	Av. 139 lb. per sq. in.
1-3-6 concrete	92 to 115 lb. per sq. in.	Av. 99 lb. per sq. in.
1-3-6 poorly made concrete	55 to 83 lb. per sq. in.	Av. 69 lb. per sq. in.
1-5-10 concrete	63 to 74 lb. per sq. in.	Av. 68 lb. per sq. in.



Beam No. 5. 1-3-6 Concrete Poorly Made.

Beam No. 27. 1-3-5½ Concrete.

Beam No. 33. 1-3-5½ Concrete.

Beam No. 47. 1-2-4 Concrete.

Beam No. 30. 1-4-7½ Concrete.

Beam No. 62. 1-5-10 Concrete.

Beam No. 63. 1-5-10 Concrete.

Beam No. 71. 1-5-10 Concrete.

Beam No. 66 1-5-10 Concrete.

Beam No. 67 1-5-10 Concrete.

FIG. 3. SKETCHES SHOWING BEAMS AFTER FAILURE.

The one beam of 1-4-7½ concrete which failed in this way gave 43 lb. per sq. in. This beam was subjected to repetitive loading. These results show the importance of using a rich concrete in the web of reinforced concrete beams which are subjected to any considerable amount of diagonal tension when there is no metallic web reinforcement or when the web reinforcement is not effective. It is probable that not enough attention has been given to this element in the design of short and deep beams.

Fig. 3 gives sketches showing the cracks which were observed in these beams. The sketches represent the position of the cracks after the failure of the beam, or after the load had reached a maximum, and do not indicate the position or extent of the cracks within the maximum load. Fig. 4 and 5 are reproduced from photographs and give the appearance of beams after failure. Generally speaking, the crack when first observed, extended from the bottom of the beam to the steel reinforcement and from the steel reinforcement diagonally a short distance toward a load point, although in some cases the diagonal crack was observed before the vertical crack was visible. Sometimes this diagonal crack was observed before the maximum load was reached and sometimes not until the maximum load had been passed, or even until after the beam had failed quite suddenly. In some of the beams the diagonal crack was seen to extend forward slowly toward the load point before or just after the maximum load was reached and then a horizontal crack grew along the level of the top of the reinforcing bars toward the support. The phenomena of final failure were frequently connected with the slipping of the reinforcing bars or with the stripping of these bars from the concrete above as was described in Bulletin No. 4 for a former series of tests. The slipping of the bars which occurred was not observed until the maximum load had been passed, and generally in these cases the crack also extended along the bars. It seemed evident to the observers that this slip did not occur before the maximum load was applied and before the existence of the diagonal crack had materially modified the conditions in the beam. Beams No. 66 and 67 (See Fig. 3 and 5) throw some light upon this matter. They were made for this purpose with very lean concrete and reinforced with corrugated bars. The condition of the beam at failure showed that the horizontal crack was due to vertical tension and that horizontal

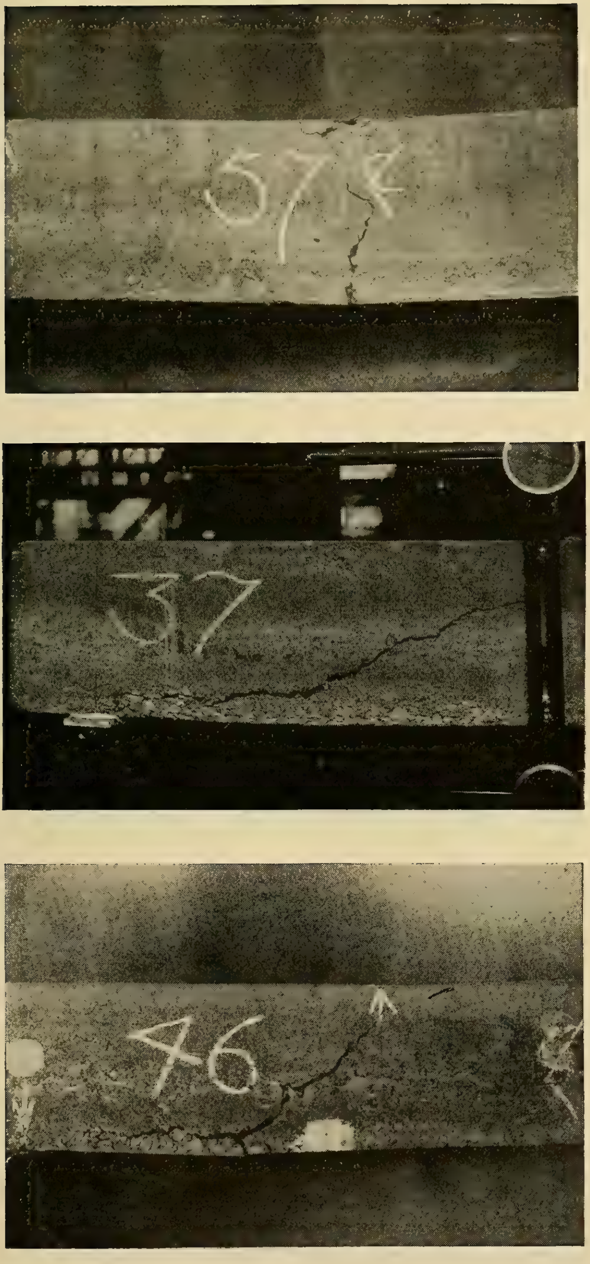


FIG. 4. VIEWS SHOWING BEAMS AFTER FAILURE.

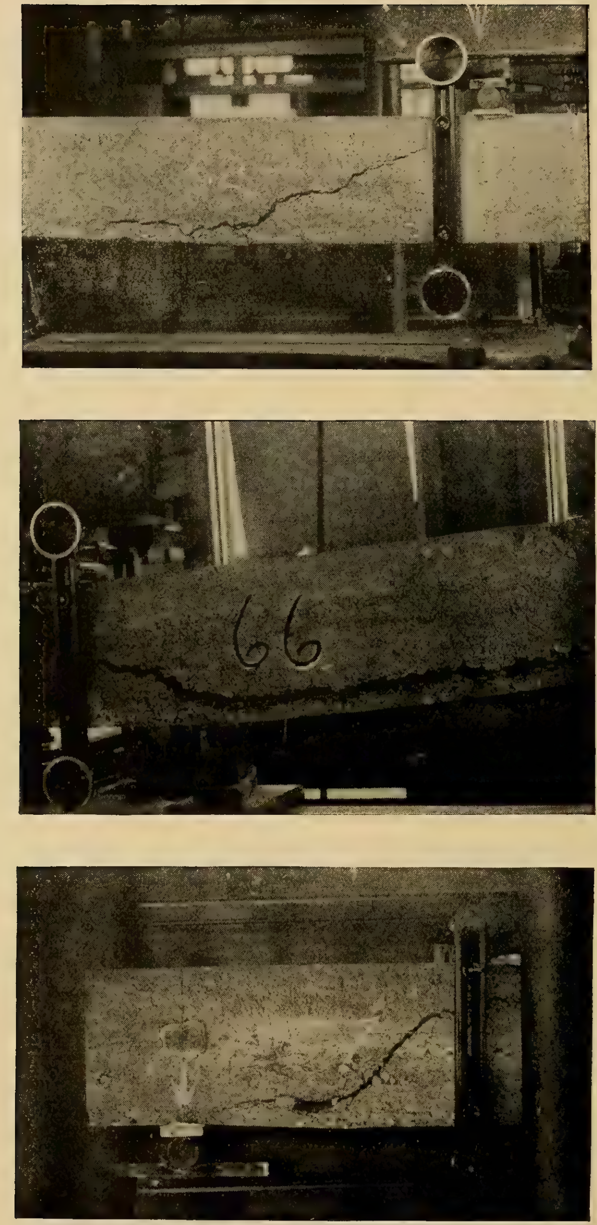


FIG. 5. VIEWS SHOWING BEAMS AFTER FAILURE.

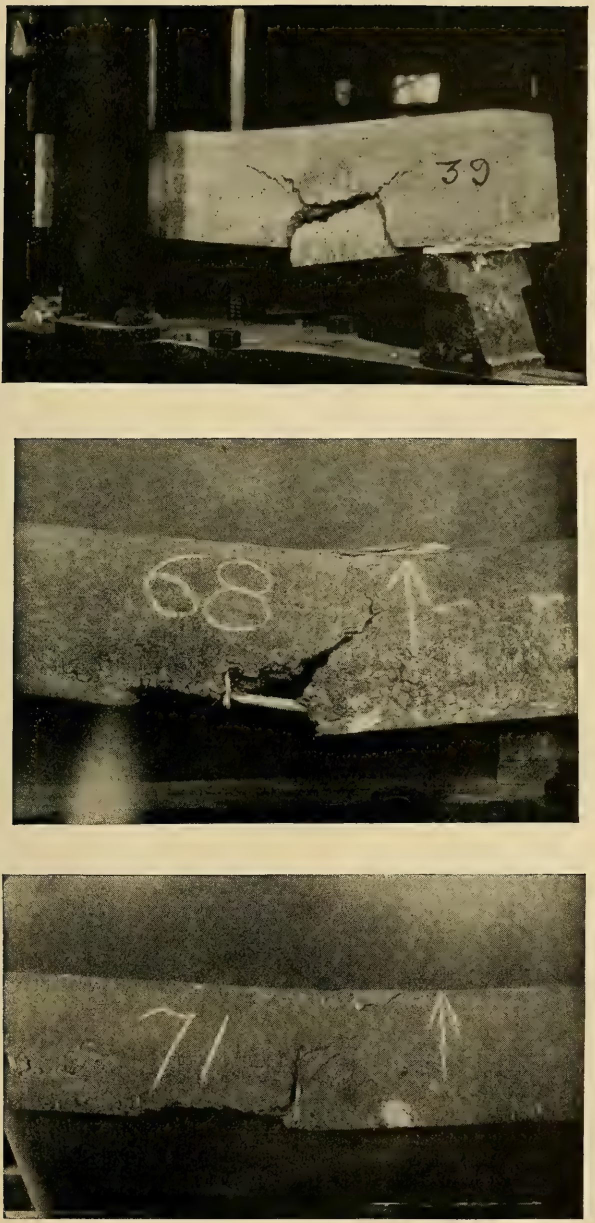


Fig. 7. Views Showing Beams after Failure.

shear or slip did not take place until after this crack had been formed. The indentations in the concrete formed by the corrugations of the bars were left in perfect condition and there was no crushing or tearing at the edges of these indentations. The bar had simply been pulled down and out of the place in which it had rested. Comparisons with the results of beams made with the same concrete and with smooth steel (Beams No. 62, 63, and 71) show values almost identical and go to indicate that slipping had no part in the critical failure of the beams made up with smooth bars. In all the cases where slipping of the bar took place the action extended progressively from the diagonal

TABLE 11.

DIAGONAL TENSION FAILURES.

BARS BENT UP.

All beams loaded at one-third points.

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v = \frac{V}{bd'}$	Manner of Failure
			1-3-6 Concrete.			
7	10	0.98	.44	8370	69	Diagonal tension.
13	10	2.21	.75	9320	80	Diagonal tension.
14	10	2.21	.66	10560	89	Diagonal tension.
17	8	0.98	.46	9150	74	Diagonal tension.
20	6	0.98	.48	11620	90	Diagonal tension.
21	6	0.98	.48	16500	126	Diagonal tension.
22	6	0.98	.49	16760	128	Diagonal tension.
Av.	......	....	....	....	95	
			1-3-5½ Concrete.			
36	10	0.98	.58	7910	64	Diagonal tension
38	8	2.21	.70	11920	99	Diagonal tension
39	8	2.21	.73	14000	115	Diagonal tension
Av.	.....	....	......	....	93	

crack toward the load point, though in some cases this action was quite sudden. From all the information available it seems to be evident that, whatever slip of the bars may have taken place, the slipping did not exist before the time of the maximum load and resulted from the changed conditions incident to the formation of the diagonal crack. It therefore seems evident that the failure of these beams should be credited to diagonal tension.

Table 11 gives results of beams with reinforcing bars bent up, or inclined in the outer thirds of the length of the beam. The values of v are calculated by equation (18) and with d as 10 in., and hence do not give the actual amount of the shear. The amount and position of this bending have been described under "5. Test Beams". Fig. 6 gives the sketches of the appearance of the crack after final failure or after the maximum load had been passed. The view of Beam No. 39, shown in Fig. 7, is from a photograph of the beam after failure. Generally speaking, the vertical portion of the crack from the bottom of the beam to the reinforcement formed first, and was due to the failure of the concrete in tension. The diagonal crack then grew toward the load point, generally forming before the maximum load was reached, and the growth of the crack along the reinforcing bar generally

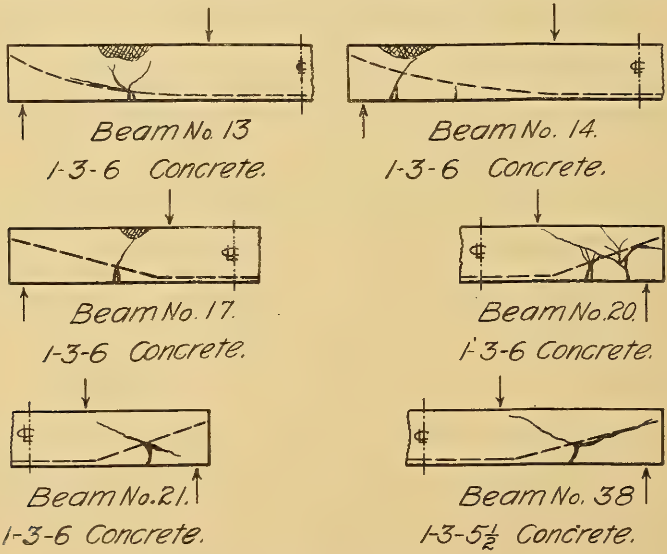


FIG. 6. SKETCHES SHOWING BEAMS AFTER FAILURE.

followed. It was expected that this method of bending up bars would give a higher value for the vertical shear as calculated by equation (18) in beams failing by diagonal tension than that found in beams with the bars horizontal, but in this the results were disappointing. In a few beams the values ran higher. Comparing Beam No. 21 with Beam No. 22 it will be seen that no difference was observed whether part of the bars were bent up and part left horizontal, or all were bent up. The values of v for these beams were among the highest found with this quality of concrete. In the phenomena of failure it appeared that the element of slip was present, though it is not known that this slip occurred before failure. Calculations indicate that the bond stress developed at the end of the bars must have been considerable. It should be noted that none of the bars were anchored at the ends.

Table 12 gives the beams in which failure occurred by tension in the steel or compression of the concrete. It will be seen that the dimensions of these beams were such that the diagonal tension developed (as measured by the vertical shear) at the time of failure was less than was found with beams which failed in diagonal tension and hence that the strength of the concrete in diagonal tension had not been reached. Beam No. 35 which failed in compression, was of 1-4-7½ concrete, a very lean mixture. Beam No. 10 was one of the beams with poorly made concrete and in this case the inferior quality of the concrete was especially noticeable.

In Beams No. 68, 69, 70, 71, 72, and 73, U-shaped stirrups of ½-in. mild-steel round rods were placed in a vertical position and enveloped the horizontal reinforcing bars. The longitudinal spacing was inadvertently made 12 in. Beam No. 69 failed by tension in the steel at a value of v of 129 lb. per sq. in., which is below the resistances developed in the beams of 1-2-4 concrete which failed in diagonal tension, and the efficiency of the stirrups was not determined. Beams No. 72 and 73 also failed by tension in the steel at values of v below what was found in beams of 1-3-5½ concrete of the same quality which failed in diagonal tension. Beam No. 70 failed by compression of the concrete, but a diagonal crack had formed, before the maximum load was applied, at a value of v which is high for 1-3-5½ concrete, and the stirrups seemed to be effective in preventing sudden failure after the

TABLE 12.

MISCELLANEOUS FAILURES.

All beams loaded at one-third points.

Beam No.	Span ft.	Per cent Reinforcement	k	Maximum Applied Load lb.	Vertical Shearing Stress lb. per sq. in. $v = \frac{V}{bd'}$	Manner of Failure
1–4–7½ Concrete.						
35	12	0.98	.68	7500	64	Compression.
1–3–6 Concrete.						
4	12	0.98	.48	9360	78	Tension
10	12	0.98	.64	9430	77	Compression.
12	10	2.21	.79	10200	89	Compression
Av.	......	....	......	....	81	
1–3–5½ Concrete						
32	12	0.98	.57	9800	81	Tension.
74	10	0.98	.52	11950	95	Tension.
26	8	0.98	.51	14030	109	Tension.
Av.	......	....	......	....	95	
1–2–4 Concrete.						
28	12	0.98	.36	9900	82	Tension.
31	12	0.98	.48	9400	79	Tension.
Av.	......	....	......	....	80	

maximum load was passed. Beam No. 68 (Fig. 7) failed in diagonal tension at a value of v of 115 lb. per sq. in. At a load of 13 000 lb. the diagonal crack extended 7 in., and at the maximum load, 14 220 lb., it extended to a point under the load point and was $\frac{1}{32}$ in. wide. The load fell off very slowly, and the stirrups prevented sudden failure. Beam No. 71 (Fig. 7) failed by diagonal tension at a value of v nearly the same as the companion beams of 1-5-10 concrete which did not have stirrups. The load dropped off rapidly after the maximum load was reached and the stirrups seemed to have little effect. It is clear that the spacing of the stirrups in these beams caused them to be inefficient, the distance apart being too great, and besides, the stirrups were not properly placed in the beam. Further tests are now in progress, in which the dimensions of the beam and the size and spacing of the stirrups are expected to bring out the effectiveness of this method of metallic web reinforcement.

15. *Summary.*—The following summary of parts of the foregoing discussion is given:

1. For beams proportioned to give failure by tension in the steel reinforcement (i. e., when neither compression of concrete nor diagonal tension causes failure), those made with the richer concrete carried higher loads. The beams with 1-2-4 concrete carried loads greater by, say, 10%, than those with 1-3-5½ concrete.

2. In beams which failed by tension in the steel, the resisting moment developed was found to be about the same for loads applied at two points more or less far apart and at eight points (approaching a uniform load), thus confirming the general applicability of the ordinary beam theory to simple beams without end restraint or horizontal restraint for any of the ordinary methods of loading. For center loading the resisting moment developed ran 10% higher and in former tests even greater. This excess indicates a different distribution of stresses for center loading from that assumed in the ordinary beam theory.

3. The tests with repetitive loading are not conclusive and show the need of further investigation in this direction. The manner of failure in general was the same as may be expected with beams of the same reinforcement and same quality of concrete loaded progressively to final failure. Whether the maximum load carried in the case of this repetitive loading is less than would have been the case with progressive loading is not known, but there are some indications that the maximum load was less than it would have been without repetition. The in-

crease in the deflections of the beam with repetition of the load was marked, and the set in the beam was considerable. It should be remembered that the repetitive load was a considerable proportion of the maximum load finally applied and much higher than ordinary working loads. It seems evident that the amount of reinforcement for which the elastic tensile strength of the steel may be considered to balance the compressive strength of the concrete, (conveniently called the "balanced reinforcement"), should be taken at a lower percentage in beams subjected to a repetition of load than is found necessary in the case of beams tested by means of a gradually applied load, and that for the ordinary conditions of fabrication and use the "balanced reinforcement" selected should be much less than that determined by test beams.

4. The manner of failure depends not only upon the relative dimensions of depth and length of beam and the amount of reinforcement, but also upon the richness and strength of the concrete.

5. The loads carried by beams failing by diagonal tension depended both upon the richness of the concrete and upon its quality as influenced by the methods of mixing and storing. Poorly made concrete gave a vertical shearing stress averaging 71 lb. per sq. in. as compared with 99 lb. per sq. in. for well made concrete of the same mixture. The value, 138 lb. per sq. in., for the 1-2-4 concrete shows the advantage of the richer mixture.

6. Failure by diagonal tension generally occurs without warning, resembling somewhat in this respect the failure of unreinforced concrete beams. On account of the variability of concrete and its unreliability in resisting tensile stresses, relatively low diagonal tensile stresses (high factor of safety), as measured by the vertical shearing stresses, should be specified, unless there is effective metallic web reinforcement. The values allowed by many building ordinances seem too high to secure safety under the condition of ordinary building operations. Short, deep beams and beams restrained at the ends require that special attention be given to web stresses.

7. Slipping of bars and stripping of bars may accompany final failure of beams which fail by diagonal tension. It appears that slipping or stripping did not take place in the beams having the reinforcing bars horizontal until after the maximum load was

reached and the presence of diagonal cracks had modified the distribution of stresses. At these maximum loads the calculated bond resistance developed was low as compared with the bond strength of the steel and concrete. The beams reinforced with deformed bars carried no higher loads than those with plain bars. The beams with bars bent up or inclined toward the ends gave quite variable results, but in general the values were even lower than those with the bars in a horizontal position. There is some probability that slipping occurred in these tests at or before the maximum load and that anchoring the ends of the bars would have been beneficial. The results for the beams having vertical stirrups showed that the stirrups as used were not efficient in taking web stresses.

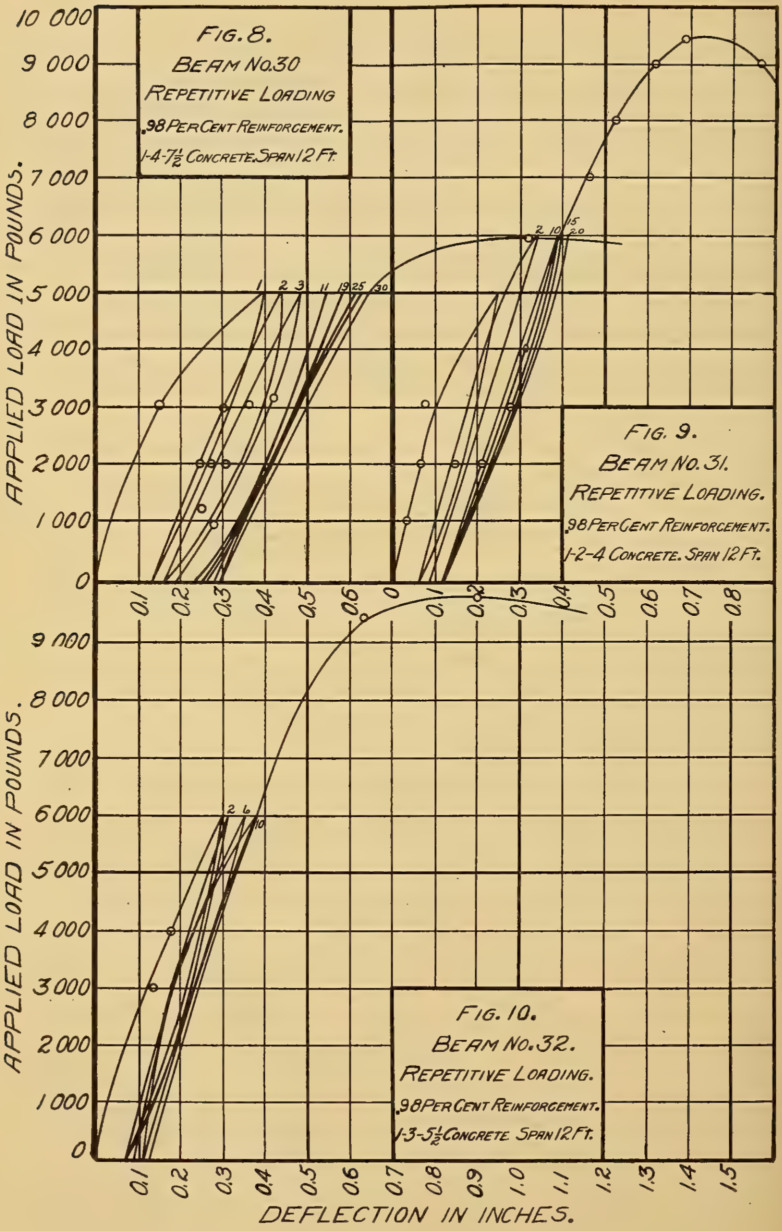


FIG. 8.
BEAM NO. 30
REPETITIVE LOADING
.98 PER CENT REINFORCEMENT.
1-4-7½ CONCRETE. SPAN 12 FT.

FIG. 9.
BEAM NO. 31.
REPETITIVE LOADING.
.98 PER CENT REINFORCEMENT.
1-2-4 CONCRETE. SPAN 12 FT.

FIG. 10.
BEAM NO. 32.
REPETITIVE LOADING.
.98 PER CENT REINFORCEMENT.
1-3-5½ CONCRETE SPAN 12 FT.

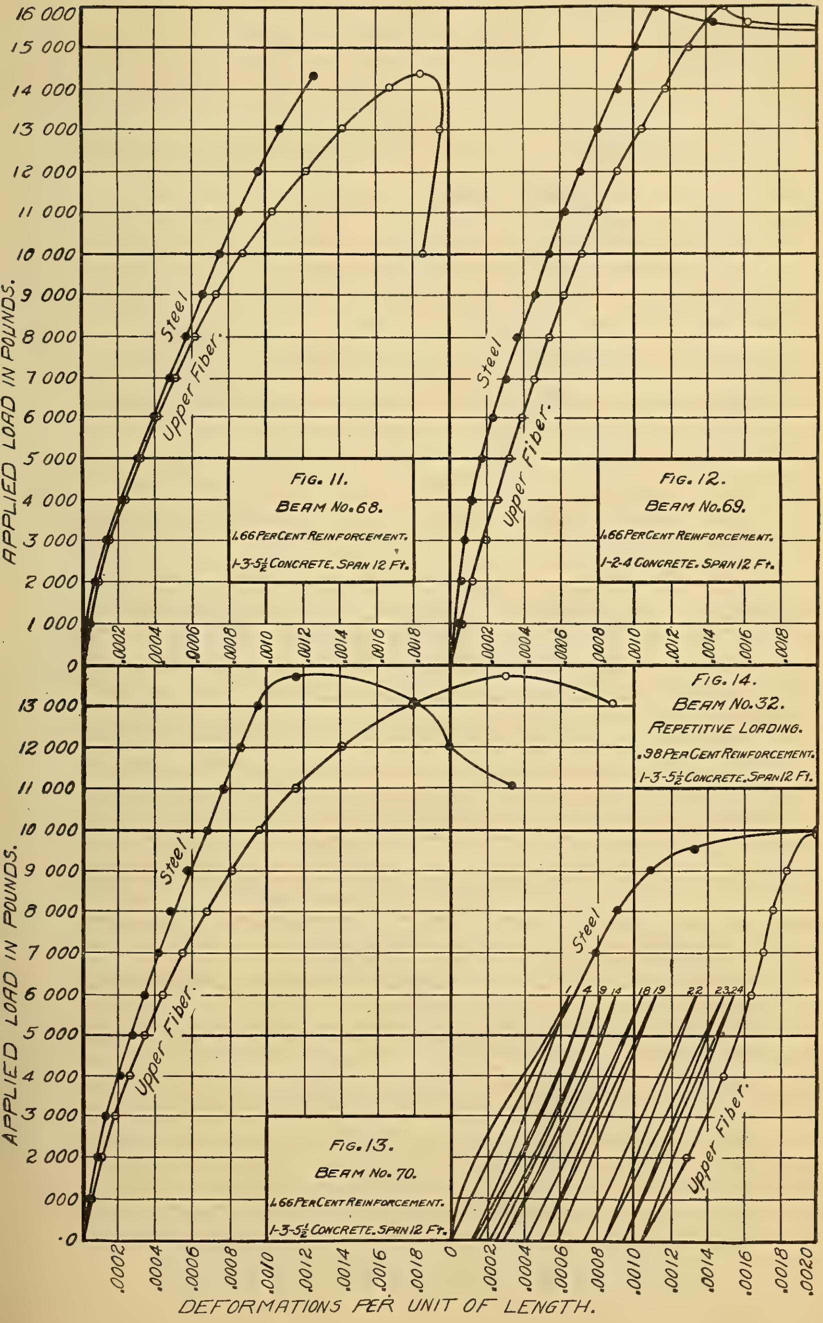


Fig. 11.
Beam No. 68.
1.66 Per Cent Reinforcement.
1-3-5½ Concrete. Span 12 Ft.
Fig. 12.
Beam No. 69.
1.66 Per Cent Reinforcement.
1-2-4 Concrete. Span 12 Ft.
Fig. 13.
Beam No. 70.
1.66 Per Cent Reinforcement.
1-3-5½ Concrete. Span 12 Ft.
Fig. 14.
Beam No. 32.
Repetitive Loading.
.98 Per Cent Reinforcement.
1-3-5½ Concrete. Span 12 Ft.
Steel
Upper Fiber.
1 4 9 14 18 19 22 23 24
APPLIED LOAD IN POUNDS.
16 000
15 000
14 000
13 000
12 000
11 000
10 000
9 000
8 000
7 000
6 000
5 000
4 000
3 000
2 000
1 000
0
.0002
.0004
.0006
.0008
.0010
.0012
.0014
.0016
.0018
.0020
DEFORMATIONS PER UNIT OF LENGTH.

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Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905.

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BULLETIN NO. 15

HOW TO BURN ILLINOIS COAL WITHOUT SMOKE

BY

L. P. BRECKENRIDGE

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS
PUBLISHED BY THE UNIVERSITY

THE ENGINEERING EXPERIMENT STATION was established by action of the Board of Trustees December 8, 1903. It is the purpose of the Station to carry on investigations along various lines of engineering, and to study problems of importance to professional engineers and to the manufacturing, railway, mining, constructional and industrial interests of the state.

The control of the Engineering Experiment Station is vested in the heads of the several departments of the College of Engineering. These constitute the Station Staff, and with the Director determine the character of the investigations to be undertaken. The work is carried on under the supervision of the Staff; sometimes by a Research Fellow as graduate work, sometimes by a member of the instructional force of the College of Engineering, but more frequently by an investigator belonging to the Station Corps.

The results of these investigations will be published in the form of bulletins, and will record mostly the experiments of the Station's own staff of investigators. There will also be issued from time to time in the form of circulars, compilations giving the results of the experiments of engineers, industrial works, technical institutions and governmental testing departments.

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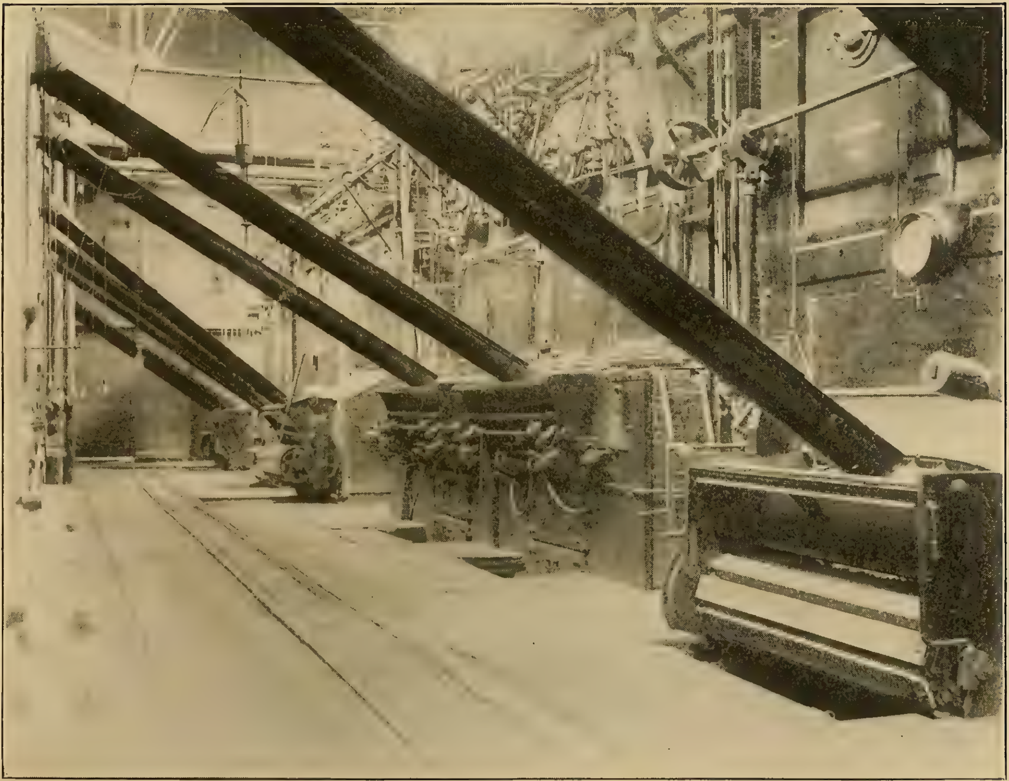

View in Central Heating Station University of Illinois

UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

ULLETIN No. 15 AUGUST 1907

HOW TO BURN ILLINOIS COAL WITHOUT SMOKE

Y L. P. BRECKENRIDGE, DIRECTOR OF THE ENGINEERING EXPERIMENT STATION

It is the intention to discuss in this paper the fundamental princi-
es that apply to smokeless furnace construction and operation and to
ustrate by means of units in actual operation several ways in which
ese principles have been satisfactorily applied. With a clear idea
the principles involved, with a knowledge of the character of
e coal to be used and the capacity at which it is desired to drive the
rnace, there should be little difficulty in designing, constructing and
erating stationary boiler furnaces which under ordinary conditions
service will operate with high economy and which will burn Illinois
al without smoke.

The writer does not profess to be acquainted with all the methods
at have been found satisfactory for the purpose of smoke prevention,
ither does he consider it advisable to describe in detail all the fur-
ces that are claimed to be capable of burning coal without smoke.
ll that can be attempted here will be a description of those furnaces
ith which the writer is personally familiar or which have been ex-
nined while in operation and have been found to give satisfactory re-
lts.

Perhaps the one thing that has most delayed progress toward suc-
ss in the smokeless operation of furnaces is the fact that in the past
much in their operation has depended upon the human element.
will doubtless be found that those plants smoke less that are
echanically operated. Such operation is by no means a sure preven-
e of smoke, as certain constructive features are now well recog-

nized as necessary even with good mechanical stokers if smokeless combustion is to be obtained.

This discussion will be confined to those settings and furnaces that have been found to be most satisfactory in burning Illinois coals without smoke. From a study of the tests of the various coals of the United States, as presented in the reports of the United States Geological Survey fuel testing plant, it seems safe to say that engineers now have sufficient information available to enable them to design boiler furnaces that will burn any coal without smoke. The progress made in this direction during the last five years is surely encouraging and it is confidently believed that the time will soon come when no power plant can offer as an excuse for a smoky chimney the plea that no appliances are available which can be depended upon for smoke prevention. The problem of smoke prevention is the problem of perfect combustion. There is no such thing as smoke consumption and this term should never be used. There is such a thing as perfect combustion and this means smokeless combustion.

THE PRINCIPLES OF SMOKELESS COMBUSTION

The subject of combustion should be familiar to all mechanical engineers. However, as a basis for a thorough understanding of the problems of smoke prevention, a brief review of the principles of combustion may prove acceptable in connection with this subject. In fact, the complete solution of the smoke problem consists in providing furnaces, so constructed and so capable of operation as to make combustion nearly perfect.

Combustion may be defined as a rapid chemical combination, resulting in heat and light. The combining elements are: (a) Oxygen, which is usually derived from atmospheric air; (b) Either carbon or

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This bulletin was prepared at the suggestion of the Conference Committee on Fuel Tests, the members of which are given below. It should be stated, however, that the Committee is in no way responsible for the opinions expressed by the writer.

H. Foster Bain, Director State Geological Survey, Urbana, representing the State Geological Survey; A. Bement, Consulting Engineer, Chicago, the Western Society of Engineers; Edwin H. Cheney, President Fuel Engineering Co., Chicago, the Building Managers' Association of Chicago; F. H. Clark, Supt. Motive Power Burlington Road, C. B. & Q. Ry., Chicago, the Western Railway Club; Adolph Mueller, President H. Mueller Mfg. Co., Decatur, Ill., the Illinois Manufacturers' Association; Carl Scholz, President Coal Valley Mining Co., Chicago, the Illinois Coal Operators' Association; Wm. L. Abbott, Chief Operating Engineer Chicago Edison Co., Chicago, the Board of Trustees, University of Illinois; L. P. Breckenridge, Director Engineering Experiment Station, University of Illinois, Urbana, Ill.

ydrogen, or a compound of the two. Sulphur sometimes appears ith carbon and hydrogen, and also combines with oxygen. The subance that is formed by the chemical union is called the product of mbustion; and the heat that is produced by the combustion of a unit eight (one pound) of the fuel is called the heat of combustion. This usually measured in British thermal units (B. t. u.). One B. t. u. is e amount of heat required to raise the temperature of one pound of ater one degree Fahrenheit. The following table relates to the mplete combustion of hydrogen, carbon and sulphur:

COMPLETE COMBUSTION OF THREE ELEMENTS

		PRODUCT OF COMBUSTION		
Element	Chemical Symbol	Name	Chemical Symbol	Heat of Comb'st'n B. t. u. per lb.
Hydrogen	H	Water	H_2O	62000
Carbon	C	Carbon Dioxide	CO_2	14500
Carbon	C	Carbon Monoxide	CO	4400
Sulphur	S	Sulphur Dioxide	SO_2	4000

In the complete combustion of carbon, the product of combustion, will be observed, is carbon dioxide (CO_2). Each atom of carbon ombines with two atoms of oxygen. If sufficient oxygen is not proided, it may happen that each carbon atom will combine with but one xygen atom, thus forming carbon monoxide (CO). As a result of is incomplete combustion, the heat developed is only 4400 B. t. u. he carbon monoxide may itself combine with oxygen, according to e formula

$$CO + O = CO_2$$

nd the heat developed will be the difference, 14 500 — 4400 = 10 100 . t. u. per pound of carbon in the carbon monoxide.

A knowledge of the relative weights of these elementary atoms ives a direct means of computing the amount of oxygen required for ombustion. The weights are as follows:

H	1	O	16
C	12	S	32

n the burning of hydrogen to H_2O, two atoms of hydrogen, each of eight 1 combine with one atom of oxygen, weight 16; hence he ratio of oxygen to hydrogen is 16 : 2 = 8 : 1, i.e., 8 lb. f oxygen are required for the combustion of 1 lb. of hydrogen, and there results 8 + 1 = 9 lb. of water. This combination with others may conveniently be presented in a tabular form as follows:

THE COMBUSTION OF IMPORTANT ELEMENTS WITH OXYGEN

Elements	Chemical Equation	Relative Weights	Weights pounds
Hydrogen to H_2O	$2H + O = H_2O$	$(2 \times 1) + 16 = 18$	$1 + 8 = 9$
Carbon to CO_2	$C + 2O = CO_2$	$12 + (2 \times 16) = 44$	$1 + 2\ 2/3 = 3\ 2/3$
Carbon to CO	$C + O = CO$	$12 + 16 = 28$	$1 + 1\ 1/3 = 2\ 1/3$
Sulphur to SO_2	$S + 2O = SO_2$	$32 + (2 \times 16) = 64$	$1 + 1 = 2$
Methane or Marsh Gas	$(CH_4) + 4O =$ $CO_2 + 2H_2O$	$(12 + 4) + (4 \times 16 =$ $12 + (2 \times 16) + 2\ (2 + 16)$	$1 + 4 = 2.75$ $+ 2.25$

When the oxygen needed for combustion is taken from the atmosphere, the nitrogen always present must be taken into consideration. Nitrogen takes no part in the combustion but mingles with the products of combustion, absorbs heat from them and passes away with them. Approximately, it takes 4.25 lb. of air to furnish 1 lb. of oxygen; the remaining 3.25 lb. are nitrogen. When the combustion of the different elements takes place in air the resulting relative weights are modified on account of the presence of the nitrogen. This is exhibited below.

THE COMBUSTION OF IMPORTANT ELEMENTS IN AIR

Elements	Chemical Equation	Relative Weights	Weights pounds
Carbon in Air	$C + Air =$ $CO_2 + 7.43N$*	$12 + (2 \times 4.25 \times 16) =$ $44 + 104$	$1 + 11.33 =$ $3.67 + 8.66$
Marsh Gas in Air	$CH_4 + Air =$ $CO_2 + 2H_2O + 14.86N$	$(12 + 4) + (4 \times 4.25 \times 16) =$ $44 + (2 \times 18) + 208$	$1 + 17 =$ $2.75 + 2.25 + 13$

* The atomic weight of nitrogen is 14.

It will be seen from the above table that to burn 1 lb. of carbon 11.33 lb. of air must be supplied. The 8.66 lb. of nitrogen contained in the weight of air pass away with the 3.67 lb. of carbon dioxide (CO_2) formed by the combustion. For the complete combustion of 1 lb. of marsh gas, 17 lb. of air are required. Chemical combinations of carbon and hydrogen, the so-called hydrocarbons, play an important part in the burning of coal, particularly in those coals of large volatile content. The most important of these are the following:

Hydrocarbon Element	Chemical Symbol	Heat of Combustion
Methane or Marsh Gas	CH_4	24020 B. t. u.
Ethylene or Olefiant Gas	C_2H_4	21930 B. t. u.
Acetylene	C_2H_2	21850 B. t. u.

At the ordinary temperature and atmospheric pressure, a pound of air has a volume of about 13.1 cu. ft. Using this value, the following (round numbers) are obtained for the theoretical amount of air required for the complete combustion of various fuels:

AMOUNT OF AIR REQUIRED TO BURN ONE POUND OF VARIOUS FUELS

Kind of Fuel	Carbon	Hydrogen	Sulphur	Methane
Weight of Air (lb.)	11.33	34	4.25	17
Volume of Air (cu. ft.)	150	445	55	220

The heat developed by the combustion is absorbed by the products of combustion, and as a result, the temperature of these gases rises in a marked degree. Thus when carbon is burned in air, the 14 500 B. t. u. developed should heat the 3⅔ lb. of CO_2 and 8⅔ lb. of nitrogen from an initial temperature of perhaps 60° to a calculated final temperature of about 4500°. The actual final temperature, for reasons to be explained presently, is considerably lower. The products of combustion act as a vehicle, carrying the heat developed by combustion to its final destination.

It may perhaps be profitable to picture an ideal perfect combustion, and then inquire in what ways actual combustion falls short of the ideal. The given fuel, composed of carbon, various volatile hydrocarbon gases, and perhaps sulphur, is to be burned in air. Theoretically, each atom of the fuel finds and seizes upon the number of oxygen atoms with which it will combine. Each atom will meet with two oxygen atoms at a temperature sufficiently high for ignition. They will combine, and the resulting CO_2 will pass out of the furnace, carrying with it the heat arising from the combustion; likewise with the hydrogen and sulphur atoms. No more air will be delivered than is just sufficient to furnish the exact number of oxygen atoms, and no carbon or hydrogen atoms will pass out of the furnace without finding oxygen atoms with which they can combine.

Actual combustion deviates from ideal conditions in many respects. If only the theoretical amount of air is supplied, on account of the difficulty of properly mingling the fuel and air, some of the fuel atoms will not find oxygen atoms, and will escape uncombined. Or some of the carbon may burn to carbon monoxide instead of to carbon dioxide, and the CO will escape without further combustion. It is found in practice that to insure complete combustion, an excess of air must be furnished. This excess is usually 50 per cent, and may reach 100 per cent; i.e., while only 11.3 lb. of air are required for the complete combustion of 1 lb. of carbon, it is usually necessary to furnish 18 to 24 lb. Since the heat of combustion is distributed throughout the excess of air introduced into the furnace as well as the products of combustion, the furnace temperature is lowered by the presence of the extra air.

In another important particular, the actual state of affairs is likely to be quite different from the ideal combustion outlined above. Carbon and oxygen atoms will not unite unless a certain temperature, the ignition temperature, is reached. In parts of the furnace, the temperature may fall below the ignition point because of the inrush of an excess of air, or because of cold bounding surfaces. As a result, carbon particles, even in the presence of plenty of oxygen, will refuse to burn. Let us further illustrate these principles by the use of some familiar examples.

If the reader understands why a torch smokes, why a flat wick is used in a lamp, why a circular burner is provided for the Argand or student lamp, why a chimney is provided for oil lamps, why a fish tail flame was used in early gas lighting, or why a vacuum bulb is provided for the incandescent lamp; if he understands these things, and most of us do, he will, it is believed, be able easily to see what conditions are necessary for perfect combustion in boiler furnaces.

If any fuel is to be burned without smoke it must be supplied with the correct amount of air. The torch smokes because the large round wick brings up oil, especially in the center, to which air cannot be supplied. If the air supply through the center tube of our student lamp is shut off a smoking flame results. The candle with the small wick is an advance over the candle supplied with the large wick. The flame from the flat wick has an extended surface for air supply, while the circular burner not only has maximum surface for air supply, but the air coming up through the center of the tube is heated, making it still better suited to aid combustion and burn a large amount of oil. Thus it is that we have successfully solved the problem of burning a large oil supply without smoke. If we try to increase the oil consumption and turn up our lamps too high, they smoke, and because they are in the room with us we immediately turn them down. Furnaces burning coal sometimes smoke just because they are forced too hard, and because the top of the chimney is not in our room, but in the public's pure air we do not turn them down, but let them smoke.

Air correct in amount is a necessity for complete combustion and the simple experiments which we have all been making with our oil and gas flames should be sufficient evidence to us that only when we are able to supply our furnaces with the correct amount of air shall we be able to control the smoke which it is so easy to make. How much easier our problem would be if, as in the lamp, we could see all that was taking place and could regulate all by a simple knurled brass handle.

SMOKE PRODUCTION

Having briefly outlined the essential features of perfect combustion let us turn our attention to the conditions present when the combustion is imperfect, usually resulting in smoke. The products of combustion, carbon dioxide, steam and sulphur dioxide are colorless gases. If nothing except these gases escaped from a chimney, there would be no smoke problem. Visible smoke is due to the volatile hydrocarbons which all bituminous coal contains to a greater or less extent, and which are driven off when the coal is heated. The percentage of volatile matter in coal varies widely; thus in the eastern anthracites, it may be as low as 3 per cent, while in the western lignites, it may rise as high as 50 per cent. The larger the percentage of volatile matter, the more liable is the coal to produce smoke, and the more difficult is smoke prevention.

When coal is heated in the furnace, the volatile content, consisting largely of methane and ethylene, is driven off. If the volatile gases should not enter any region of high temperature, they would simply pass out of the chimney with the products of combustion. But, as a rule, the gases are ignited, and burn, much as ordinary illuminating gas burns. The phenomena connected with the combustion of volatile gases are, however, not simple. There are grounds for supposing that a hydrocarbon, at a sufficiently high temperature, is decomposed into its elements. If oxygen is present, the hydrogen at once combines with it. The carbon particles will not combine with oxygen except under favorable conditions; if there is not a sufficient supply of air, or if it is not at a sufficiently high temperature, the carbon particles refuse to combine and are carried along with the products of combustion to be afterwards deposited as soot, or appear at the top of the chimney as smoke. This may be shown by an equation as follows:

$$C_2H_4 + 2\,O = 2H_2O + 2\,C$$

Ethylene and oxygen form water vapor and soot.

The mere explanation of the formation of black smoke suggests immediately the means that should be employed to prevent such formation. Evidently sufficient air must be furnished to burn the carbon particles liberated from the hydrocarbon gases; this air must be at a high temperature. If too great a volume of cold air is admitted to the furnace, and if at the same time the bounding surfaces of the combustion space are boiler plates or tubes of relatively low temperature, the result will be a low temperature in the space above the fuel, and it will be impossible to burn the carbon content of the volatile matter. To insure proper combustion, after the gases are driven from the coal, they should intimately mingle with sufficient air in a chamber, in which high temperature can be maintained. The heating surface of the boiler should not be permitted to come in contact with these gases until combustion has been completed.

Losses Due to Smoking Chimneys

The destruction of property or the effect upon the health of the community due to the smoke nuisance are matters upon which there is not an opportunity to dwell in this article. Both of these subjects are now matters of common every-day knowledge to the residents of our American cities. Something should be said, however, about the fuel losses due to smoking chimneys. The absence of smoke by no means indicates perfect combustion. It may simply mean excessive air dilution and this means uneconomical operation. Statements are frequently seen in the daily press to the effect that one-quarter or one-third of the fuel burned goes off in the black smoke issuing from the chimney. Such statements are very far from the truth. It is doubtful if the black carbon particles which issue from chimneys and which we call soot ever amount to one per cent of the fuel burned in any furnace. It takes but a small amount of soot to give a dense black color to smoke. If it were to save only these soot particles we could not afford expensive stoker and furnace settings. The appearance of black smoke is fortunately the signal of incomplete combustion and the losses due to this cause are many times the losses due to the carrying away of the small soot particles. This matter is well stated by that practical and clear writer, Wm. H. Booth, as follows:

"It is customary to speak of smoke and the smoke nuisance as though black smoke were the only feature of imperfect combustion that demanded a remedy. But it cannot be too strongly emphasized that the visible impurities of the waste gases from factory chimneys are the least harmful part of their constituents; and that the invisible gases, which too often escape as the result of imperfect combustion, are far more detrimental in their effects upon vegetation and upon the health of the community. These invisible gases consist of unaltered hydrocarbons and of carbon monoxide; their presence is due either to deficiency of air, or to the lack of the requisite temperature in the combustion area. Smoke is the visible sign of the presence of these deleterious gases. It is, therefore, a useful signal of something wrong in the combustion process. Smoke ought to be attacked, not only because it brings dirt and depression in its train, but because its emission is accompanied by that of gases which are directly detrimental to the health of all living things, and at the same time carry away much heat from the plant of the fuel user. Both on humanitarian and economic grounds its suppression is called for*."

If there is a deficient air supply part of the carbon atoms will not find enough oxygen atoms with which to combine and there will be a considerable part of the escaping gases leaving the chimney as carbon monoxide (CO) instead of being burned to carbon dioxide (CO_2). For each pound of carbon burned only to carbon monoxide (CO) there will be a loss of approximately 10 000 heat units and this constitutes the great source of loss so frequently referred to as the loss due to incomplete combustion. This loss may readily amount to 5 per cent of the total heat in the coal. The density of the accompanying smoke may or may not be an indication of the proportion, though the loss due to carbon monoxide in perfectly smokeless chimney gases in practice will usually not exceed 0.05 of one per cent. Smokelessness is a relatively safe indication that the total heat has been liberated. Unfortunately it gives no indication of the degree of efficiency with which the heat is being utilized. The problem from the standpoint of the operator demands smokelessness with a minimum air supply. Losses due to sensible heat in the stack gases while seldom rising higher than 32 per cent of the total heat may be as low as 10 to 12 per cent without smoke or incomplete combustion. These figures are found in the fuel test reports of the United States Geological Survey under Illinois coals. The following tabulation will serve to indicate

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SMOKE PREVENTION AND FUEL ECONOMY, Booth and Kershaw—Archibald, Constable & Co., London, 1904.

how the heat generated in a boiler furnace may be distributed when operating under poor, average and best conditions.

AN APPROXIMATE HEAT DISTRIBUTION FROM ILLINOIS COAL

	PERCENTAGE OF HEAT		
	1 Poor Condition	2 Average Condition	3 Best Condition
1. Absorbed by the boiler	50.0	65.0	75.0
2. Carried away in dry chimney gas es	24.0	16.0	10.0
3. Radiation and unaccounted for osses	15.0	12.0	10.0
4. Moisture formed by burning of hydrogen	4.0	3.5	3.0
5. Evaporating moisture in coal	2.0	2.0	1.5
6. Incomplete combustion of carbon	5.0	1.5	0.5
Total heat	100.0	100.0	100.0

The per cents given in the last column of this tabulation are seldom attained in present practice, but they are by no means impossible. They represent conditions for which we should continually strive. Losses of 20 to 25 per cent are not unusual, both with and without smoke. Too little air is wrong. Too much air is wrong. Absolutely complete combustion can be obtained using Illinois coal burned on an automatic stoker with as low as 30 per cent excess of air. The question is one of proper furnace construction to meet the requirements of the fuel coupled with a good fireman and an intelligent use of instruments which will tell him at all times the conditions of the combustion and draft.

It has been stated elsewhere, what relation should exist between the weight of coal burned and the weight and volume of air required for perfect combustion. It is sometimes more striking, however, to bring out this point, by using a larger unit than the single pound. Take for instance, a 1000 horse-power boiler plant. Many such plants would burn 5000 lb. of coal an hour; the air supplied would weigh 100 000 lb. and the volume of this air at a chimney temperature of 500 degrees Fahr. would be about 2 400 000 cu. feet. The chimney for this plant must therefore discharge nearly 105 000 lb. (52.2 tons) of gases into the atmosphere each hour; that is, for each ton of coal burned the chimney will discharge from 18 to 22 tons of gases. To keep these gases of the right color and composition for 90 per cent of the time is the problem to be solved. Chimneys will still be needed when the so-called smoke problem is solved. The popular solution consists in changing the color of the gases flowing from the chimney from black to a light gray, that is from No. 5 to No. 1 on the Ringelmann chart. At least this is all that the public and the smoke inspector will demand. But even when this problem is well solved our chimneys must continue to discharge immense volumes of heated gases, and these gases will often carry with them fine particles of ash which cannot be burned; all of which may produce something of that disagreeable haze which floats over manufacturing cities. The dense black smoke from chimneys is certainly a nuisance. It can be stopped for about 90 per cent of the time, and if this is accomplished a most wonderful improvement will be observed in the atmosphere of our cities. If smoke is not largely prevented in large cities it will be because of laxity in carrying out the provisions of the law, combined with the indifference of power plant owners, rather than because of a lack of furnaces and boiler settings which are available for successfully burning bituminous coal without smoke.

THE OBSERVATION OF SMOKE

From the point of view of the general public there are but two kinds of chimneys, those that smoke and those that do not smoke. The emission of black smoke for a short period say of three minutes of an hour is sometimes enough to leave the impression on the casual observer that the chimney smokes all the time. It is therefore important that there should be some way devised for estimating the relative blackness of smoke and some plan adopted for recording the length of time during which smoke of varying degrees of blackness is emitted from chimneys. Numerous schemes have been proposed to accomplish this purpose. One of the most scientific of these plans is that invented by Professor Ringelmann of Paris.*

"In making observations of the smoke proceeding from a chimney, four cards ruled like those in the cut, (See Fig. 1), together with a card printed in solid black and another left entirely white, are placed in a horizontal row and hung at a point about 50 feet from the observer and as nearly as convenient in line with the chimney. At this distance the lines become invisible, and the cards appear to be of different shades of gray, ranging from very light gray to almost black. The observer glances from the smoke coming from the chimney to the cards, which are numbered from 0 to 5, determines which card most nearly corresponds with the color of the smoke, and makes a record accordingly, noting the time. Observations should be made continuously during say one minute, and the estimated average density during that minute recorded, and so on, records being made once every minute. The average of all the records made during a boiler test is taken as the average figure for the smoke density during the test, and the whole of the record is plotted on cross section paper, (See Fig. 2), in order to show how the smoke varied in density from time to time. A rule by which the cards may be reproduced is given by Professor Ringelmann as follows:

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* Trans. A. S. M. E., Vol. XXI, Dec., 1899.

Card 0. All white.

Card 1. Black lines 1 mm. thick, 10 mm. apart, leaving spaces 9 mm. square.

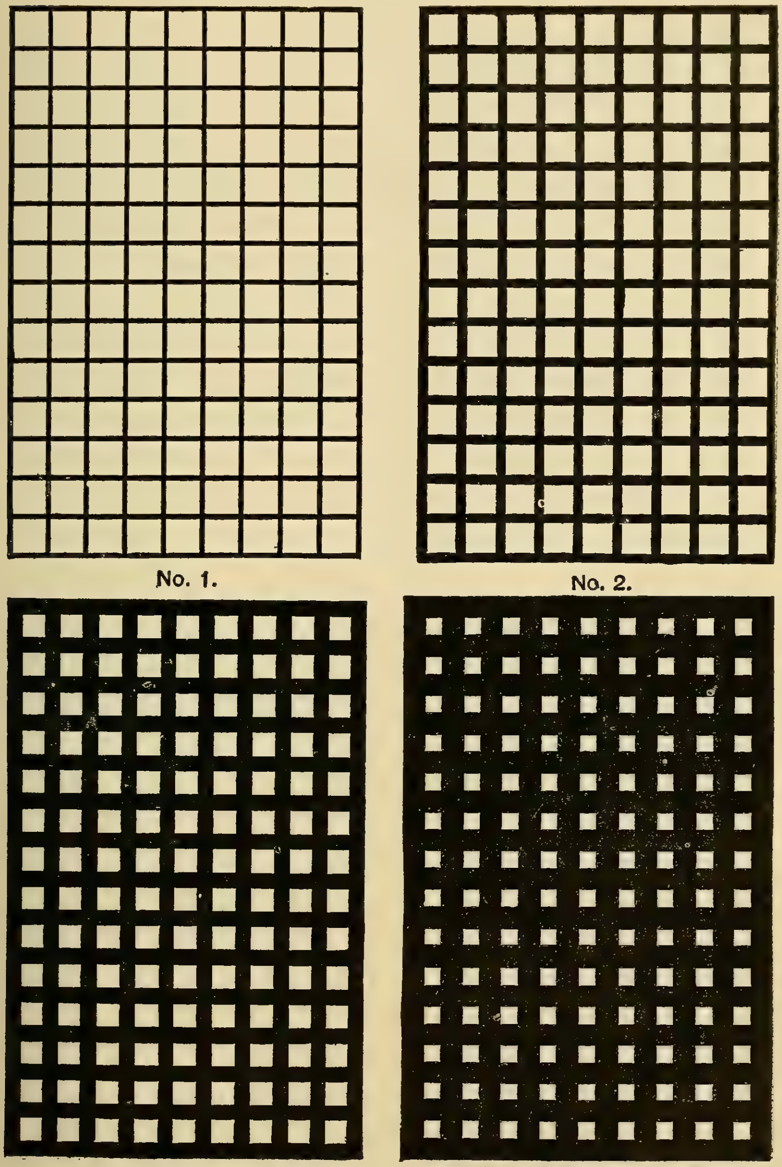
Card 2. Lines 2.3 mm. thick, spaces 7.7 mm. square.
Card 3. Lines 3.7 mm. thick, spaces 6.3 mm. square.
Card 4. Lines 5.5 mm. thick, spaces 4.5 mm. square.
Card 5. All black.

The cards as printed on page 13 are much smaller than those used by Professor Ringlemann. The thickness and spacing of the lines are in the same proportion, but reduced to one-half size."

The smoke observations reported in the government fuel tests made by the Technologic Branch of the U. S. G. S. are all based on the Ringelmann chart. This chart has, therefore, been adopted in the reports of fuel tests made by the Engineering Experiment Station and the numbers used in its bulletins refer to this scale.

An observer soon becomes skilled in taking smoke records and when several observers have been trained to the system their records are very nearly alike. In fact one soon becomes so familiar with the scale of densities that he no longer needs the actual charts for comparison, but may be trusted to take the record without the aid of the chart. An attempt has been made to illustrate smoke densities in the plate shown in Fig. 2. Many difficulties surrounded the preparation and printing of such a plate and it is inserted with some misgivings. It is believed, however, that if it should prove successful it will give some reader a better idea of the value of the scale numbers 1 to 5, used to denote smoke densities than will the pictures of the Ringelmann charts.

When a record of a smoking chimney is desired numerous ways are adopted for showing its behavior. For many records a column of the scale density numbers is sufficient, set of course opposite the inter-



No. 1.

No. 2.

No. 3.

No. 4

FIG. 1 THE RINGELMANN SCALE FOR GRADING THE DENSITY OF SMOKE

vals of time which are deemed short enough for the purpose for which the observations are taken. Where many records must be taken the work may be facilitated by using some of the simple mechanical devices which have been perfected for this purpose. These usually consist of a drum rotated by clockwork, and a marking pen movable by hand leaving a record on suitable paper that has been wrapped on the drum. The form of smoke chart record adopted in the work of the Engineering Experiment Station is shown in Fig. 3. This form needs little explanation. The scale density numbers are those at the left of the sectioned plates and these densities are approximately shown at the top under their respective numbers. It is intended that the time scale shall be chosen and placed along the zero horizontal lines. This time scale will vary according to the character of the tests being made. Graphic charts are for many purposes much preferable to any other scheme for exhibiting the continuous operation of chimneys supposed or claimed to be smokeless, and they are particularly useful when comparing several stacks for the same interval of time, or when changes made in a setting are being studied.

FURNACES AND BOILER SETTINGS SUITABLE FOR BURNING ILLINOIS COAL WITHOUT SMOKE

General Principles of Construction.—Various writers on boilers and furnaces have enunciated the principles of smoke prevention in different ways. In "Steam Boiler Economy," Wm. Kent says:

"Coal can be burned without smoke provided:

(a) The gases are distilled from the coal slowly.

(b) That the gases when distilled are brought into intimate contact with very hot air.

(c) That they are burned in a hot fire-brick chamber.

(d) That while burning they are not allowed to come in contact with comparatively cool surfaces, such as the shell or tubes of a steam boiler; this means that the gases shall have sufficient space and time in which to burn before they are allowed to come in contact with the boiler surfaces."

Mr. A. Bement, who has made a careful study of the smoke problem, suggests* some slight modifications of the Kent form of statement. He says: "Professor Kent's requirements may be modified as to two features as follows:

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*The Suppression of Industrial Smoke with Particular Reference to Steam Boilers.—Journal Western Society of Engineers. Dec., 1906.

(a) That the evolution of gas from the coal shall proceed uniformly.

(b) That the gases which are distilled uniformly from the coal shall enter a fire-brick chamber of either sufficient length to allow the flames to become entirely consumed naturally or that the chamber be provided with such auxiliary mixing and baffling devices as will cause the gases to be artificially mixed together before the exit of the chamber is reached."

Further discussing the subject, Mr. Bement presents clearly the importance of uniformity of the evolution of the volatile gases and points out that the lack of uniformity of the evolution of the gases when coal is hand-fired must be supplemented by efficient mixing devices such as fire brick piers or special baffling. The matter presented in this bulletin only corroborates the testimony presented in Mr. Bement's article, and except for the fact that possibly this publication may reach a larger number of manufacturers throughout the state than will Mr. Bement's paper, it need hardly have been presented.

Any fuel may be burned economically and without smoke if it is mixed with the proper amount of air at a proper temperature.

This is the statement of the smoke problem that the writer has sometimes presented. We have then three forms of presentation of the problem. To the fuel expert any one of these forms is sufficiently clear. It does, however, require some familiarity with the problem to understand just what these statements mean. To the owner who is trying to make his plant stop smoking or who must decide between several types of furnaces or boilers that he wishes to install, all these carefully stated formulas are often meaningless and provoking. What the owner wishes to know is what furnaces and boilers are in the market that are really suitable for burning Illinois coals without smoke. It is in many ways unfortunate that there are so many independent manufacturers of furnaces and boilers. Both furnace and boiler should preferably be sold and installed by the same company. It may not be long before this will be so. At least, the boiler companies should furnish complete plans with their boilers including the furnace. Possibly

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The Peabody Atlas—Shipping Mines and Coal Railroads in the Central Commercial District of the United States, with Chemical, Geological and Engineering Data. A. Bement, 1906. 16¾×18 in. 149 pp. An extensive and valuable work relating to the central coal fields of the United States, published by the Peabody Coal Co., Chicago. The section devoted to smokeless furnaces and smoke suppression contains many excellent illustrations which should be of great help to engineers who wish to be fully informed on this subject.

this matter might be left to the consulting engineers, but many firms buy their boilers directly from the boiler companies and follow the same setting plans that have been developed and found efficient with anthracite or other eastern coals, and often these are entirely unsuited for Illinois coals having such a high volatile content.

Much confusion has doubtless arisen and many disappointments have followed because of the fact that furnaces found entirely suitable for eastern coals have been tried further west without modification and have proved themselves failures. Some of the furnaces described in this paper would not be suitable for use with eastern coals. The use of the Dutch oven furnace, which was common before the chain grate stoker, doubtless paved the way for the various forms of automatic stokers which made use of the good points recognized in the old forms of the Dutch oven furnaces.

The history of the growth and development of the smokeless furnace could well form the subject for an engineering article. It is hoped that some one may soon make the steps in such a development a matter of record. The writer certainly makes no claims for discovery of invention in this connection. He has, however, experimented with many methods of firing, many devices for smoke prevention and many variations in furnace construction, not only in Illinois, but also in Pennsylvania and in New England.

The boiler plant at the University of Illinois at Urbana, Illinois, consists of 2000 horse-power in nine units. During the last two years this plant has been operated practically without objectionable smoke fully 90 per cent of the time. Over 200 separate boiler tests have been made in this plant to determine the economy of operation and for a study of furnace conditions. Many changes have been made in the constructive features of the furnaces and boiler baffling with the view of studying the smoke problem. Many varieties of coals have been tested for smokelessness. The character of the installation is indicated below.

Boiler No. 1. Babcock & Wilcox (Special high-pressure boiler 275 lb.), equipped with B. & W. chain grate stoker, usual vertical baffling, capacity 150 H. P. This unit can be run without smoke at capacities from 50 to 120 per cent.

Boiler No. 2. Babcock & Wilcox standard boiler (150 lb.) equipped with B. & W. chain grate stoker, usual vertical baffling, capacity 150 H. P. This unit can be run without smoke at capacities from 50 to 120 per cent.

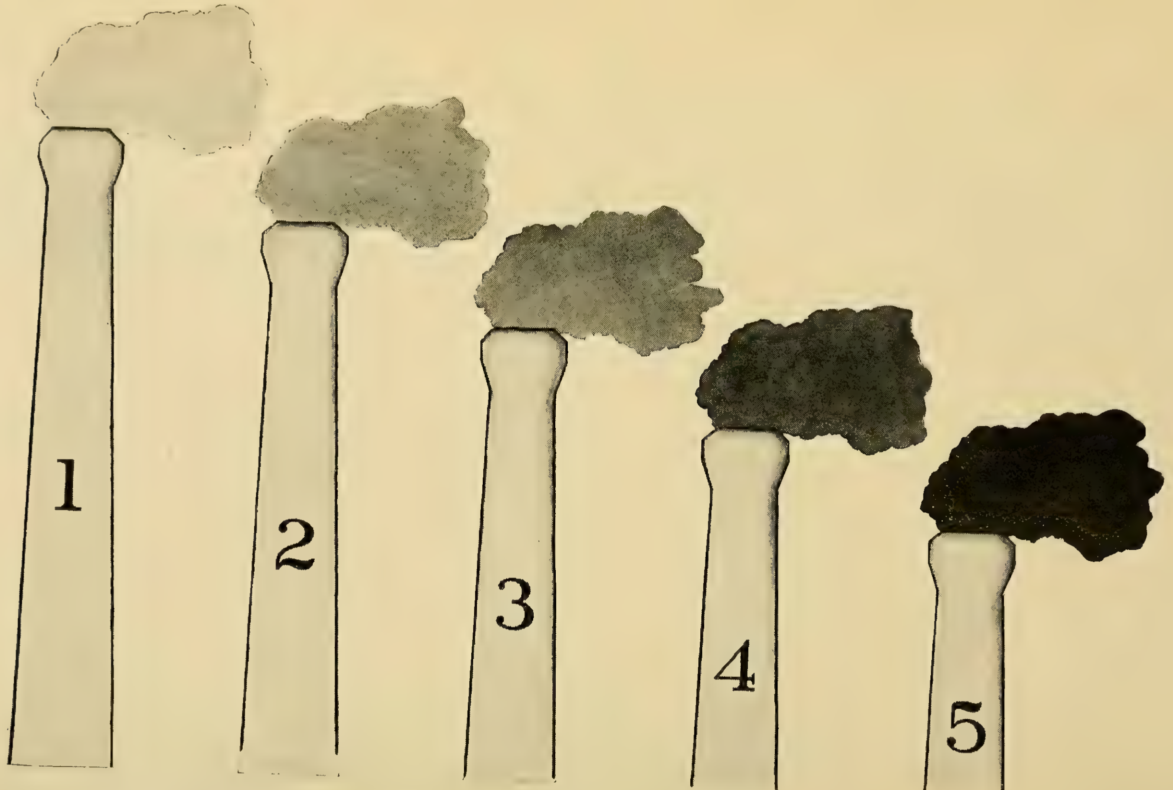

Fig. 2 Chimneys Illustrating Smoke Densities Approximating the Ringelmann Smoke Charts

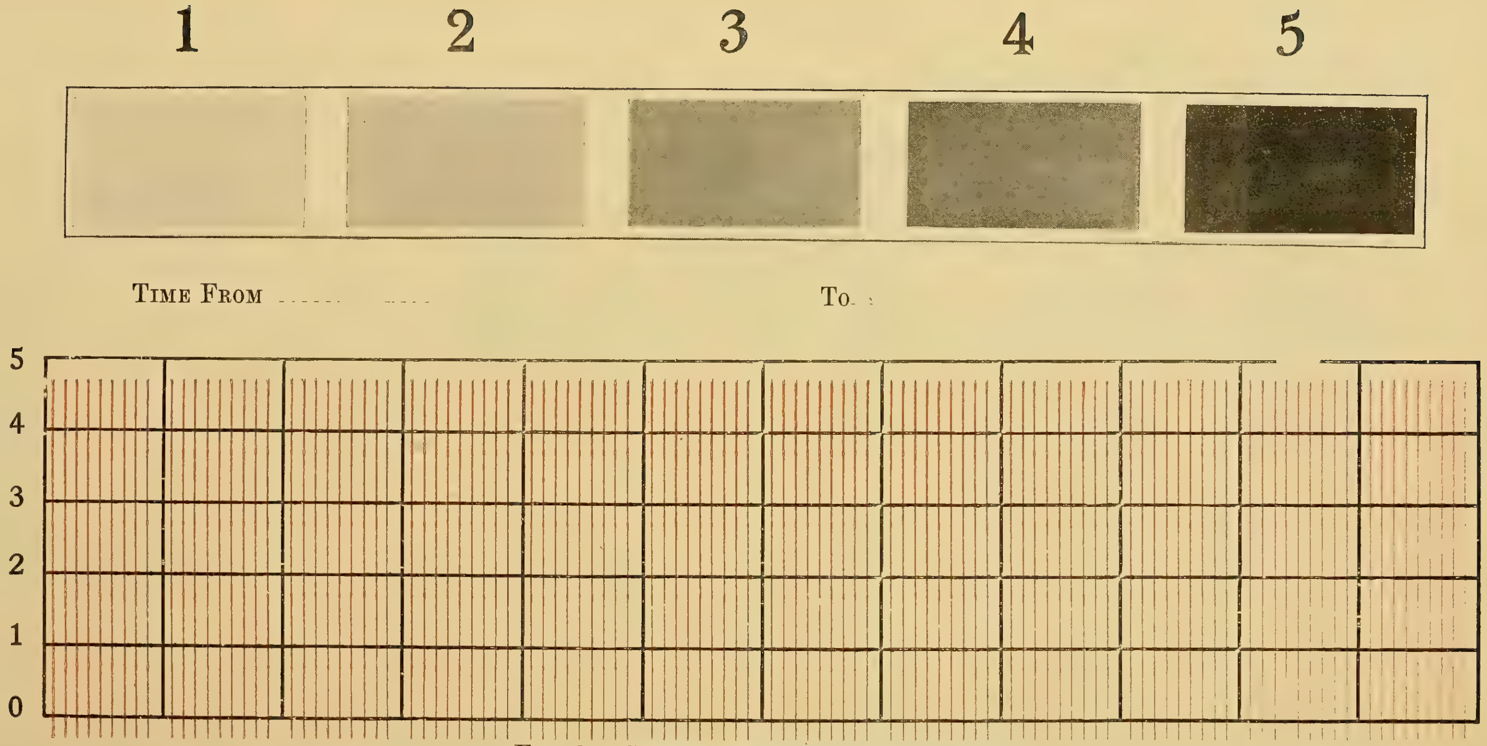

Fig. 3. Section of Smoke Chart Record

Boiler No. 3. Stirling standard boiler (150 lb.) equipped with Green chain grate stoker, usual baffling and combustion arches, capacity 260 H. P. Setting shown in Fig. 4. This unit can be run without smoke at capacities from 50 to 140 per cent.

Boiler No. 4. National water tube boiler (150 lb.), formerly equipped with Murphy furnace but now equipped with Green chain grate stoker: usual vertical baffling, capacity 250 H. P. This unit can be run without smoke at capacities from 50 to 120 per cent. When equipped with Murphy furnace, this unit was smokeless except when cleaning fires.

Boiler No. 5. Babcock & Wilcox standard boiler (150 lb.), equipped with Roney stokers, usual vertical baffling, capacity 220 H. P. Setting shown in Fig. 5. This unit when handled carefully can be run up to capacity without smoke above No. 2 on the smoke chart. It requires careful attention and at capacities above 100 per cent cannot be run without objectionable smoke. This boiler is an exact duplicate of unit No. 6.

Boiler No. 6. Babcock & Wilcox standard boiler (150 lb.), equipped with Roney stokers, special baffling making tile roof furnace. Boiler and stoker duplicate of unit No. 5. Capacity 220 H. P. Setting shown in Fig. 6. This unit can be run without smoke at capacities from 50 to 100 per cent.

Boiler No. 7. Stirling standard boiler (150 lb.) equipped with Stirling bar grate stoker, usual baffling and combustion arches, same as Boiler No. 3, capacity 260 H. P. This unit can be run without smoke at capacities from 50 to 140 per cent.

Boiler No. 8. Stirling standard boiler (150 lb.). Exact duplicate of boiler No. 7, capacity 260 H. P.

Boiler No. 9. (Engineering Experiment Station special test boiler), Heine standard boiler (150 lb.) equipped with Green chain grate, usual combustion arch, tile roof furnace, adjustable water back at bridge wall, capacity 210 H. P. Setting shown in Fig 7. This unit can be run without smoke at capacities from 50 to 140 per cent.

A study of the various units mentioned above reveals the fact that any one of the four well known types of boilers may be set over at least three well known types of automatic stokers and be operated without objectionable smoke. The opinion of the writer, doubtless held by most engineers, is that the boiler has very little to do with the smoke problem, except perhaps that some types of boilers lend themselves more easily to the necessary furnace construction, which is of the utmost importance when perfect combustion is desired.

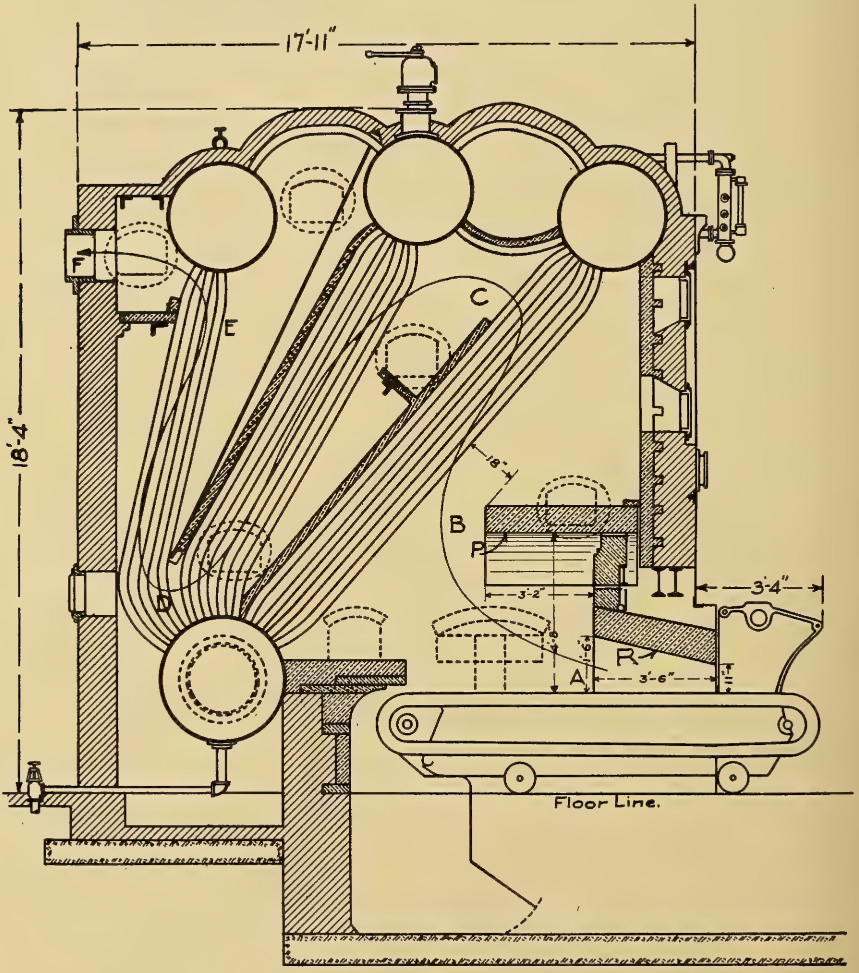

Fig. 4 Stirling Standard 260 H. P. Boiler Equipped with Chain Grates

(Operates easily without smoke at capacities from 50 to 140 per cent).

DESCRIPTION OF THE SETTINGS SHOWN BY THE FIGURES

In the figures which accompany this article there are shown several types of boilers and furnaces that have been found suitable for burning Illinois coals without smoke. These settings have been operated under the immediate observation of the writer. Each figure is supplied with a title which is at the same time a description of the setting and an indication of the possibility of its smokeless operation under varying conditions of service. The drawings themselves are sufficient to illustrate the conditions that must be fulfilled to secure smokelessness when using boilers of these types under usual service capacities. The stack draft available for boilers No. 1 to 8 is 0.75 in. to 1.0 in. supplied by a brick chimney 150 feet high with an inside diameter of 6 feet. Boiler No. 9 is supplied with an economizer and

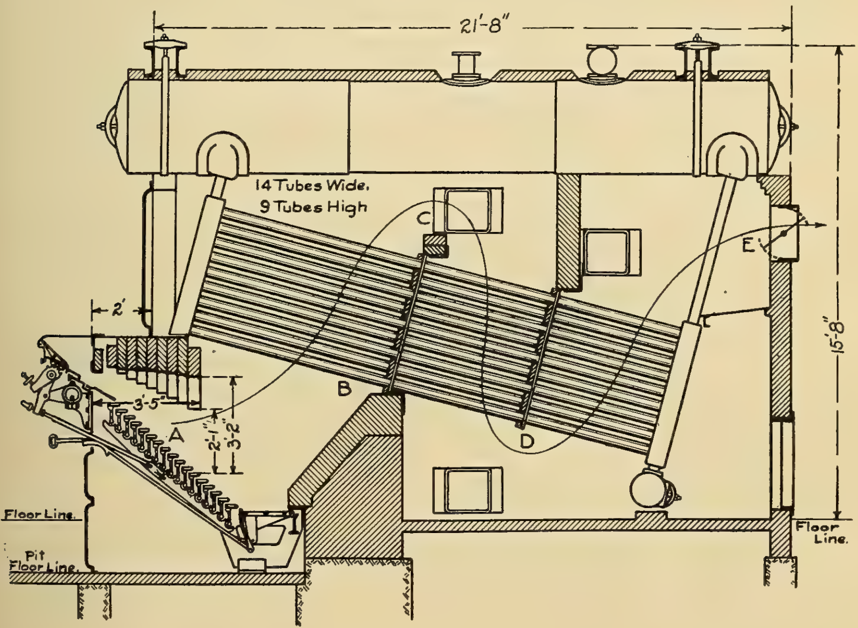

FIG. 5 BABCOCK & WILCOX 220 H. P. BOILER EQUIPPED WITH RONEY STOKERS, USUAL VERTICAL BAFFLING

(With careful handling may be run up to capacity without smoke above No. 2 on Smoke Chart; above 110 per cent capacity can not be run without objectionable smoke except by expert firemen.)

an induced draft fan permitting higher rates of combustion than are possible in the other settings.

In all of these settings it has been found necessary to pay particular attention to the coking arch under which the fresh charge of coal is being delivered. It seems to be true that over chain grates this arch should be inclined upwards more for a light than for a heavy draft. It is also believed that the arch of the Roney stoker should be kept well down toward the grates. The dimensions given in Fig 5 have given good results in these settings.

When Illinois coal is being burned in any furnace it is essential that the volatile products of combustion should be uniformly distilled from the coal and mixed with sufficient air at a high temperature. To accomplish this, particularly to maintain a high temperature, the mingling air and products of combustion must be kept away from the tubes or plates of the boiler which are comparatively cool, and which would therefore cool the gases before complete combustion had taken

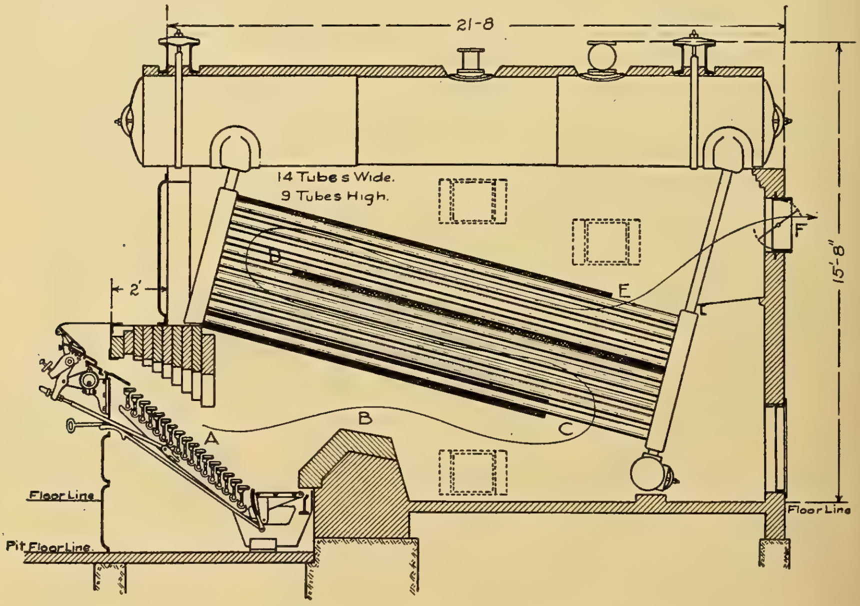

FIG. 6 BABCOCK & WILCOX 220 H. P. BOILER EQUIPPED WITH RONEY STOKER

(Setting as in Fig. 2, but arranged with different baffling, forming a tile roof furnace. This unit can be run at capacities from 50 to 100 per cent without smoke.)

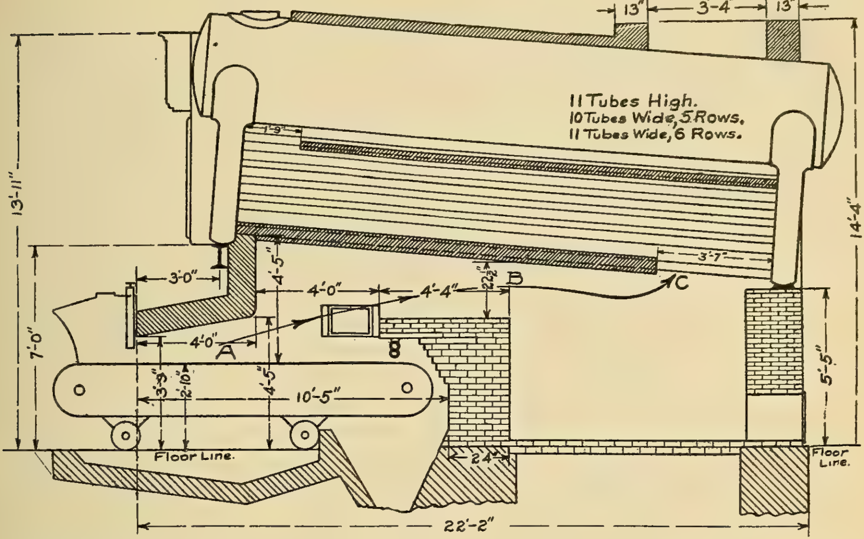

Fig. 7 Heine Standard 210 H. P. Boiler Equipped with Green Chain Grate, Combustion Arch, Tile Roof Furnace, Adjustable Water-back at Bridge Wall

(This is the Engineering Experiment Station Boiler furnished with induced draft. It can be run easily without smoke at capacities from 50 to 140 per cent; almost impossible to make smoke with this setting under any condition of operation.)

place. It is for the reason just mentioned that such forms of furnaces as shown in Fig. 6, 7, and 8 are planned. Special forms of fire clay tile are designed to be hung on the water tubes of boilers that are directly over the fire. The tiles form the roof of the furnace and prevent the hot gases, which are still not properly mixed, from coming in contact with the cooler tubes which are surrounded by the tiles. These tiles are made in various forms, three types being shown in Fig. 9. The kind shown at C makes a smooth flat roof for the furnace and has been found very satisfactory.

The length of a tile roof furnace or the distance of flame traveled before reaching the cool tube surface must evidently depend upon the total volume of volatile products distilled from the coal on the grate in a given time, which must be mixed with the air supply before the cooling tubes are reached. The higher the coal is in volatile content or the higher the rate of combustion per square foot of grate area, the longer will be this flame length. The coals that have been burned without smoke in the furnaces shown have had a combustile volatile content of from 30 to 40 per cent and they have been burned at a rate of from 16 to 40 lb. of coal per square foot of grate per hour. It will now be understood why such settings as shown in Fig. 5 cannot be depended upon for smokelessness. The cold tubes of the boiler are too near the flame, the temperature of the flame is reduced before combustion is complete and smoke results. There are coals low in combustible volatile matter that may easily be burned in such furnaces without smoke. In fact, when not forced, this furnace will burn many kinds of Illinois coals, but only with the greatest care have we been able to burn the usual grades of Illinois coals at desirable rates of combus-

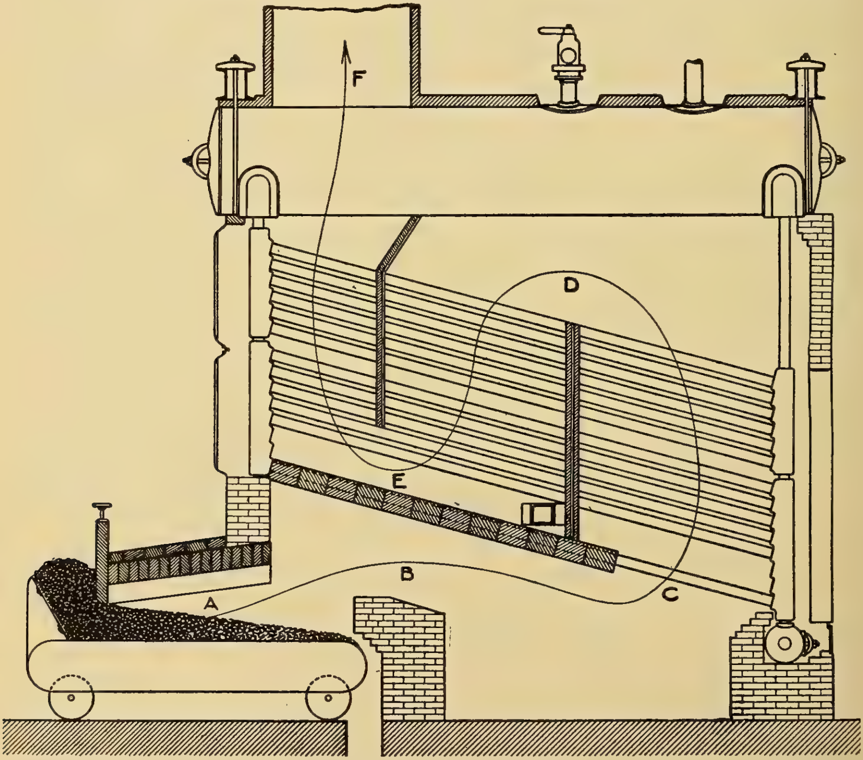

FIG. 8 SMOKE-PROOF STEAM GENERATOR DESCRIBED BY A. BEMENT, W.S.E. PAPER, OCTOBER, 1906.

(A satisfactory plan when proper attention is given to the draft available and to the areas of the gas passages.)

tion (24 to 30 lb.) without objectionable smoke, (No. 2 to 3½ on the smoke chart).

When the baffling shown in Fig. 6 was installed a series of smoke trials was arranged with this setting and the setting shown in Fig. 5. The results shown by these trials were disappointing and unsatisfactory. Lack of sufficient draft made it impossible to run setting Fig. 6 above its rated capacity. Under this condition there was little difference in the smoke from the two furnaces. This result suggests that changes in baffling of existing plants must be made with careful consideration of draft areas available or capacities may be cut down to an undesirable extent. The capacity possible with the setting shown in Fig. 6 proved to be about 90 per cent to 98 per cent while the capacity of the settings shown in Fig. 5 with the same coal was from 100 per cent to 120 per cent. The setting of Fig. 6 should prove satisfactory with higher draft pressures or by increased areas possible with a larger number of tubes in the vertical rows. From what has already been said about cool tubes, an inspection of the settings in Fig. 4, of a Stirling boiler over a chain grate stoker, might convey the impression that such a setting would produce smoke because of the absence of tile on the tubes below the path of the flame A B C. This setting has, however, proved to be a satisfactory and smokeless one. Looking at the progress of combustion through the side openings provided in the three settings of this character in the University plant, it is seen that the flame rolls along over the edges of the two arches shown and is by these arches completely mixed. No part of the tubes below the flame path A B C is enveloped by the flames; therefore the tubes have little cooling effect. It is also very evident that the lower ends of the tubes next the furnace fulfill their duty as heat transmitters by taking care of a very considerable part of the radiant heat which is available in large quantities at that point. Boilers of this type are now being manufactured at almost any desired height, the necessary capacity being furnished by increased width. They have proved very satisfactory, smokeless furnaces being easily arranged under them.

The Heine boiler set over a Green chain grate shown in Fig. 7 is the boiler installed by the Engineering Experiment Station for fuel tests with Illinois coals. This boiler is exactly like the Heine boiler used in the government fuel tests at St. Louis.* In this setting, how-

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*The characteristic feature of these two settings is a tile roof furnace, originating in 1901 with W. L. Abbott, at the Harrison St. station of the Commonwealth Edison Co., Chicago.

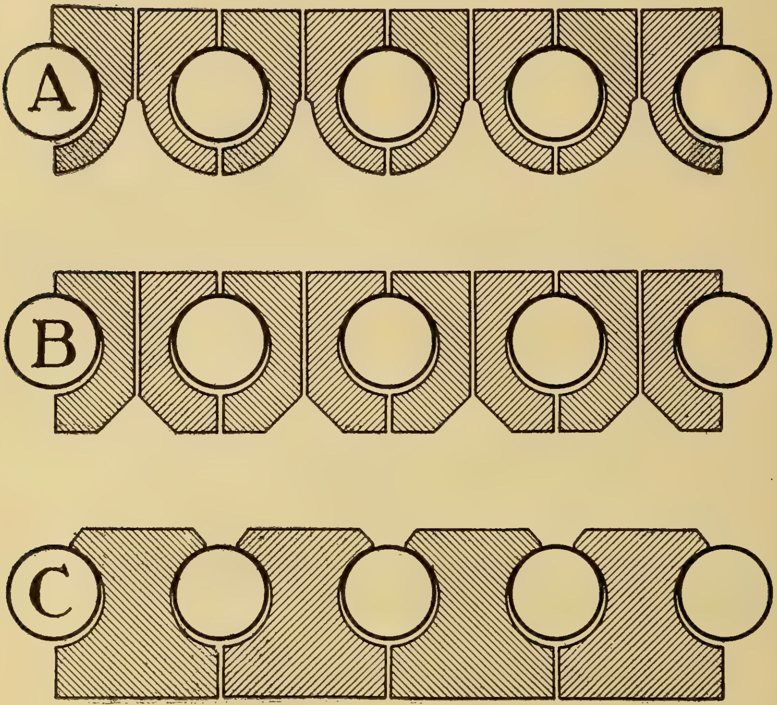

Fig. 9 Three Types of Tile Roofs for Boiler Furnaces

ever, we have a stoker furnace while the government boilers were set with hand-fired furnaces, evidently necessary where coals from many sections of the country were to be tested. The setting shown in Fig. 7 is provided with an economizer and a large induced draft fan making high rates of combustion possible. This setting is smokeless under the many and varied conditions of operation which have thus far been applied to it. Over 100 trials of about 10 hours' duration of various kinds of Illinois coal have been made in this setting. In all of these trials the smoke record has been "No smoke all day." This is the setting to which the writer has referred in several recent talks by saying that in this setting it has been impossible to make smoke. In order to justify any such statement, attention is called to the results of a series of tests made with this setting for the express purpose of smoke production if such was possible under apparently adverse conditions.

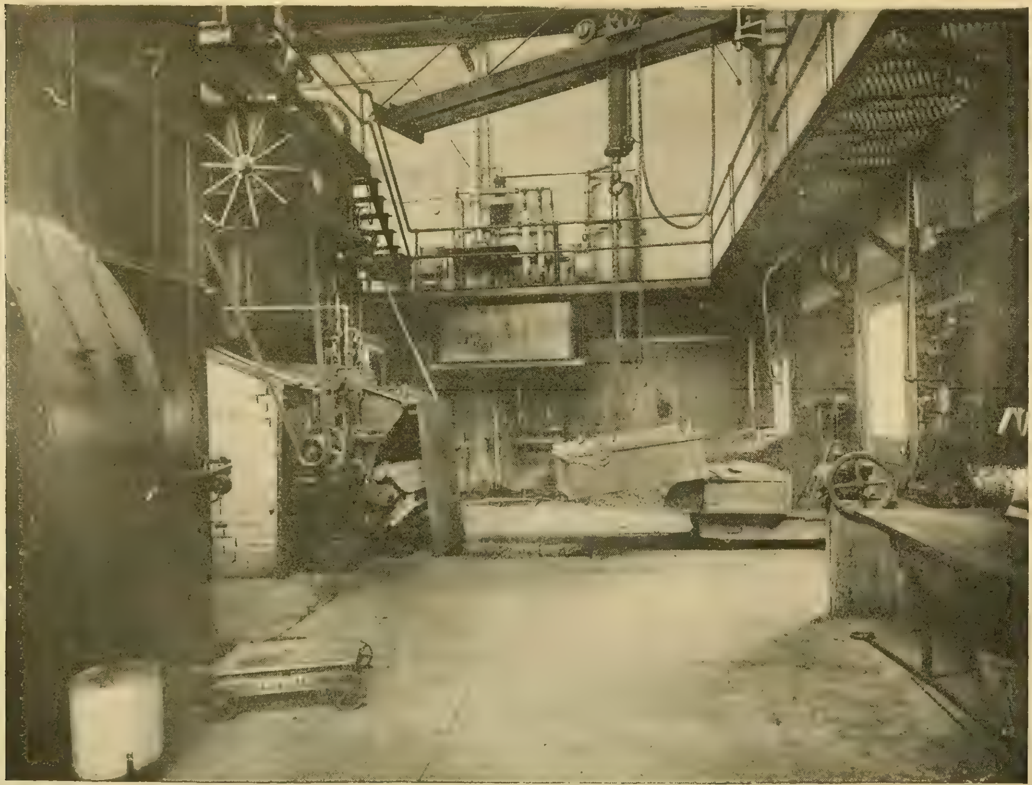

Fig. 10 General View of Engineering Experiment Station Fuel Testing Plant
210 H. P. Heine Boiler, Weighing Tanks, Ash Car and Hoist, Induced Draft

Those who desire to grasp the simple yet fundamental principles of smokeless combustion for Illinois coals should thoughtfully consider just how this furnace fulfills the conditions of perfect combustion, as stated on page 15 in the words of Kent, Bement and the writer. A detailed account of the process in this furnace may well bear repetition here.

The fresh coal, fairly uniform in size, advances slowly from the hopper along on the grate toward the furnace where the temperature is very high. The combustible volatile matter is continually being distilled from the coal, more and more rapidly, but with much uniformity, while it is passing under the combustion arch. Some of the necessary air flows in through the coal in the hopper, more through the grate under the arch, but by far the most flows through the redhot coals on that part of the grate beyond the arch. This air is thus heated and made ready for combining with the volatile products flowing from beneath the arch, and all together mix and roll along on the bottom of the tiles forming the roof of the furnace.

The bottom row of boiler tubes is covered with suitably formed tiles, which prevent the still actively mingling gases from being cooled by coming in contact with the tubes, and so the combustion processes go on until completed before reaching C, a point before the gases pass in among the cooling tubes. The tiles in the adjoining rows touch each other so that no gases pass between them. In such a furnace as this the smoke problem is settled somewhere between B and C. When the total volume of combustible material distilled from the coal is quite large in a given interval of time, the flame length may reach to C, while for smaller volumes the flame may end at B. Thus it is that the flame length indicates the end of the combustion processes and gives us valuable information concerning the proportions of smokeless furnaces.

It is important in the operation of this or any form of mechanical stoker that abundant opportunity be afforded for inspecting the condition of the fires and the progress of combustion. Appliances for continually indicating the composition of the escaping gases should be considered a necessity in all large and well managed plants.

The character of the draft apparatus available with this setting made it possible to make sudden changes in the rate of combustion. The rate of travel of the chain grate was easily adjusted by a throttle governor on the independent steam engine in driving the grate. The rate of combustion could be calculated very closely from the thickness of the fuel bed and the rate of grate feed, by reference to the large number of tests already made with this unit. These special smoke tests were run on four different days, and on each day eight changes in condition of operation were made. The thicknesses of fire used were four, five, six and seven inches. The capacities varied from 60 to 150 per cent of the rated capacity (210 H. P.) of the boiler. The character and the size of the coal used are indicated in the tables. The results of these tests amply justify the conclusion that this type of furnace may be depended upon to operate even under adverse conditions without objectionable smoke, for out of a total of 32 tests in only six cases could any smoke be seen coming from the stack, and in three of these six smoke did not exceed No. 1 on the smoke chart. Where the most smoke was made, it was only No. 1½ and 2½, and here it will be observed that the draft had been suddenly reduced in the breeching, amounting to the condition of closing the damper with a brisk fire.

TABLE 1

Special Smoke Tests of Illinois Coal with a Chain Grate Stoker, Tile Roof Furnace and Heine Water Tube Boiler, with Varying Rates of Combustion. February 5, 1907. Coal--Washed, Size No. 4[a].

ITEM	Test No.	1	2	3	4	5	6	7	8
Time, Start		8:10	9:00	9:30	9:56	1:00	1:30	2:00	2:35
Time, Stop		9:00	9:30	9:56	1:00	1:30	2:00	2:35	2:47
Thickness of Fuel Bed		4	4	4	4	4	4	4	4
Draft Pressure in Furnace		.092	.110	.180	.095	.098	.057	.050	.080
Draft Pressure in Breeching		.430	.600	1.260	.580	.500	.250	.090	.240
Lbs. Coal Burned per sq.ft.of Grate Surface as Fired		31.4	39.3	45.5	—	35.8	28.3	23.4	—
Do. Dry Coal		27.8	34.8	40.4	—	31.7	25.5	20.7	—
Mean per cent of Rated Capacity Developed		112	126	155	—	135	109	91	—
Flue Gas Temperature, °F		629	671	762	661	659	600	521	560
Temperature over grate		2500	2550	2620	—	2530	2390	2310	—
Temperature back of Bridge Wall		2470	2550	2710	—	2490	2390	2310	—
Per cent CO_2		12.0	12.8	12.2	—	12.8	12.8	14.4	—
Smoke		0	½	1	0[b]	0	1½	2½	0

(*a*) Moisture in coal as fired, 11.3 per cent; ash, 7.5 per cent.
(*b*) Grate stopped for repairs.

TABLE 2

SPECIAL SMOKE TESTS OF ILLINOIS COAL WITH A CHAIN GRATE STOKER, TILE ROOF FURNACE AND HEINE WATER TUBE BOILER, WITH VARYING RATES OF COMBUSTION. February 9, 1907.—Washed, Size No. 2[a].

ITEM	Test No.	1	2	3	4	5	6	7	8
Time, Start		8:03	9:00	10:00	11:00	12:00	1:00	2:00	3:00
Time, Stop		9:00	10:00	11:00	12:00	1:00	2:00	3:00	4:00
Thickness of Fuel Bed		6	6	6	6	6	6	6	6
Draft Pressure in Furnace		.126	.154	.178	.122	.12	.088	.060	.120
Draft Pressure in Breeching		.56	.73	.96	.56	.50	.23	.11	.35
Lbs. Coal Burned per sq. ft. of Grate Surface as Fired		27.5	28.2	34.9	28.5	23.7	21.5	21.0	31.3
Do. Dry Coal		24.5	25.5	31.3	25.4	21.5	19.3	18.7	27.7
Mean per cent of Rated Capacity Developed		92	109	119	112	111	89	67	99
Flue Gas Temperature, °F		671	679	707	663	642	574	516	562
Temperature over Grate		2375	2485	2420	2340		2280	2235	2465
Temperature back of Bridge Wall		2305	2385	2340	2360		2210	2090	2475
Per cent CO_2		10.1	10.1	9.1	10.0	9.8	11.6	11.4	11.8
Smoke		0	0	0	0	0	$\frac{1}{8}$	0	$1\frac{1}{2}$[b]

(*a*) Moisture in coal as fired, 11 per cent; ash, 7.97 per cent.

(*b*) Much unconsumed coke forced over back end, thus increasing the ratio of volatile carbon in the fuel actually consumed.

The results of these experiments are given in Tables 1, 2, 3 and 4, and they certainly demonstrate the possibilities of smokeless combustion of some typical Illinois coals.

It is not within the scope of this bulletin to enter into any lengthy discussion of the results as shown by these tables. It should, however, be stated that this setting has been operated not only without smoke, but also with economy, as may be seen by inspection of Table 5. The complete record of the results of the economy tests made under this boiler will form the subject of a separate bulletin soon to be published.

RELATION BETWEEN CAPACITY AT WHICH BOILERS ARE DRIVEN AND THE TENDENCY OF THEIR FURNACES TO SMOKE

There is a rate of driving a boiler furnace which, if exceeded, will result in producing smoke. This rate varies for different types of furnaces and for the various methods of baffling or kinds of mixing piers used. It also varies for different kinds of coals. A strong draft will make a large air supply possible, and allow thicker fires or finer coal to be used. The possibility of sufficient air makes a furnace smokeless when an insufficient supply caused by a weak draft might cause the production of smoke. There is an always increasing tendency to drive boilers and furnaces a little harder. Boilers that were purchased for 1000 H. P. a few years ago are now being forced to 1400 and 1600 H. P. This means of course larger grate areas and a much higher rate of combustion per unit of grate area. As far as the boilers are concerned, they seem to be ready and willing to transmit about the same proportion of the heat available to the boiler as ever. It is doubtful if constructive furnace details have kept pace with the demands for the higher rates of combustion and at the same time have

TABLE 3

Special Smoke Tests of Illinois Coal with a Chain Grate Stoker, Tile Roof Furnace and Heine Water Tube Boiler, with Varying Rates of Combustion. February 11, 1907. Coal—1½-inch Screenings. Ordinary Grade for Tests, 1, 2, 3, 4, 5[a]. Poor Grade for Tests, 6, 7, 8.

ITEM	Test No.	1	2	3	4	5	6	7	8
Time, Start		8:05	9:00	10:00	11:00	12:00	1:00	2:00	3:00
Time, Stop		9:00	10:00	11:00	12:00	1:00	2:00	3:00	4:00
Thickness of Fuel Bed		5	5	5	5	5	7	7	7
Draft Pressure in Furnace		.202	.252	.40	.23	.108	.302	.355	.58
Draft Pressure in Breeching		1.21	1.30	—	1.12	.23	1.48	1.21	—
Lbs. Coal Burned per sq. ft. of Grate Surface as Fired		32.7	35.4	39.3	30.2	12.4	46.5	26.8	26.2
Do. Dry Coal		27.4	30.0	33.2	25.5	10.5	—	—	—
Mean per cent of Rated Capacity Developed		87	103	106	84	56	106	75	68
Flue Gas Temperature, °F		674	704	718	650	542	674	646	625
Temperature over Grate		2360	2400	2615	2489	2180	2530	2425	2485
Temperature Back of Bridge Wall		—	—	—	—	—	—	—	—
Per cent CO_2		6.8	7.6	6.6	7.2	6.2	7.0	4.7	4.3
Smoke		0	0	0	0	0	0	0	0

(*a*) Moisture in coal as fired, 14.23 per cent; ash, 15.4 per cent.

TABLE 4

Special Smoke Tests of Illinois Coal with a Chain Grate Stoker, Tile Roof Furnace and Heine Water Tube Boiler, with Varying Rates of Combustion. February 12, 1907. Coal—1½-inch Screenings, selected [a].

ITEM	Test No.	1	2	3	4	5	6	7	8
Time, Start		8:00	9:00	10:00	11:00	12:00	1:00	2:00	3:00
Time, Stop		9:00	10:00	11:00	12:00	1:00	2:00	3:00	4:00
Thickness of Fuel Bed		6	6	6	6	6	5	5	5
Draft Pressure in Furnace		.18	.25	.20	.10	.06	.195	.10	.05
Draft Pressure in Breeching		1.00+	1.5+	.93	.36	.21	1.20	.57	.15
Lbs. Coal Burned per sq. ft. of Grate Surface as Fired		28.3	32	28.3	23.9	16.8	31.0	26.5	22.6
Do. Dry Coal		24.2	27.3	24.2	20.4	14.3	26.5	22.6	19.3
Mean per cent of Rated Capacity Developed		89.8	85	91	85	74	100	92	80
Flue Gas Temperature °F		649	662	639	588	544	676	624	540
Temperature over Grate		2450	2525	2450	2435	2525	2305	2470	2490
Temperature Back of Bridge Wall		—	—	—	—	—	—	—	—
Per cent CO_2		7	5.9	7	9.9	9.7	7.3	8.7	11.8
Smoke		0	0	0	0	0	0	0	0

(a) Moisture in coal as fired, 14.6 per cent; ash, 9.48 per cent.

provided efficient smoke preventing plans. In other words when boilers are forced much over 30 per cent of their rated capacity the probability of smoke from their furnaces rapidly increases. Nothing inherent in the boiler itself is at all responsible for smoking furnaces, for as elsewhere pointed out it is the total amount of volatile combustible evolved in the furnace per unit of time that imposes duties upon those constructive features of the furnace and combustion chamber which are designed to furnish the proper amount of suitably warmed air to the furnace and to provide for its effective mingling before the products of combustion reach the cooling surfaces of the boiler. It would seem necessary, therefore, that until further advances are made in furnace design and methods of operation, when a plant has reached the point of perhaps 140 per cent of its rated capacity, additional boilers ought to be added if only in the interest of a less amount of smoke. In

TABLE 5

Results of Evaporative Tests with Illinois Coals on a Chain Grate Stoker

Experiment Station Boiler with Tile Roof Furnace. Same Setting as Those Used for Special Smoke Tests in Tables 1 to 4.

Number........	47	50	91	96	107	127	131	134
2. Duration of Trial.....	8.32	8.32	8.30	8.25	8.10	7.02	8.15	7.37
3. Size and Condition of Coal (1)..........	¾—¼(w)	¾—¼(w)	1½—0 (2)	(1¾—1)	1—¾(w)	⅜—0(w)	⅞—⅜(w)	⅞—⅜(w)
4. Per cent Moisture in Coal............	10.72	11.23	11.88	7.14	9.40	21.25	19.83	19.29
5. Per cent Ash in Coal..	7.31	7.64	16.89	8.42	8.21	12.58	7.68	7.19
6. Per cent Carbon in Ash Free, Dry Coal.	60.41	60.41	53.95	60.41	62.43	55.94	57.39	56.35
7. Per cent Volatile Matter in Ash Free, Dry Coal......	39.59	39.59	46.05	39.59	37.57	44.06	42.61	43.65
8. B. t. u. per Pound Coal as Fired.......	11898	11740	10036	12305	11922	9350	10274	10391
9. Thickness of Fuel Bed at Grate, inches.	6	4	6	5	6	5	5	5
10. Draft Pressure in Furnace, in. of water	0.127	0.085	0.258	0.112	0.120	0.387	0.117	0.161
11. Per cent CO_2 in Flue Gases (3)......	12.60	12.28	7.00	9.52	10.99	7.10	11.46	11.38
12. Excess Air Used, per cent............	43.54	47.16	150.1	83.4	56.4	143.8	57.4	57.6
13. Temperature of Escaping Flue Gases, °F	599.0	612.0	613.4	606.0	599.0	624.0	574.0	665.0
14. Dry Coal per sq. ft. Grate Surface, per hour	22.48	22.35	26.41	22.10	20.49	27.62	20.60	27.55

TABLE 5.—(Continued)

1. Trial Number..........	47	50	91	96	107	127	131	134
15. Carbon in Ash and Refuse, per cent of Dry Coal...........	4.00	3.95	2.23	2.23	1.56	5.46	1.75	1.89
16. Steam Pressure Gage..	150.0	150.0	146.9	150.4	147.5	149.5	149.4	148.3
17. Per cent of Builders' Rated Horse-power Developed..........	109.4	109.3	103.1	101.8	99.9	99.5	101.4	133.8
18. Equivalent Evaporation from and at 212° F. per lb. Coal as Fired............	8.24	8.23	6.52	8.12	8.37	5.38	7.49	7.44
19. Equivalent Evaporation from and at 212° F. per lb. Ash Free, Dry Coal.....	10.51	10.60	9.42	9.86	10.34	8.70	10.53	10.33
20. Efficiency of Boiler..	69.93	70.75	64.60	65.32	69.00	60.18	71.76	70.61
21. Efficiency of Boiler including Grate	66.89	67.70	62.75	63.60	67.82	56.26	70.39	69.16

(*1*) Item 3—Size is given in inches. (w) refers to washed coal.
(*2*) Sixty per cent of this coal was below ¼ inch.
(*3*) The flue gases were free from carbon monoxide and smoke.

confirmation of this particular point the following discussion* is worth presenting. "The work at the government coal testing plant has been somewhat in the direction of smoke abatement, although the principal object of the work has been the determination of the value of different fuels for steaming purposes. Whenever we tested a soft coal we tried to eliminate the smoke. We have reduced the smoke 50 per cent or more, and often we burn soft coal without smoke. We have a good fireman, and he is watched all the time. Our furnace has a fire clay tile roof with a smooth lower surface, and a large combustion chamber; it is equipped with a plain grate of 40 sq. ft. We fire about fifty pounds of coal at a time on one-half of the grate area, and every two or three minutes. The furnace has three fire doors and we fire alternately in the front of the first and the third doors and in the rear of the middle door at one time, and then the rear of the first and third doors, and the front of the middle door at the next time. In this manner the distilled volatile matter is divided into three streams and this division facilitates its mixing with the hot air coming from the uncovered portions of the fuel bed.

"Another point in connection with the hand-fired furnace is that it is usually forced to burn more volatile matter than it was built for. We have often found that coal burned slowly, say at the rate of 20 pounds per square foot of grate area per hour, made no smoke at all. However, when the rate of combustion was increased to 26 pounds or more, smoke was produced, even though as great care was used in firing, and the firing was as frequent. This fact is in accordance with Mr. Bement's paper, where he states that the combustion space of a furnace is of a certain capacity, and that one should not try to burn more coal in it than the furnace was built for. The fact is that whenever a furnace is run above its capacity, it is bound to make smoke.

"I have recently noticed an interesting fact, that with a chain grate stoker in a Heine boiler furnace, no smoke was made when the rate of combustion was 42 pounds. If the same coal were burned in a hand fired Heine boiler furnace, smoke would probably be produced at the rate of combustion of 26 pounds. This seems to indicate that with the chain grate stoker more of the coal burns on the grate as fixed carbon and less of the combustible is driven off and burned as volatile matter, than is the case with a hand-fired furnace. This is probably due to the fact that with the chain grate stoker the coal is fed in gradually and is, therefore, heated gradually, while with the hand-firing, the coal is thrown on the top of the white-hot fire and is heated rapidly; this rapid heating is the cause of more combustible being distilled off as volatile matter and more smoke being produced.

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*Jour. W. S. E., Dec. 1906. "Suppression of Industrial Smoke." A. Bement. Discussion by H. Kreisinger, U. S. G. S.

"In conclusion I will say that the hand-fired furnace should be the last one to be considered for preventing smoke. In the first place it is rather expensive to operate, and the results are doubtful because too much dependence has to be placed on the fireman."

This discussion has been quoted because it definitely states a rate of combustion (20-26 lb.) as being about the limit for smokeless hand-firing in a certain type of furnace. It also gives good reasons why this rate may be nearly doubled on a chain grate stoker and not produce smoke. It might be well to point out, however, that while the average rate of combustion in the hand-fired furnace appears to be 20-26 lbs. for a short interval just after firing fresh coal, the rate is much higher, equal perhaps to the rate of 42 lb. of the chain grate stoker. It is entirely reasonable to suppose that furnaces will be planned that will provide for smokeless consumption of coal at much higher rates of combustion, but until they are available power plant owners should not be allowed to force boilers to such a point above capacity as will cause the furnaces to become troublesome smoke producers.

SPECIAL MIXING DEVICES, PIERS, WING WALLS AND JETS

The method of introducing the coal into the furnace will in all cases have much to do with the type of furnace which must be used to stop smoke. When the coal is fed into the furnace mechanically, as it is when stokers are used, one of the most essential elements of successful smoke prevention has been met, namely, slow and uniform gas evolution. When coal is fed by hand into a hot furnace, six shovelfuls more or less at one time, this same element has been ruthlessly neglected. The gas evolution can neither be slow nor uniform. It is also very doubtful if the proper amount of air at the right temperature will be supplied. It is to assist in correcting such known violations of correct methods of coal supply to furnaces that certain devices have been introduced which are intended to correct these initial mistakes. A steam jet is such a mixing device. When properly applied it does not let the volatile gases and air supply escape from the furnace until very intimately mixed. It has, however, two very serious objections, noise and large steam consumption. From 5 per cent to 8 per cent of the steam made is often used in these jets. The success of the steam jet in preventing smoke, due almost entirely to its ability to properly intermingle the fuel gases and the air supply, immediately suggests the accomplishment of the same intermingling in other ways. Such ways have been devised. One of the early plans was proposed by Professor Kent in his wing wall furnace. Other plans have followed as typified by the Wooley furnace, and the Kuss mixing piers. These methods of mixing are efficient and economical and are especially to be considered in connection with hand-fired furnaces. The particular arrangement to be adopted will be determined by several factors, viz.; (a) The composition of the coal to be used. (b) Size of coal. (c) Method of firing. (d) The rate of gas evolution.

SMOKE PREVENTION WITH THE HORIZONTAL FIRE TUBE BOILER

The horizontal fire tube boiler is still much in use in the smaller units of from 50 to 150 horse-power. It was brought into prominence in the early days of American steam boiler practice as the natural successor of the plain cylinder and flue boilers, all of which were externally fired. It soon became the standard type of boiler throughout manufacturing New England where in many places it still retains its position on account of its cheapness and its economical operation with all grades of anthracite and many grades of bituminous coals, especially those containing a high fixed carbon content.

With the coals just mentioned the combustion takes place mostly on the grate or at a short distance above it. Many plants have been installed with this type of boiler, in which the grate has been placed not more than 14 to 16 inches beneath the boiler. These plants have burned anthracite coal successfully. With the introduction of the West Virginia bituminous coals containing small amounts of volatile combustible matter the grates were lowered under this type of boiler to 24 and 30 inches with good effect. Still, with either coal, much the greater part of the heat was generated on or near the grate and the heat made available for transmission was largely radiant heat. The plates directly over the fire itself transmitted a correspondingly large part of the heat of the coal to the water in the boiler. The satisfactory performance of the fire tube boiler with eastern coals together with its availability made it naturally the boiler to be adopted by manufacturers moving westward with the center of population. It is easily seen, however, that with Illinois coals carrying 30 to 40 per cent of volatile combustible matter and burned at rates which produce flame lengths of from 5 to 20 feet, this type of boiler as usually set is by no means adapted for the smokeless consumption of this kind of fuel. There is, in fact, no better method of producing dense black smoke with Illinois coal than to install a horizontal fire tube boiler with the usual furnace, and hand fire such a plant with run-of-mine coal. In such an outfit all the fundamental principles laid down elsewhere in this paper are disregarded. The method of introducing the coal directly into the hot furnace, in fine dust and large lumps, prevents slow or uniform distillation of the gases; the air supply through open doors, through holes in the fire, or through a fuel bed of varying thicknesses is neither correct in quantity nor is much of it properly heated; the mingling products of combustion come in contact with the cool surface of the plates of the boiler, reducing the temperature of the gases below the ignition temperature before combustion is completed.

Having in view these defects of the usual plan of operating the fire-tube boiler with Illinois coal, many ways suggest themselves by which these faults may be corrected. It is possible to burn Illinois coal without smoke with fire tube boilers, but the furnace requires special treatment and such settings are not common. The plans usually proposed are either low-set stokers or extended Dutch oven furnaces. When hand-firing is adopted the wing wall furnace or other form of mixing baffles or piers is of great assistance. With any of these devices careful firing is very necessary for satisfactory results. The twin brick arch furnace which keeps the gases away from the boiler plates altogether is an effective smoke preventive. Careful firing with low rates of combustion (12 to 16 lb.) per square foot of grate, assisted by automatically controlled air supply, will often enable these settings to be run so as to escape the fines of city smoke inspectors, but they can hardly be compared as to smokelessness with such settings as are illustrated in this paper. Horizontal tubular boilers are often adopted on account of their cheapness, and when such is the case the addition of any special furnace constructions or any special devices to aid in smoke prevention, is seldom given any consideration. From what has been said it is doubtless evident why this type of boiler is so frequently found to be one of our worst smoke offenders.

BURNING ILLINOIS COAL IN LOCOMOTIVE BOILERS

The writer is compelled to admit that he does not know how to burn Illinois coal in a locomotive boiler under the usual operating conditions without making smoke. Careful firing is the most effective way of reducing the amount of smoke made, but no fireman can be found skillful enough to meet the exigencies which are always arising and to operate a locomotive boiler in service without smoke even 75 per cent of the time when using Illinois coal. It is not possible here even to enumerate the many devices which have been tried to prevent smoke production on locomotive boilers. The high rates of combustion on the grate of this boiler, often reaching 150 lb. and sometimes 200 lb. per square foot of area, produce temperatures which soon destroy all forms of fire resisting material or other constructions not backed up by water-jacketed metals. Were it not, however, for the many and sudden changes in the conditions of operation imposed by the very duties for which the locomotive is designed, all other obstacles might possibly be overcome and the locomotive become smokeless. Fortunately we can clearly see how to eliminate the smoking locomotive from the thickly populated districts by the use of the electric locomotive which now is able to meet the demands placed upon it for speed, acceleration, and tractive effort. It is also pointed out that the large power plant will now produce the electricity needed for this service with greater economy and at the same time burn Illinois coal without smoke.

HOW TO HAND FIRE ILLINOIS COALS SO AS TO REDUCE THE PRODUCTION OF SMOKE

There are many small power plant units that are hand-fired which smoke badly. The construction of many of these furnaces is such that it is almost impossible to operate the plant without smoke. Still something might be done to reduce smoke if the fireman exercised more care in firing. Whatever can be done by the fireman in the way of properly introducing the fuel into the furnace is just so much gained and it relieves the auxiliary mixing devices or baffles if such exist from just so much work later on. The best method of hand-firing for smokelessness is also the best method for attaining economy. There are three generally recognized methods of hand-firing: (a) The Spreading, (b) The Coking and (c) The Alternate. The first is satisfactory and generally used for anthracite; the second for coking coals and the last for non-coking coals. It is the alternate method that is best suited to Illinois coals. This method is described as follows. The fuel bed area is divided into equal parts two, four, or six depending on the size of the entire surface. The fresh coal is fired alternately on one-half of these areas at a time at such intervals as may be necessary to hold the steam pressure. Depending on the rate of driving, these intervals will vary from one to five minutes. For small areas first one-half the surface of the fuel bed is covered and then perhaps three minutes later the other half. This method allows much of the air supply to come through the bright fuel bed and thus become heated and suitable for mixing with the highly volatile content which is being rapidly driven from the freshly fired coal on the other side. Just because fresh fuel has been spread over one part of the fuel bed, the air most needed at that moment cannot as easily flow through it, and another part of the fuel bed should be left free for its passage at that time. When the fuel bed area is very large, some checker board system of firing may be adopted which, when alternately fired and left free for air passage, will result in a large reduction in the amount of smoke produced by the too common method of spreading the coal over the entire surface at each firing. It must not be forgotten that a large supply of warm air is needed immediately after fresh fuel is spread over a part or all of the fuel bed; this is best supplied as just explained, but it may be advantageous to provide for still more air by leaving the fire doors open slightly just after each firing. There are several devices on the market which provide for an air supply over the fire, which are turned on with the opening or closing of the fire door and which can be arranged to close at the end of any desired time depending upon the rate of driving and frequency of firing found desirable. The firing of small amounts of coal at frequent intervals produces less smoke than firing large amounts at longer intervals. The latter method, however, usually proves less tiresome to the fireman and is for that reason more frequently adopted.

The Distance Between the Boiler and the Grates

Having in mind the horizontal fire tube boiler, the distance from the bottom of the boiler to the grates should be from 30 to 34 inches. At this distance the flame from Illinois coals will sweep along the bottom of the boiler and much smoke will result. Still it must be borne in mind that a large part of the heat to be obtained from the burning of the fixed carbon part of these coals is transferred to the shell in the form of radiant heat and for this purpose the grate should be near the boiler. While it is necessary in preventing smoke, that the flames be kept from the cooling surfaces of boilers, this cannot be accomplished by simply lowering the grates under a horizontal fire tube boiler, as the writer has unfortunately been incorrectly quoted as having stated. For boilers of this type some form of furnace extending partly or entirely in front of the boilers and either hand fired or fed by stokers of the Murphy type would undoubtedly furnish a satisfactory solution to the smoke problem. As elsewhere pointed out the smoke problem is not usually satisfactorily solved with Illinois coals and the horizontal fire tube boiler. The small unit hardly warrants an automatic stoker. Desire for a cheap plant prevents any special form of furnace and so it is that this kind of a setting frequently proves to be a troublesome smoker.

THE PREPARATION OF COAL FOR SMOKELESS BURNING

When the coal fed into a furnace is fairly uniform in size it is much easier to burn it without smoke than when it is of different sizes. In all the settings described in this article as smokeless, the coal burned has been of such uniformity as to meet this requirement. The standard commercial sizes are all that are required, such as No. 1, 2, 3, 4, or 5. Take, for instance, the chain grate stoker; the very principle of its operation, complete consumption of the coal while it travels the length of the furnace, makes it very evident that if small pieces of coal are just consumed, the very large pieces will not be consumed. Just to what extent it will pay to size coal for regular use is not yet very clear, but experiments reported by Mr. W. L. Abbott in a paper before the Western Society of Engineers (Sept. 1906), makes it evident that the influence of variation in size of the fuel used is of much more importance than has heretofore been generally believed. In these tests the capacity as well as the efficiency of the plant tested increased rapidly with the size of the coal used between the average sizes of 0.12 inches and 0.30 inches, after which both these factors dropped again. The following table indicates the general results obtained.

Effect of variation in size of Illinois coal screenings.

Size in inches	Horse-Power	Efficiency
.15	350	53
.20	525	60
.25	650	65
.30	725	65
.35	625	63
.40	550	60
.45	500	58
.50	525	58

Whether these results are generally applicable or whether they apply only to the conditions existing in the operation of a single plant, it is difficult to say. The plant in which these tests were made corresponds so closely to the plants described in the paper that it would seem very desirable that those operating such plants should take advantage of the results obtained and be sure that the mere matter of inattention to size should not be responsible for a capacity or efficiency loss of from five to ten per cent.

The washing of coal, which removes a considerable part of the ash and sulphur, has proved very advantageous to many plants, especially where capacity has been an important consideration. The washing itself, however, does not make coal burn without smoke. As explained elsewhere the total volume of volatile combustible distilled per hour from each square foot of grate area must determine those furnace proportions necessary, with the various methods of coal supply to the furnace, which will with any kind of coal prevent smoke. The writer desires to mention in this connection that since he began to use washed nut coal in his hot water residence heater and in his kitchen range black smoke is seldom seen coming from his chimneys. Previous to the use of this coal the kitchen range pipe required cleaning at least once, sometimes twice each year. It is now three years since this pipe was cleaned. No soot now gathers on the underside of the stove lids as was formerly the case. A fire is easily maintained eight to twelve hours in the residence heater. Experiments with two types of residence heaters, which are now available for test purposes, are in progress by the Experiment Station, and it is hoped that more exact information concerning the relative value of various coals used in this kind of furnace will soon be available. The writer confidently believes that we shall soon know how to burn Illinois coals without smoke in residence heating boilers as readily and surely as we now know how to burn it under power plant boilers.

Briquetted coal offers some opportunity for smoke reduction in certain kinds of furnaces, and certain railroads have reported favorably on its use inside city limits. It does not appear that it can compete with raw coal, certainly not in Illinois, if any consideration is to be given to the cost. There are conditions arising where the question of cost need not be considered,—such for instance as on naval vessels in time of war. For this reason a series of tests is now being arranged between the steam engineering department of the U. S. Navy and the technologic branch of the United States Geological Survey to determine the comparative value of raw and briquetted coal on board several types of naval vessels. In these tests the question of smokeless operation will be a question of careful consideration.

Powdered fuel has been used with much success in places where the coal has not been too high in volatile combustible content, and where the cost of the coal ordinarily used exceeded five or six dollars a ton. Powdered fuel can be burned without smoke and it can be burned with excellent economy. It cannot, however, be cheaply reduced to a powder.

The writer will welcome suggestions relating to the smoke problem from engineers throughout the country, also drawings, blueprints, etc., and complete and reliable data pertaining to smokeless furnace construction.

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. (Out of print.)

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905.

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Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906.

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman. 1906.

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906.

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Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906.

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Bulletin No. 12. Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907.

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Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907.

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1907.

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BULLETIN NO. 16

A STUDY OF ROOF TRUSSES

BY

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UNIVERSITY OF ILLINOIS
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BULLETIN NO. 16 AUGUST 1907

A STUDY OF ROOF TRUSSES

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The investigation described in this bulletin had for its original object the determination of a formula for the weight of roof trusses more accurate than those now in existence. As the investigation progressed, however, other topics arose and some interesting results were secured, which it is believed will be of value to architects and engineers. Very little study has been devoted to roof trusses in comparison with the thorough treatment of bridge trusses by eminent writers. The chief result of the work has been the devising of a method to save time and labor by presenting data in a form most convenient for comparison. This system will be found convenient in calculating and designing roof trusses to satisfy given conditions, whether constructed of wood and steel, or entirely of steel.

I. METHOD OF INVESTIGATION

In the determination of weights, general mathematical methods may be readily applied to most forms of bridge trusses, especially those with parallel chords; these are, however, less valuable for roof trusses where far more varied conditions must be arbitrarily limited in order to make such methods applicable. The results are then of doubtful worth. A more practical method of investigation was therefore chosen. For a single common type of truss, Fig. 2, nearly fifty trusses of varied span, rise, and distance apart were calculated and designed in the same general way. Next the weight of each truss was carefully computed, and if this materially differed from the assumed weight of the truss, the necessary corrections were made in the sectional dimensions and weight of members.

The verticals were steel rods with upset ends; all other members were long leaf pine timbers. Splices in the tie-beam and its connection with the principal were made with vertical steel fish plates and through bolts. A purlin rested on each apex of the principal and supported the rafters on which was laid $\frac{7}{8}$-in. matched sheathing covered by a painted tin roof.

In accordance with the usual custom of engineers the roof was assumed to support a snow load and wind pressure at the same time, although the writer believes that this extreme condition rarely occurs. The assumption, however, provides some surplus strength for contingencies, such as unusual snowfall, very violent winds, etc.

II. Conditions Assumed

The spans of the trusses were assumed to vary by 20-ft. intervals from 20 to 200 ft.

The rise of the trusses was varied by 5-ft. intervals from $\frac{1}{10}$ to $\frac{1}{4}$ the span.

The distance between trusses was varied by 5-ft. intervals from 10 to 30 ft.

The horizontal panel length was varied from 10 to 25 ft.

The number of purlins per panel was varied from 1 to 5.

III. Loads Supported by a Truss

1. *Permanent loads*

The following are the permanent loads assumed:

Painted tin covering	2 lb. per sq. ft.
Long leaf pine lumber	4 lb. per sq. ft. B. M.
Steel (see Cambria, etc.)	480 lb. per cu. ft.
Cast iron	450 lb. per cu. ft.
Weight of truss	Assumed

The weights of trusses were first assumed in accordance with Merriman and Jacoby's formula; but the following formula was deduced from the results of this investigation and is found to agree more closely with the computed weights of the trusses examined:

$$W = \frac{S}{25} + \frac{S^2}{6000}$$

in which S = span in feet
and W = weight of truss in lb. per sq. ft. of horizontal projection of the roof.

2. Snow load

The snow load varies with latitude, but was here assumed at 20 lb. per sq. ft. of horizontal projection of roof for location of Chicago. Denoting by i the angle of inclination of roof surface with the horizontal, we have $20 \cos i$ = snow load in lb. per sq. ft. of inclined roof surface.

3. Wind pressure normal to roof

The formula for the normal wind pressure most commonly employed in England and the United States is that of Hutton, viz.:

$$P_n = P \sin i^{(1.84 \cos i - 1)}$$

in which P denotes the pressure on a vertical surface and P_n the normal pressure.

A review of Hutton's apparatus and experiments (Hutton's mathematical papers) casts serious doubts upon the accuracy of this formula. The complex form is also objectionable. Other formulas are proposed by different authors. In Fig. 1 are shown the values of P_n given by various formulas reduced to a common basis of 30 lb. per sq. ft. on a vertical plane. A comparison of these values shows that different formulas give widely different results.

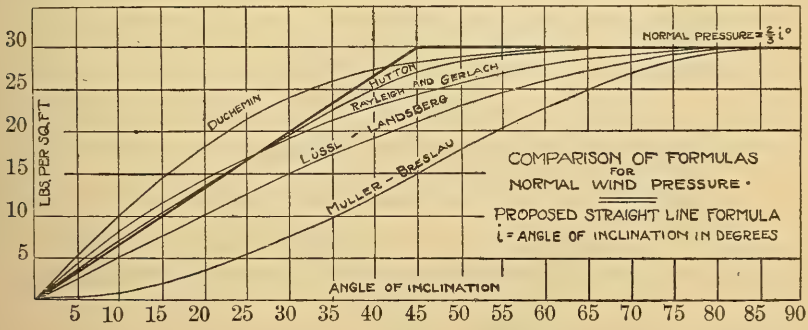


FIG. 1

Müller of Breslau is the greatest living authority on graphic statics and probably on the theory of bridges and roofs; his formula, however, gives the smallest values for normal wind pressures. It is evident that very little is certainly known concerning the relation between horizontal and normal wind pressures; hence the following empirical formulas were here adopted as being sufficient and convenient in use:

Taking the angle i in degrees,

$P_n = \frac{2}{3}\ i$, for $P = 30$ lb. per sq. ft. horizontal pressure.
$P_n = \frac{8}{9}\ i$, for $P = 40$ lb. per sq. ft. horizontal pressure.
$P_n = \frac{10}{9}\ i$, for $P = 50$ lb. per sq. ft. horizontal pressure.

These formulas are applicable for values of i less than 45° for higher inclinations, the normal and horizontal pressures are equal. They were believed to be original, but it has since been found that similar formulas had already been published.

IV. Application of the Method to a Truss

The treatment of a specimen truss will most clearly explain the method employed.

1. Assumptions

Let the following program be assumed:

Span of truss = 200 ft.; rise = 50 ft.; trusses set 20 ft. apart on centers; 20 panels of 10 ft. each; $i = 26.5°$. Roof covered with painted tin, laid on matched sheathing, supported by rafters resting on one purlin fixed on each apex of principal. Vertical rods, splice plates, and bolts to be of steel; other members to be long leaf pine timbers.

The following values are readily found:

Panel length of principal = 11.18 ft.
Panel area of roof surface = 223.6 sq. ft.
Snow load on roof surface = 20 cos i = 17.9 lb. per sq. ft.
Normal wind pressure on roof surface = $\frac{2}{3}\ i$ = 17.7 lb. per sq. ft.

2. Sheathing

To determine the maximum safe length L of sheathing between rafters we proceed as follows:

Vertical load on sheathing = 2 lb. (tin) + 4 lb. (sheathing) + 17.9 lb. (snow) = 23.9 lb. per sq. ft.

This load is resolved into a normal component, acting at right angles to surface of sheathing, and into a component parallel to the surface. The latter may be neglected, since it is resisted amply by the edgewise strength of the sheathing. The normal component is 23.9 cos i = 21.4 lb. per sq. ft. Adding to this the normal wind pressure, we get for the total normal load on the sheathing

$$21.4 + 17.7 = 39.1 \text{ lb. per sq. ft.}$$

Let L = distance in ft. between centers of rafters
t = thickness of sheathing in inches.

For safety against breaking the sheathing we have

$$L = 43.2 \frac{t}{\sqrt{w}} = \frac{43.2 \times .875}{\sqrt{39.1}} = 6.05 \text{ ft.}$$

For deflection limited to $\frac{1}{360}$ of length,

$$L = 12.9 \frac{t}{\sqrt[3]{w}} = \frac{12.9 \times .875}{\sqrt[3]{39.1}} = 3.33 \text{ ft.}$$

Hence the rafters can not be spaced more than 3.33 ft. between centers.

3. *Rafters*

Assuming 2-in. by 6-in. rafters, the maximum safe distance between their centers is determined as follows:

Let e = distance between centers of rafters in inches;

$\frac{I}{c}$ = section modulus = 12 for 2-in. by 6-in. cross-section.

I = moment of inertia = 36 for 2-in. by 6-in. cross-section.

The values of the section modulus and moment of inertia may be found without calculation by means of Tables 1 and 2.

The vertical load on rafters is 2 lb. (tin) + 4 lb. (sheathing) + 2 lb. (rafters) + 17.9 lb. (snow) = 25.9 lb. per sq. ft.

Hence the normal load on the rafters is 25.9 cos i + 17.7 (wind) = 40.9 lb. per sq. ft.; and the load parallel to the rafters acting lengthwise and producing longitudinal compression in them is 25.9 sin i = 11.6 lb. per sq. ft.

For safety against breaking,

$$e = \frac{11200\ I}{wL^2c} = \frac{11200 \times 12}{40.9 \times 11.18^2} = 26.3 \text{ in. between centers of rafters.}$$

TABLE 1

TABLE OF VALUES OF SECTION MODULUS $\frac{I}{c}$

For Rectangular Wooden Timbers

inches

	1	2	3	4	6	8	10	12	14	16	18
2	1	1	2	3	4	5	7	8	9	11	12
4	3	5	8	11	16	21	27	32	37	42	48
6	6	12	18	24	36	48	60	72	84	96	108
8	11	21	32	43	64	85	107	128	149	171	192
10	17	33	50	67	100	133	167	200	233	267	300
12	24	48	72	96	144	192	240	288	336	384	432
14	33	65	98	131	196	261	327	392	457	523	588
16	43	85	128	171	256	341	427	512	598	683	768
18	54	108	162	216	324	432	540	648	756	864	972

TABLE 2

TABLE OF VALUES OF SECTION MOMENT OF INERTIA I

For Rectangular Wooden Timbers

inches

	1	2	3	4	6	8	10	12	14	16	18
2	1	1	2	3	4	5	7	8	9	11	12
4	5	11	16	21	32	43	53	64	75	85	96
6	18	36	54	72	108	144	180	216	252	288	324
8	43	85	128	171	256	341	427	512	149	171	192
10	83	167	250	333	500	667	833	1000	1167	1333	1500
12	144	288	432	576	864	1152	1440	1728	2016	2304	2592
14	229	457	686	915	1372	1829	2287	2744	3201	3659	4116
16	341	683	1024	1365	2048	2731	3413	4096	4779	5461	6144
18	486	972	1458	1944	2916	3888	4860	5832	6804	7776	8748

For safe limit of deflection,

$$e = \frac{30223 \cdot I}{wL^3} = \frac{30223 \times 36}{40.9 \times 11.18^3} = 19.2 \text{ in. between centers of rafters.}$$

Since the rafters are dressed smaller than 2 in. by 6 in., it is best to space them 16 in. between centers instead of 19.2 in., thus making 15 equal spaces per bay of the roof.

The deflection $\triangle$ of the rafters at the middle of their length is

$$\triangle = \frac{3\ wL^4e}{2720000\ I} = \frac{3 \times 40.9 \times 11.18^4 \times 16}{2720000 \times 36} = .315 \text{ in.}$$

The total longitudinal load on one rafter is

$$11.6 \times 1\tfrac{1}{3} \times 11.18 = 173 \text{ lb.}$$

One-half this, or 86.5 lb., acts on the cross-section at mid-length of the rafter. If the rafter is straight, this compression would be uniformly distributed over the cross-section, thus giving $\frac{86.5}{12} = 7.2$ lb. per sq. in. compression. But after the rafter has deflected .315 in., the compression in the upper fibers of the cross-section is increased to

$$p' = p\left(1 + 6\frac{\triangle}{d}\right) = 7.2\left(1 + 6\ \frac{.315}{6}\right) = 9.5 \text{ lb. per sq. in.,}$$

where p = uniform compression in lb. per sq. in. at mid-length, and d = depth of rafter in inches.

If the load is expressed in tons, we have

p' = 9.5 lb. per sq. in. = .0048 ton per sq. in. of cross-section. The compression p' must be deducted from the safe fiber stress of long leaf pine timbers in tons per sq. in. to obtain the safe value for this case. Taking 0.7 tons per sq. in. as this safe stress, we obtain, therefore, .7000 — .0048 = .6952 ton per sq. in. as the net safe fiber stress for long leaf pine timbers.

Let this be denoted by F; then for safety against breaking and compression:

$$e = \frac{16000\ F\ I}{wL^2c} = \frac{16000 \times .6952 \times 12}{40.9 \times 11.18^2} = 26.0 \text{ in.} = \text{maximum}$$

distance between centers of rafters in inches.

It is evident from this analysis that the parallel loading and the longitudinal compression thereby caused in rafters may generally be neglected except for very steep roofs.

4. *Purlins*

The weight of the purlin being unknown is at first neglected. The total loading on a purlin may be resolved into two components, one normal to the roof, the other parallel thereto.

Denoting these components by W and W' respectively, we have

$$W = \frac{223.6 \times 40.9}{2000} = 4.57 \text{ T., the normal loading,}$$

$$W' = \frac{223.6 \times 11.6}{2000} = 1.30 \text{ T., the parallel loading.}$$

For normal loading

$$\frac{I}{c} = 2.14\ W L = 2.14 \text{ x } 4.57 \text{ x } 20 = 195.2$$

$$I = .795\ W L^2 = .795 \text{ x } 4.57 \text{ x } 20^2 = 1453$$

For parallel loading

$$\frac{I}{c} = 2.14\ W L = 2.14 \text{ x } 1.30 \text{ x } 20 = 55.6$$

$$I = .795\ W L^2 = .795 \text{ x } 1.30 \text{ x } 20^2 = 413$$

From Tables 1 and 2 we select a cross-section for the purlin such that the values for $\frac{I}{c}$ and I (edgewise for normal loading and flatwise for parallel loading) are not less than those just found. A cross-section 8 in. x 14 in. is found to satisfy the four conditions just determined.

The weight of the purlin is therefore

$$\frac{8 \text{ x } 14 \text{ x } 4 \text{ x } 20}{12} = 747 \text{ lb.} = .374 \text{ T.}$$

The normal component of the weight is

$$.374 \cos i = .33 \text{ T.}$$

and the parallel component of the weight is

$$.374 \sin i = .17 \text{ T.}$$

Adding these to previous values, we find $W = 4.90$ T. and $W' = 1.47$ T.

Using these new values of W and W', we obtain for $\frac{I}{c}$ and I the following results:

For normal loading

$$\frac{I}{c} = 2.14 \text{ x } 4.90 \text{ x } 20 = 209.3$$

$$I = .795 \text{ x } 4.90 \text{ x } 20^2 = 1558$$

For parallel loading

$$\frac{I}{c} = 2.14 \text{ x } 1.47 \text{ x } 20 = 59.6$$

$$I = .795 \text{ x } 1.47 \text{ x } 20^2 = 443$$

Referring again to Tables 1 and 2, we find that a purlin 8 in. x 14 in. is still amply sufficient to support both its load and its own weight.

5. *Weight of truss*

Using the formula given in Art. 3, we obtain for the weight of truss

$$= \frac{S}{25} + \frac{S^2}{6000} = \frac{200}{25} + \frac{200^2}{6000} = 14\tfrac{2}{3} \text{ lb. per sq. ft. of}$$

horizontal projection of roof.

6. *Computation of apex loads*

Permanent loads:

Covering, sheathing and rafters,	223.6 x 8 =	1789 lb.	
Purlin, as before		747 lb.	
Truss	200 x 14⅔ =	2933 lb.	
		5469 lb. =	2.735 T.
Snow load	200 x 20 =	4000 lb. =	2.000 T.
Wind load	223.6 x 17.7 =	3958 lb. =	1.979 T.

7. *Computation of loading on one-half truss*

Permanent loading	2.735 x 9½ =	25.98 T.
Snow loading	2.000 x 9½ =	19.00 T.
Wind loading	1.979 x 9½ =	18.80 T.

8. *Truss diagram, stress diagrams, and stress sheet*

Having the loads, the truss diagram is drawn, and the stress diagrams are then drawn for permanent and wind loadings

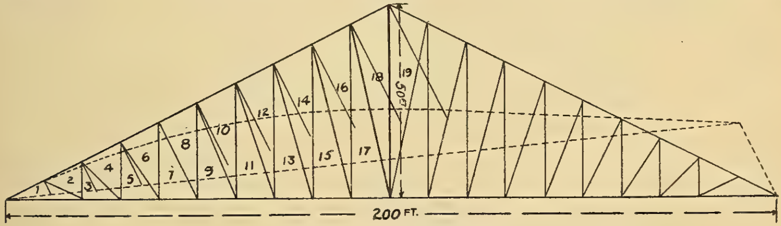
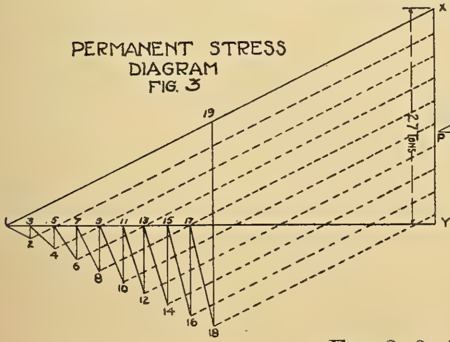


FIG. 2 TRUSS DIAGRAM

PERMANENT STRESS DIAGRAM FIG. 3



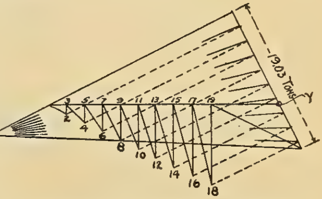


FIG 4 WIND STRESS DIAGRAM

FIG. 2, 3, 4

STRESS SHEET

Member	Permanent Stress	Snow Stress	Wind Stress	Maximum Stress	Center Length	Sectional Dimensions	Center Length Weight, lb.	Weight of Connections	Excess of Perm. Stress	Corrected Maximum Stress	Corrected Sectional Dimensions
X 1	—57.7	—42.3	—25.5	—125.5	11.18	14 by 14	1460		2.2	123.3	No change
X 2	54.8	40.1	24.0	118.9	11.18	14 by 14	1460		2.1	116.3	No change
X 4	51.6	37.8	22.5	111.9	11.18	14 by 14	1460		2.0	109.9	No change
X 6	48.6	35.6	21.0	105.2	11.18	14 by 14	1460		1.8	103.4	No change
X 8	45.5	33.3	19.6	98.4	11.18	14 by 14	1460		1.7	96.7	No change
X 10	42.6	32.2	18.2	93.0	11.18	14 by 14	1460		1.6	91.4	No change
X 12	39.5	28.9	16.6	85.0	11.18	14 by 14	1460		1.5	83.5	No change
X 14	36.4	26.6	15.1	78.1	11.18	14 by 14	1460		1.4	76.7	No change
X 16	33.3	24.4	13.6	71.3	11.18	14 by 14	1460	—229 lb. wood	1.3	70.0	No change
X 18	30.3	22.2	12.5	65.0	11.18	14 by 14	1460	+ 68 lb. steel	1.2	63.8	No change
Y 1	+51.6	+37.7	+28.5	+117.8	10.00	14 by 18	1640		2.0	115.8	No change
Y 3	48.9	35.8	26.3	111.0	10.00	14 by 18	1640		1.9	109.1	No change
Y 5	46.2	33.8	24.1	104.1	10.00	14 by 18	1640		1.8	102.3	No change
Y 7	43.4	31.8	21.9	97.1	10.00	14 by 18	1640		1.7	95.4	No change
Y 9	40.8	29.8	19.7	90.3	10.00	14 by 18	1640		1.6	88.7	No change
Y 11	38.0	27.8	17.5	83.3	10.00	14 by 18	1640		1.4	81.9	No change
Y 13	35.2	25.8	15.2	76.2	10.00	14 by 18	1640		1.3	74.9	No change
Y 15	32.4	23.7	13.0	69.1	10.00	14 by 18	1640	+ 336 lb. wood	1.2	67.9	No change
Y 17	29.8	21.8	10.9	62.5	10.00	14 by 18	1640	+ 2150 lb. steel	1.1	61.4	No change

1 2	—3.1	—2.3	—2.5	—7.9	11.18	14 by 3	314		.1	7.8	No change
3 4	3.8	2.7	3.2	9.7	14.20	14 by 3	393		.1	9.6	No change
5 6	4.9	3.6	4.1	12.6	18.00	14 by 4	672		.2	12.4	No change
7 8	6.1	4.4	4.9	15.4	22.40	14 by 6	1254		.2	15.2	No change
9 10	7.3	5.3	5.9	18.5	26.90	14 by 6	1505		.3	18.2	No change
11 12	8.7	6.3	7.0	22.0	31.70	14 by 8	2367		.3	21.7	No change
13 14	10.0	7.3	8.0	25.3	36.50	14 by 8	2725		.4	24.9	No change
15 16	11.3	8.2	9.2	28.7	41.30	14 by 8	3168		.4	28.3	No change
17 18	12.5	9.2	10.3	32.0	46.30	14 by 10	4320	— 761 lb. wood	.5	31.5	No change
2 3	+1.3	+1.0	+1.1	+3.4	10.00	1, $\frac{13}{16}$ '' o	35		.0	3.4	No change
4 5	2.7	2.0	2.2	6.9	15.00	2, $\frac{13}{16}$ '' o	106		.1	6.8	No change
6 7	4.0	3.0	3.4	10.4	20.00	2, 1 '' o	214		.2	10.2	2, $\frac{15}{16}$ '' o
8 9	5.4	4.0	4.3	13.7	25.00	2, $1\frac{1}{8}$ '' o	338		.2	13.5	No change
10 11	6.8	5.0	5.5	17.3	30.00	2, $1\frac{1}{4}$ '' o	500		.3	17.0	No change
12 13	8.2	6.0	6.6	20.8	35.00	2, $1\frac{3}{8}$ '' o	707		.3	20.5	No change
14 15	9.5	7.0	7.7	24.2	40.00	2, $1\frac{1}{2}$ '' o	962		.4	23.8	2, $1\frac{7}{16}$ '' o
16 17	10.9	8.0	9.0	27.9	45.00	2, $1\frac{9}{16}$'' o	1173		.4	27.5	No change
18 18	24.6	18.0	10.1	52.7	50.00	3, $1\frac{3}{4}$ '' o	1224	+ 1648 lb. steel	.9	51.8	No change

Total weight=51342+3212 lb.=54554 lb. Less 174 lb.

Pine L. L.
Maple
Aver.

SAFE RESISTANCE OF
WOODEN POSTS AND STRUTS
IN TONS

N. CLIFFORD RICKER

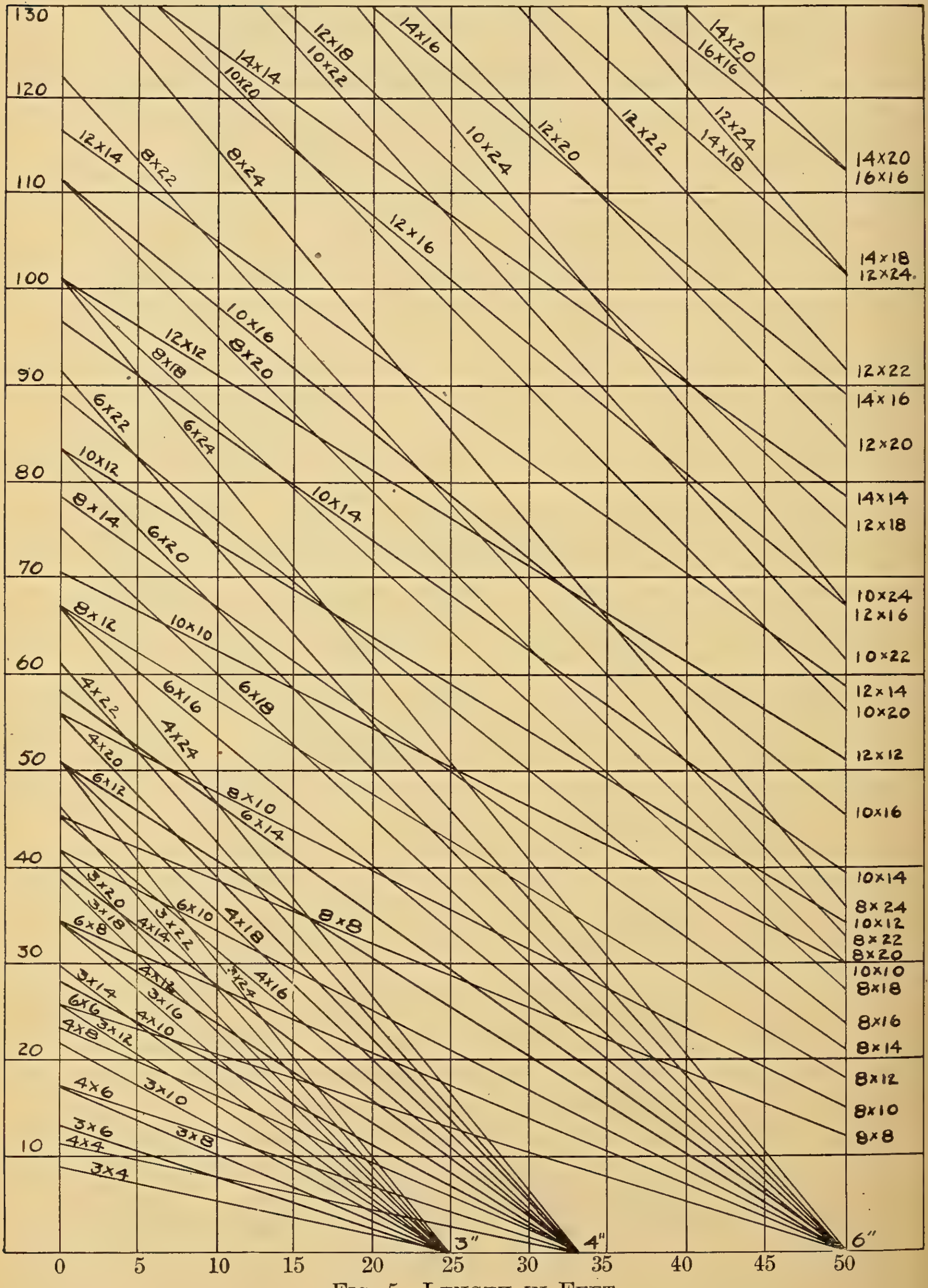


FIG. 5 LENGTH IN FEET

The magnitude of each stress is measured and written in the proper column and space of the stress sheet.

Since the diagram for snow stresses would be exactly similar to that for permanent stresses, it need not be drawn, but the snow stresses can be quickly found with a good slide rule, using the following proportion:

25.98 : 19.00 = permanent stress : snow stress.

It is best to tabulate the permanent, snow and wind stresses separately, to permit taking any combination of them as the maximum stress on the members.

Maximum stresses are here found by taking the sum of the stresses on each member.

9. *Sections of members*

Principals and struts are in compression, and their sectional dimensions are most conveniently found by the graphical table, Fig. 5, based on a modified form of Stanwood's formula for yellow pine columns, viz.:

$$p = .700 - .084 \frac{L}{d}$$

where p = safe compression in tons per sq. in.
L = free length of timber in feet
d = least side in inches

(a) Principals

The length of the principal is 11.18 ft.; and from the stress sheet the maximum compression in X 1 is 125.5 tons. Hence by the table (Fig. 5) the section of principal should be 14 in. x 14 in. For convenience in construction and better appearance, the principals are made of uniform section throughout their entire length.

(b) Struts

The struts are most conveniently framed and look best if made the same breadth as principal, so as to be flush on each side. They are here 14 in. wide, and their safe thickness or depth is found by the table.

(c) Tie-beam

The tie-beam is to be made the same breadth as principals and struts. The safe resistance of long leaf pine to tension is taken as 0.6 ton per sq. in. Then if b is the breadth of the tie-beam in inches and d the depth, we have

$$d = \frac{\text{maximum tension}}{0.6 \times b} = \frac{117.8}{0.6 \times 14} = 14.0 \text{ in.}$$

Assuming that 3 horizontal rows of 1-inch bolts are to be used in splices, we should make the total depth 14 + 3 = 17 in., or say 18 in.

(d) Vertical steel rods

The table for rods, nuts, washers, etc., is based on a safe tensile stress of 7.5 tons per sq. in. for net section of rods, whose ends are upset to standard diameters. The bearing areas and diameters of washers are based on a maximum safe pressure of $\frac{1}{3}$ ton per sq. in. on long leaf pine.

TABLE 4

Table of Steel Rods for Long Leaf Pine Timbers

Diameter of Rod inches	Safe Strength tons	Upset Ends inches	Sq. Nuts inches	Cast iron Round Washers inches	Area of Washers sq. in.	Drop inches	Weight 1 in. Rod pounds	Wt., Two Ends, Nuts and Washers
$\frac{1}{2}$	1.47	$\frac{3}{4}$ by $4\frac{1}{2}$	$\frac{3}{4}$ by $1\frac{3}{8}$	$\frac{3}{4}$ by $2\frac{9}{16}$	4.93	$2\frac{1}{2}$	.67	3.48
$\frac{9}{16}$	1.87	$\frac{3}{4}$ by $4\frac{1}{2}$	$\frac{3}{4}$ by $1\frac{3}{8}$	$\frac{3}{4}$ by $2\frac{13}{16}$	6.13	$2\frac{1}{2}$	.85	3.96
$\frac{5}{8}$	2.30	$\frac{7}{8}$ by $4\frac{1}{2}$	$\frac{7}{8}$ by $1\frac{5}{8}$	$\frac{7}{8}$ by $3\frac{1}{8}$	7.59	$2\frac{1}{4}$	1.04	5.76
$\frac{11}{16}$	2.78	1 by $4\frac{1}{2}$	1 by $1\frac{3}{4}$	1 by $3\frac{7}{16}$	9.23	2	1.26	7.70
$\frac{3}{4}$	3.32	1 by $4\frac{1}{2}$	1 by $1\frac{3}{4}$	1 by $3\frac{3}{4}$	10.85	2	1.50	8.54
$\frac{13}{16}$	3.89	$1\frac{1}{8}$ by $4\frac{3}{4}$	$1\frac{1}{8}$ by 2	$1\frac{1}{8}$ by $4\frac{1}{16}$	12.77	2	1.76	11.56
$\frac{7}{8}$	4.51	$1\frac{1}{4}$ by $4\frac{3}{4}$	$1\frac{1}{4}$ by $2\frac{1}{4}$	$1\frac{1}{4}$ by $4\frac{3}{8}$	14.88	$1\frac{3}{4}$	2.04	15.00
$\frac{15}{16}$	5.18	$1\frac{1}{4}$ by $4\frac{3}{4}$	$1\frac{1}{4}$ by $2\frac{1}{4}$	$1\frac{1}{4}$ by $4\frac{9}{16}$	16.40	$1\frac{3}{4}$	2.35	16.34
1	5.89	$1\frac{3}{8}$ by 5	$1\frac{3}{8}$ by $2\frac{3}{4}$	$1\frac{3}{8}$ by 5	19.27	$1\frac{3}{4}$	2.67	21.76
$1\frac{1}{16}$	6.65	$1\frac{3}{8}$ by 5	$1\frac{3}{8}$ by $2\frac{3}{4}$	$1\frac{3}{8}$ by $5\frac{1}{4}$	21.57	$1\frac{3}{4}$	3.01	23.42
$1\frac{1}{8}$	7.46	$1\frac{1}{2}$ by 5	$1\frac{1}{2}$ by 3	$1\frac{1}{2}$ by $5\frac{9}{16}$	24.30	$1\frac{1}{2}$	3.38	28.86
$1\frac{3}{16}$	8.31	$1\frac{1}{2}$ by 5	$1\frac{1}{2}$ by 3	$1\frac{1}{2}$ by $5\frac{7}{8}$	26.85	$1\frac{1}{2}$	3.77	30.84
$1\frac{1}{4}$	9.20	$1\frac{5}{8}$ by $5\frac{1}{4}$	$1\frac{5}{8}$ by $3\frac{1}{4}$	$1\frac{5}{8}$ by $6\frac{3}{16}$	29.84	$1\frac{1}{2}$	4.17	37.40
$1\frac{5}{16}$	10.15	$1\frac{3}{4}$ by $5\frac{1}{4}$	$1\frac{3}{4}$ by $3\frac{1}{2}$	$1\frac{3}{4}$ by $6\frac{1}{2}$	33.03	$1\frac{1}{4}$	4.60	44.70
$1\frac{3}{8}$	11.14	$1\frac{3}{4}$ by $5\frac{1}{4}$	$1\frac{3}{4}$ by $3\frac{1}{2}$	$1\frac{3}{4}$ by $6\frac{13}{16}$	36.00	$1\frac{1}{4}$	5.05	47.40
$1\frac{7}{16}$	12.17	$1\frac{7}{8}$ by $5\frac{1}{2}$	$1\frac{7}{8}$ by $3\frac{3}{4}$	$1\frac{7}{8}$ by $7\frac{1}{8}$	39.46	$1\frac{1}{4}$	5.52	56.56
$1\frac{1}{2}$	13.25	2 by $5\frac{1}{2}$	2 by 4	2 by $7\frac{7}{16}$	43.09	1	6.01	66.24
$1\frac{9}{16}$	14.39	2 by $5\frac{1}{2}$	2 by 4	2 by $7\frac{11}{16}$	46.51	1	6.52	69.76
$1\frac{5}{8}$	15.56	$2\frac{1}{8}$ by $5\frac{3}{4}$	$2\frac{1}{8}$ by 4	$2\frac{1}{8}$ by 8	50.44	1	7.05	78.70
$1\frac{11}{16}$	16.78	$2\frac{1}{8}$ by $5\frac{3}{4}$	$2\frac{1}{8}$ by 4	$2\frac{1}{8}$ by $8\frac{5}{16}$	54.10	1	7.60	82.60
$1\frac{3}{4}$	18.04	$2\frac{1}{4}$ by $5\frac{3}{4}$	$2\frac{1}{4}$ by $4\frac{1}{4}$	$2\frac{1}{4}$ by $8\frac{5}{8}$	58.32	$\frac{3}{4}$	8.18	95.06

Divide the maximum tension in a member by the number of rods composing it to find the stress in one rod; the diameter of the rod is then found in the table.

10. Splices in tie-beam and connections with principals

Splices in principals being in compression, they are simple halved splices 1 foot long with two or four 1-inch bolts each, and are made as near the apex as possible.

Splices in tie-beams and connections of tie-beams with principals are calculated by the following formulas:

Let t = thickness of one fish plate in inches.
R = number of horizontal rows of bolts.
D = diameter of bolt in inches.
b = breadth of tie-beam in inches.
d = depth of the tie-beam in inches.
a = distance in inches between centers of bolts or to end of timber.
N = total number of bolts in both ends of splice.
Z = maximum longitudinal stress in member at splice or connection.

(*a*) Connection of tie-beam with principal.

For the thickness of the fish plate, we have

$$t = \frac{Z}{15\ (d-R\,D)} = \frac{125.5}{15\ (18-3 \times 1)} = .56 \text{ in. to resist tension.}$$

$t = .525\ D = .525 \times 1 = .53$ in. to resist crushing against bolt.

Therefore we make these fish plates each $\frac{9}{16}$ in. thick.

$a = 5\ D = 5$ in. between centers of bolts or to end of timber.

The end of the fish plate is to be 2 in. outside the center of the bolt.

For the number of bolts, we have

$$N = \frac{Z}{.375\ Db} = \frac{125.5}{.375 \times 14 \times 1} = 24 \text{ one-in. bolts in the connection.}$$

These bolts must be symmetrically arranged about the axis of the member, so that it becomes necessary to put some $\frac{3}{4}$-in. bolts in 1-in. holes to prevent the fish plates from buckling outwards, being under compression.

(*b*) Splice in the member $Y\,5$.

Applying the same formulas, we find $\frac{9}{16}$-in. fish plates as before, and 20 one-in. bolts are required.

(*c*) Splice in $Y\,11$.

Fish plates must be $\frac{9}{16}$ in. thick, and 16 one in.-bolts are needed.

(*d*) Splice at middle of truss.

Fish plates should be $\frac{9}{16}$ in. thick, and 12 one-in. bolts are needed.

11. Calculation of center length weights of members

The center length of each member is easily computed or measured on the truss diagram and it is then written on the stress sheet. This center length is then multiplied by the weight per linear foot of the member, which is doubled to include both sides of the truss (except for the middle vertical 18 18). These weights are then written in the proper column of the stress sheet.

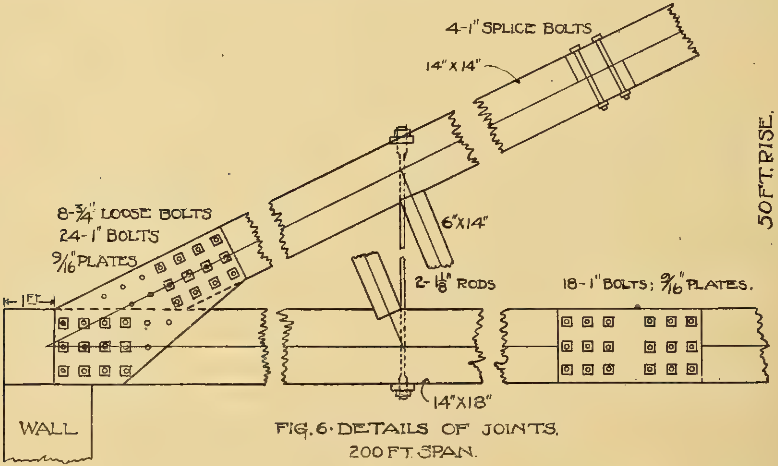


FIG. 6

12. Calculation of weights of joints and connections

The differences between the actual and center length weights of each member are next computed and written on the stress sheet.

The table of rods, etc., gives for rods their weight in pounds per linear foot, also the weight of the two upset ends, of two nuts, and of two cast-iron washers, but it does not include the portions of rod between the intersection of the center lines of the truss members and the upset end of rod. This weight must be computed and added to weight of rod ends, etc.

These weights of connections of members are written in the stress sheet.

13. *Computed weight of truss*

From the data presented in the preceding paragraphs, the total weight of the truss may now be calculated. We have

Total center length weight of members	51342 lb.
Total net weight of connections	3160 lb.
Total computed weight of truss	54554 lb.
Assumed weight of truss	58660 lb.
Deduct computed weight	54554 lb.
Excess of assumed weight over computed weight	4106 lb.

We have, therefore, excess per apex of principal =

$$\frac{4106}{20} = 205.3 \text{ lb.}$$

Whence the excess in permanent loading on one-half the truss is

$$\frac{205.3 \times 9\frac{1}{2}}{2000} = 0.975 \text{ T.}$$

This excess is $\frac{.975}{25.98} = .03\frac{3}{4}$ or $3\frac{3}{4}$ per cent of the permanent load; hence permanent stresses in the stress sheet are $3\frac{3}{4}$ per cent in excess. This excess is computed, written in the stress sheet, and deducted from the maximum stress. The corrected maximum stresses are thus found, since the snow and wind stresses on the members are unchanged.

The sectional dimensions of members are next revised for these corrected maximum stresses, the result being a slight reduction in two members only. The corresponding reduction in weight is found to be 174 pounds.

14. *Final Summary*

Net weight of wood in truss	45429 lb.
Net weight of metal, nearly all steel	8777 lb.
Total net weight of truss	54380 lb.

Hence the weight of the wood is $83\frac{1}{2}$ per cent and the weight of the metal is $16\frac{1}{2}$ per cent of the total weight of truss.

The weight of the connections is $6\frac{1}{2}$ per cent of the center length weight of the truss.

Weight per square foot of horizontal projection of roof:

Roof covering	2.12 lb.
Sheathing	4.24 lb.
Rafters	2.12 lb.
Purlins	3.74 lb.
Truss	13.58 lb.
Permanent load	25.80 lb.
Snow load	20.00 lb.
Wind load	17.70 lb. on inclined roof.

V. WEIGHTS OF TRUSSES OF DIFFERENT SPANS

Ten trusses were designed for spans of 20, 40, 60, 80, 100, 120, 140, 160, 180, and 200 ft. respectively. The trusses were set 20-ft. apart with a uniform rise of $\frac{1}{4}$ span, the panel lengths being 10 feet. Their weights were computed and plotted in Fig. 7, and the points connected by the broken line *D*. The increasing slope of this line shows that the weight of the truss increases faster than the span. The total permanent weight of the roof also increases with the span. The additional weight of the connections, however, diminishes very rapidly for spans less than 60 ft. and varies irregularly for greater spans. Other weights are constant.

VI. FORMULA FOR WEIGHT OF TRUSS

The following empirical formula for weights of long leaf pine and steel trusses is represented by the dotted line in Fig. 7, which agrees well with the computed line *D*.

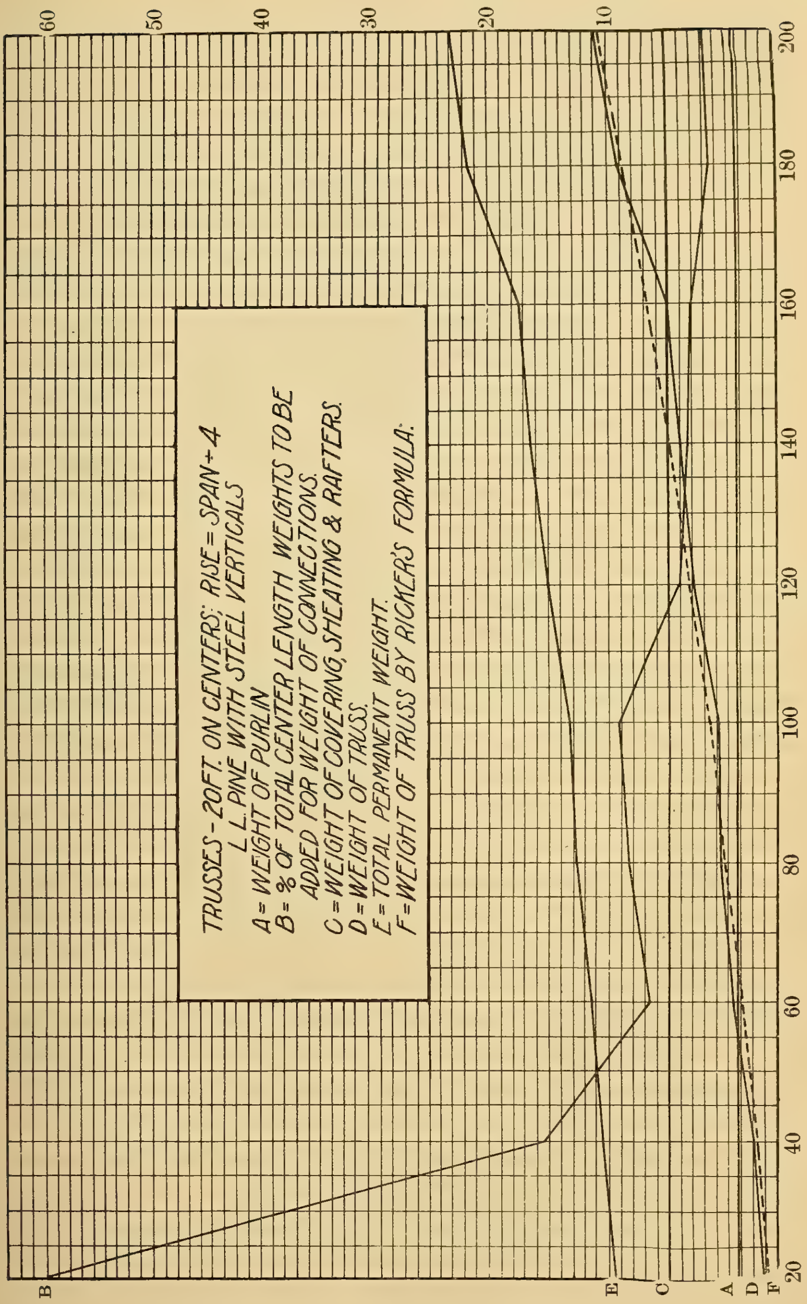
$$W = \frac{S}{25} + \frac{S^2}{6000}$$

where S = span in feet
and W = weight of truss in lb. per sq. ft. of horizontal projection of roof.

VII. WEIGHT OF WHITE PINE AND STEEL TRUSSES

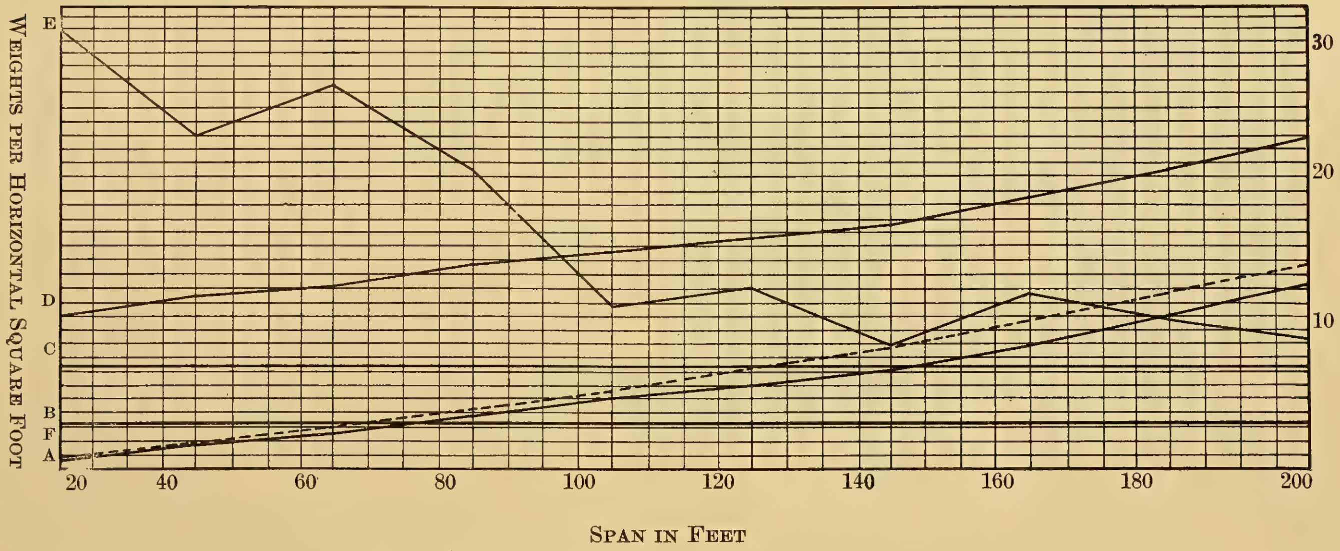
A series of trusses constructed of white pine timbers and steel verticals was also designed and computed. The results of these computations are shown in Fig. 8. The preceding formula

FIG. 7 SPAN IN FEET



WEIGHTS PER HORIZONTAL SQUARE FOOT

FIG. 8. TRUSSES 20 FT. ON CENTERS-RISE÷SPAN÷4
WHITE PINE WITH STEEL VERTICALS



B—Weight of purlin.
E—Per cent of total center length weights to be added for weight of connections.
C—Weight of covering, sheathing and rafters.
A—Weight of truss.
D—Total permanent weight.
F—Weight of truss by Ricker's formula.

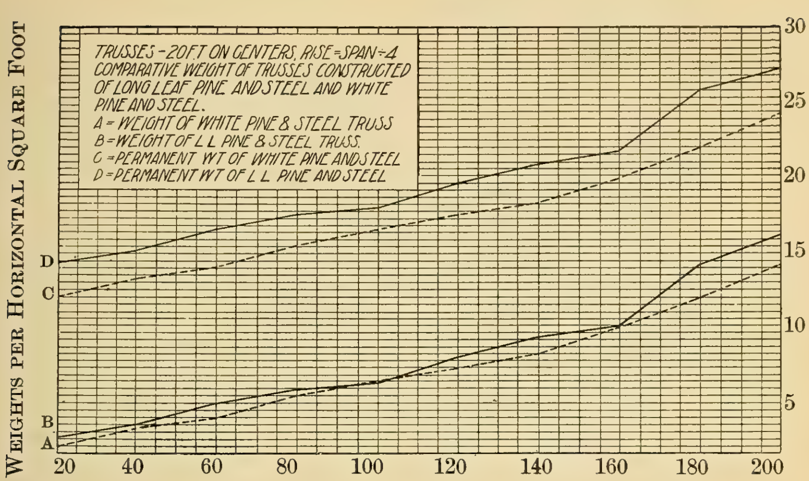


FIG. 9 SPAN IN FEET

is represented by the dotted line *F*, which here gives weights somewhat exceeding those found by computation. In Fig. 9 is shown a comparison of the weights of trusses and also of the total weights of roofs constructed of the two kinds of wood. White pine makes a somewhat lighter truss and roof than long leaf pine. However, in designing, it will be most convenient and accurate to apply the same formula and make the necessary reductions on the stress sheet.

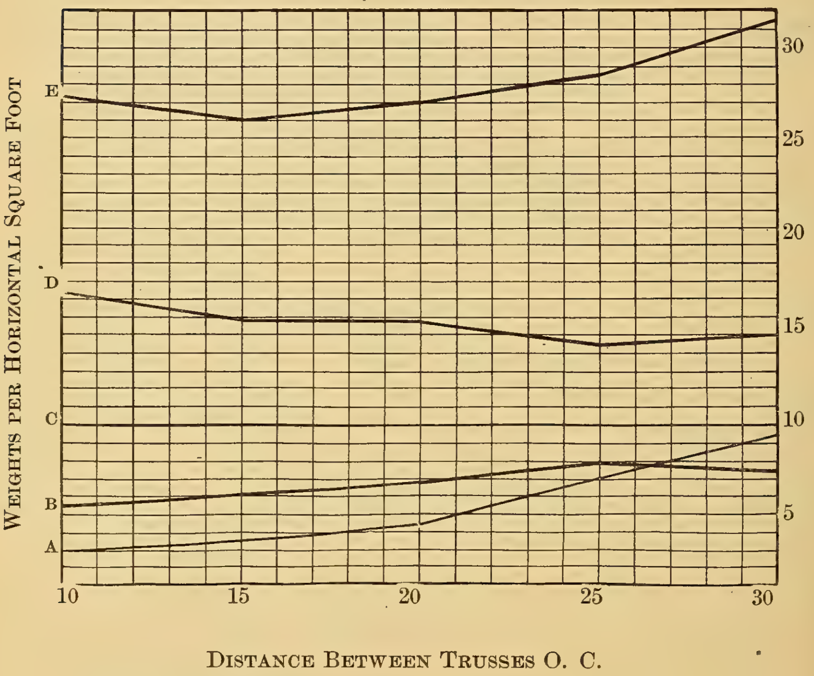
VIII. WEIGHT OF STEEL TRUSSES

Several steel trusses of different spans were also designed and computed. Their weights for spans of 100 and 200 feet were found to be about the same as those of long leaf pine and steel trusses. Therefore the formula is also applicable to steel trusses. It is very probable, however, that for short spans, steel trusses are somewhat heavier than those of wood and steel given by the formula; their connections are far more complex and certainly require the addition of a larger per cent to the center length weights of truss. But these variations are taken into account on the stress sheet.

IX. Most Economical Distance Between Trusses

Trusses of 200-ft. span and 50-ft. rise spaced 10, 15, 20, 25 and 30 ft. apart respectively were designed and their weights were computed. The results are shown in Fig. 10. The weight of covering, sheathing, and rafters remains constant; the weight of the purlins and connections increases, and the weight of the truss diminishes as the distance between the trusses increases, being a minimum for a spacing of 25 ft. The total weight of the roof, however, is a minimum when the trusses are 15 ft. apart.

Fig. 10. Trusses 200-ft. Span-50 ft. Rise
L. L. Pine with Steel Verticals

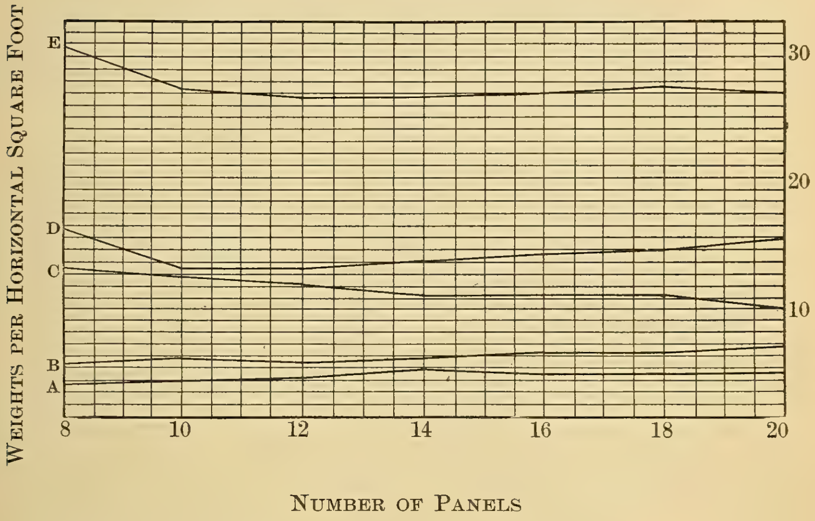


A—Weight of purlin.
B—Per cent of total center length weights to be added for weight of connections.
C—Weight of covering, sheathing and rafters.
D—Weight of truss.
E—Total permanent weight.

X. MOST ECONOMICAL SIZE OF PANELS

Trusses of 200-ft. span, 50-ft. rise, set 20 ft. apart, were divided into 8, 10, 12, 14, 16, 18 and 20 panels to determine the best length of panel. The results are given in Fig. 11 and show the following: (1) that the weight of covering, sheathing and rafters increases with the panel length; (2) the weight of the purlins diminishes; (3) the weight of the truss, and very nearly that of the entire roof, is a minimum for a panel length of 20 feet. Therefore 20-ft. panels appear to be most economical.

FIG. 11. TRUSSES 200-FT. SPAN-50 FT. RISE-20 FT. ON CENTERS
L.L. PINE WITH STEEL VERTICALS

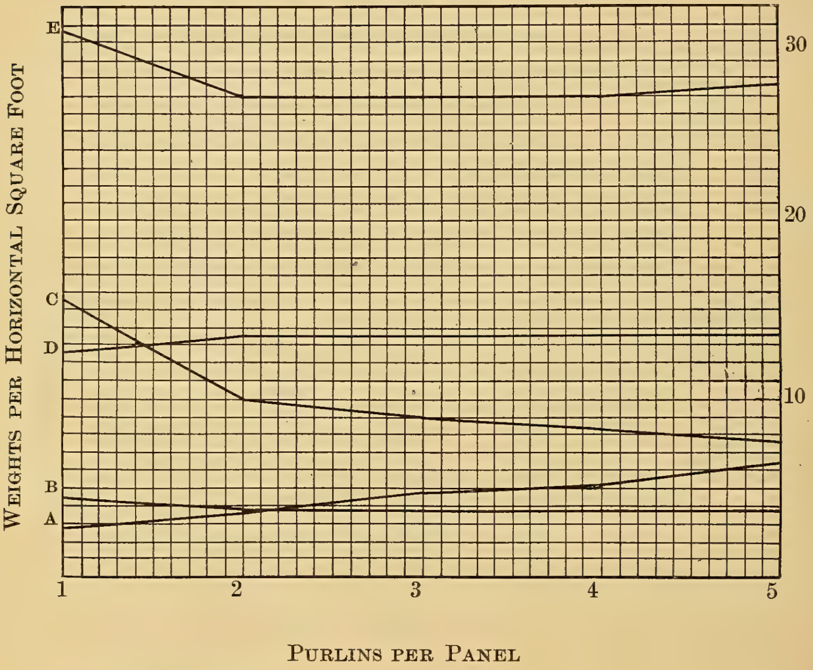


A—Weight of purlin.
B—Per cent of total center length weights to be added for weight of connections.
C—Weight of covering, sheathing and rafters.
D—Weight of truss.
E—Total permanent weight.

XI. BEST NUMBER OF PURLINS PER PANEL

For a series of trusses of like dimensions with panel lengths of 25 ft. from 1 to 5 purlins were used on each panel length of the principal, which was therefore required to resist safely the longitudinal compression and the stresses caused by the weights of the purlins and their loads. The results are plotted in Fig. 12. These show (1) that the weight of the purlins increases with their number; (2) the weight of covering, sheathing, and rafters decreases; (3) the additional weight of the connections slightly diminishes; (4) the weight of truss is increased, but is least with one purlin per panel. The total weight of the roof is least for 2, 3 or 4 purlins. No advantage results from the use of more than 2 purlins per panel of 25 ft. or of more than one for panels of ordinary size.

FIG. 12. TRUSSES 200-FT. SPAN--50 FT.-RISE-20 FT. ON CENTERS L. L. PINE WITH STEEL VERTICALS; EIGHT PANELS

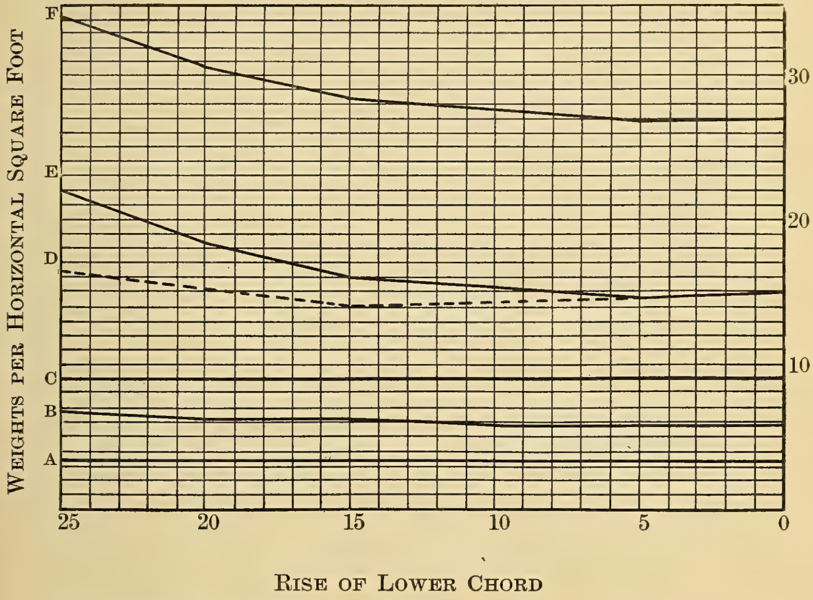


A—Weight of purlin.
B—Per cent of total center length weights to be added for weight of connections.
C—Weight of covering, sheathing and rafters.
D—Weight of truss.
E—Total permanent weight.

XII. EFFECT OF RAISING LOWER CHORD AT CENTER OF SPAN

Trusses of like dimensions were designed, excepting that the lower chord was raised 0, 5, 10, 15, 20 and 25 ft. respectively at the center of span, the rise of upper chord being 50 ft.

FIG. 13. TRUSSES—200-FT. SPAN—20 FT. O. C.—50-FT. RISE OF UPPER CHORD—LOWER CHORD RAISED



A—Weight of purlin.
B—Per cent of total center length weights to be added for weight of connections.
C—Weight of covering, sheathing and rafters.
D—Weight of truss with straight lower chord.
E—Weight of truss with raised lower chord.
F—Total permanent weight.

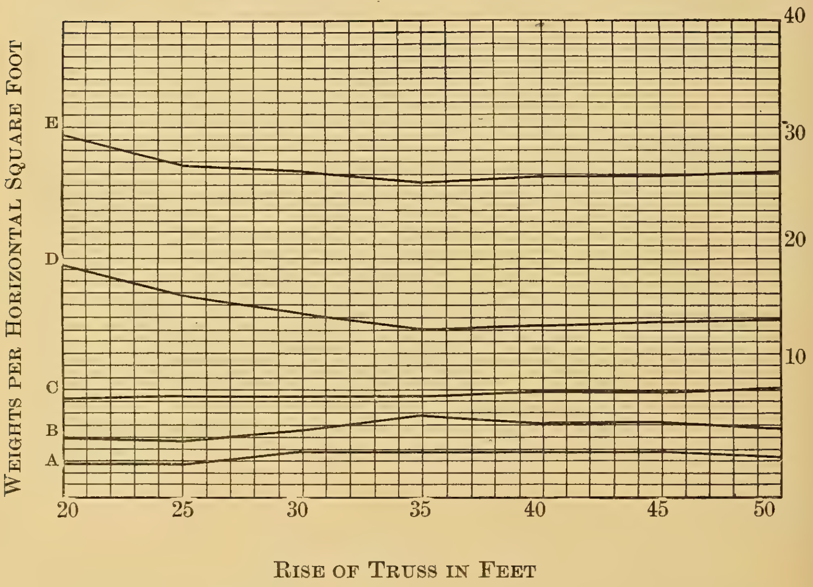
The results are given in Fig. 13. The dotted line *D* represents the weights of trusses of equal depth at the center, but having a horizontal lower chord and supporting smaller normal wind pressures on account of their lesser inclinations.

The weight of covering, sheathing, rafters, and purlins remains constant; the weights of trusses and of roofs both increase rapidly with the rise of lower chord. A comparison of curves *D* and *E* shows clearly that this raising or cambering of the lower chord is not economical and is done only for effect.

XIII. MOST ECONOMICAL RATIO OF RISE TO SPAN OF TRUSS

A series of trusses of 200-ft. span and set 20 ft. apart was also designed and computed for rises of 20, 25, 30, 35, 40, 45 and 50 ft. respectively in order to determine the best proportion of rise to span. The results are plotted in Fig. 14. As the rise increases, the weights of covering, sheathing, rafters, and purlins slightly increase; also the additional weight of the connections: the weight of trusses and that of the roof diminish, each being a minimum for a rise of 35 ft. which is practically $\frac{1}{6}$ the span,

FIG. 14. TRUSSES 200-FT. SPAN-20 FT. ON CENTERS
L. L. PINE WITH STEEL VERTICALS



A—Weight of purlin.
B—Per cent of total center length weights to be added for weight of connections.
C—Weight of covering, sheathing and rafters.
D—Weight of truss.
E—Total permanent weight.

identical with the ratio for ordinary bridge trusses. Hence the best rise is $\frac{1}{6}$ the span.

XIV. Summary of Results

(a) Comparison of formulas for normal wind pressure.
(b) System of calculation and design.
(c) Form of stress sheet.
(d) Formula for weight of truss.
(e) Comparative weights of trusses of other materials.
(f) Economical distance between trusses.
(g) Economical length of panels.
(h) Economical number of purlins per panel.
(i) No advantage results from cambering lower chord.
(j) Economical ratio of rise to span of roof trusses.

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BULLETIN NO. 17

THE WEATHERING OF COAL

BY

S. W. PARR

AND

N. D. HAMILTON

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 17 AUGUST 1907

THE WEATHERING OF COAL

BY S. W. PARR, PROFESSOR OF APPLIED CHEMISTRY AND
N. D. HAMILTON, FELLOW IN CHEMISTRY

(IN CONNECTION WITH THE ILLINOIS STATE GEOLOGICAL SURVEY)

The weathering of coal is a subject of perennial interest. Moreover, its importance is not likely to grow less in view of the trend of modern industrial conditions. Almost every circumstance attending the production and use of coal calls for greater flexibility in the current supply. The seasonal changes in temperature, the fluctuating demands for freightage, the periodical disturbances in the labor supply, all argue for the possibility of carrying at certain periods a reserve of fuel to guard against suffering or complete paralysis of industry.

It is not surprising, therefore, to learn of the increasing number of storage plants where coal may be placed in large quantities to supply the needs of great industries which would suffer, if from any cause or combination of circumstances a coal famine should ensue. An example of this is the plant of the Philadelphia and Reading Coal and Iron Company at Abrams, Pennsylvania. At this plant arrangements are made for eight piles of coal each containing 60 000 tons piled on the ground in the open and equipped with modern facilities for dumping and reloading. The storage plant also of the New York Edison Company at Shadyside, New Jersey, where 150 000 tons of coal are stored in three piles on a bed of cinders in the open, and that of the Lehigh Railroad at Wyoming, New York, with a capacity of 100 000 tons, are but a few of the instances where our industries are resorting to outdoor storage for reserve coal supplies. This method of storing is not only the practice where large quantities of coal are to be taken care of, but there are very few power and heating plants and fuel-using industries that do not find it necessary to pile more or less coal on the ground at least temporarily until room can be made for it in the coal shed or boiler house. The practice in vogue in the coal fields, among coal dealers and all consumers of comparatively small quantities varies greatly. In general, however, it may be said that the coal is either stored in covered iron or wooden bins with slanting bottoms to facilitate its removal, or in ordinary covered bins with earth, cinder or wooden floors from which it is removed with the scoop. In power and heating plants the placing of the coal bins depends upon the position of the boilers and the method used for firing and transportation of the fuel to the building. The United States Navy, which uses about 250 000 tons of coal yearly, has been equipped with large storing facilities, each compartment in the station at the New York Navy Yards having a capacity of 525 gross tons, with floors of portland cement, roof of iron and side walls of portland cement, sand and anthracite cinders. On board vessels coal is stored in whatever room happens to be available.

Aside from these prevalent methods of caring for coal reserves, the practice of storing coal under water is coming into prominence, but the working out of this plan can not be said to have gone further than the experimental stage. The English Admiralty has been experimenting with submerged coal and the Western Electric Company of Chicago has recently built two bins of 4000 and 10 000 tons respectively below the ground level. The plan is to dump the coal into the bins directly from the car and flood it with water until needed for use, when a crane fitted with grab buckets will lift it to the car again.

As one reviews the literature of the subject, it is strikingly evident that well authenticated facts and data are very meagre, with much disagreement among those who have presumed to possess knowledge in the matter. There is a general belief that coal does deteriorate, but under what conditions, to what extent, and according to what principles, are certainly open questions at the present time. The following resumé of the statements by various writers will be of advantage in gaining a present knowledge of the situation. There are included also, references which deal with the spontaneous ignition of coal, on account of the close relationship of that topic to the one in hand.

HISTORICAL REVIEW

After careful experiments, Dr. Richter in 1868[1] concluded that the weathering of coal is due to the absorption of oxygen, a part of which goes to the oxidation of carbon and hydrogen in the coal, and part is taken into the composition of the coal itself; that if the heap becomes warm, either through this process or through any other cause, the action is accelerated, but then falls off and becomes so slow that the changes effected within a year are difficult to estimate; that moisture as such has no direct influence upon the process, apart from the presence of pyrites or from the coal crumbling down more rapidly when wet than when dry, and therefore, more rapidly heating up. At a later date, he concluded that large coal was less affected than small, not because it had less surface, but because small coal was a more active absorbent of oxygen, and therefore, became more rapidly heated; that airways in heaps would have to be very numerous in order to prevent any rise in temperature.

Haedicke in 1880[2] , while assigning to pyrites the leading part in spontaneous combustion, agrees that this hypothesis does not hold good unless the temperature is allowed to rise sufficiently.

Professor Fischer of Gottingen as a result of research work prior to 1901[3] concludes that storage depreciation and spontaneous ignition are phenomena of oxidation; the part which is played by iron sulphide has been disputed, but the variances that have given rise to the uncertainty are due to the differences between the different sulphides of iron present in coal. Marcasite for example, he says, is much more weatherable than ordinary pyrites. Actual wetting is much more promotive of oxidation of the iron sulphide than heating in dry or even moist air. He also finds that many coals contain sulphur in the form of unsaturated organic compounds. He finds that those coals which rapidly absorb bromine are those which are most liable to rapid oxidation and spontaneous ignition, and as a practical test he recommends shaking a gram of the finely ground coal with 20 cc. of a half normal solution of bromine for five minutes.

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[1] Proc. Ger. Gas Association, 1900.

[2] The Gas World, April 13, 1901.

[3] The Gas World, April 13, 1901.

Then, if the smell of bromine has not disappeared, the coal may safely be put in store; if the odor has gone, the coal should be used up as soon as possible. If the oxygen is absorbed by the unsaturated organic compounds, the coal gains weight, but if absorbed by the carbon and hydrogen, this absorption causes a loss in weight due to the carbon-dioxide and water given off. Whether a coal gains weight is, therefore, dependent upon the composition of the coal. Covering wet slack coal with other coal is apt to produce spontaneous ignition; the danger here appears to arise from the sulphide of iron rather than from the organic compounds. Professor Fischer regards ventilation of the coal heap with suspicion, not because the idea is in itself wrong, but because it is not practicable to ventilate the whole heap sufficiently. He says the coal should be stored dry and kept dry under cover and in layers not too deep.

Durand, 1863[1], explains the spontaneous ignition of coal in the pit by the presence of pyrites, which, becoming heated, gives rise to combustion. Payol maintains that the main cause of spontaneous ignition is the absorption of oxygen accelerated by fine division and heat.

Jackson, 1905[2], says it is well known that coal on exposure to the weather does lose some of its volatile constituents even under ordinary conditions.

In a paper read before the German Gas Association in 1900[3], Herr Sohren said that it is no longer possible for many reasons to operate gas works with a supply of coal renewed from month to month; and that all questions affecting storage have therefore a continuously increasing importance. Undoubtedly there is a greater or less depreciation in quality of coal kept in store; and the causes of this have attracted a great deal of attention, though, on the whole, it is surprising to find to how great an extent the study of the chemistry of coal has been neglected. Questions of this nature first assumed importance in connection with the spontaneous ignition of coal in ships; in 1874 he declares that out of a list of 4485 coal laden ships, no fewer than 60 went on fire.

Lieutenant Commander J. R. Edwards of the United States Navy, 1901[4], said that experience has taught the dealers at Trinity

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[1] Journal American Chemical Society, Dec., 1900 and July, 1904.
[2] Engineering and Mining Journal, July 14, 1906.
[3] Gas World, April 13, 1901.
[4] American Society of Naval Engineers, Feb., 1901.

Building, New York, that every time coal is handled there is a depreciation of five per cent in value due to the loss in weight by the breaking up of the coal and the volatilization of the hydrocarbons. It is a fact that the best coal does not disintegrate and powder so quickly as the poorer quality. According to his theory, the hydrocarbons make it less friable.

Groves and Thorp[1] state that gases occluded in the crevices or cavities of coal escape during mining and continue to do so after storing, and that disastrous explosions on vessels carrying coal cargoes have resulted. An analysis of these exuded gases reveals their inflammable nature and suggests the probable action of the air on coal which is exposed to it for any length of time. This latter action is termed "weathering," and consists mainly in the combination of the carbon and hydrogen of the coal with the oxygen of the air, carbonic acid and water resulting. Pyrites, if present, is also oxidized and when present in large quantities causes the coal to disintegrate and oftentimes to be nearly useless. Calorific value is diminished by this exposure to the air and in some cases there is claimed to be 50 per cent loss. Oxidation may proceed so far that elevation in temperature occurs and spontaneous combustion results. Oxidation of pyrites, especially in the presence of moisture, greatly adds to the danger. To avoid the small coal, the packing of coal in lumps in vessels is proposed, but the movement of the ship would break up the coal and only delay the action. Sealing up the coal hermetically has also been suggested, but this would be impossible.

T. Rowan suggests the heating of coal to drive off the moisture before storing on board. The plan is a poor one on account of the cost and the oxidation which the heating would promote.

A. D. Parker, General Auditor, Colorado Southern Railroad, 1902[2], states that the loss in transporting coal has never been definitely determined. It consists of shrinkage, droppings, stealing, etc. Evaporation or shrinkage is inevitable. It is greater with softer coals and diminishes with density. Where coal is placed in storage, shrinkage becomes a very large item.

Mr. Stelkins, in his report before the International Navigation Congress, 1902[3], states that the tendency towards spontaneous ignition increases with the height to which coals are heaped. Stacks should not be made higher than five meters. Warm rains during and after stacking and strong compression by dumping coals from a great height all add to the danger of ignition. According to past experiences, gas flaming pit mouth coals ignite most readily, fat pit mouth slack, lean slack and nut less readily, and lump coals only very seldom. When the amount of slack increases, and the amount of stony material increases, the height to which it is safe to store coal decreases. Mr. Zorner in this same convention claimed that lightering coal renders it more liable to physical and chemical attacks and more difficult to use as fuel. Rischowske ascertained a loss of three per cent in calorific power of fresh slack coal after a storage of four months.

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[1] Chemical Technology Vol. 1, page 82-83.
[2] Railway Engineering News, May 3, 1902.
[3] American Society of Naval Engineers, Feb., 1901.

Engineering News, July 21, 1904, notes that in the New York Navy Yard space is not an item, but spontaneous combustion is an important one, hence the coal depth is limited to 18 to 25 feet, and the walls surrounding the bins are fire-proof. In each bin which contains 525 tons of coal are placed two four-inch pipes, each containing thermostats electrically connected to an annunciator. These thermostats containing tubes are arranged so as to be moved completely through the coal. A method of removing coal from the bottom of the bins is provided, and this may be done and the portion removed and redistributed over the top to prevent fire.

F. M. Griswold, of the Home Insurance Company, 1904[1], says that spontaneous ignition is more marked in free burning or so-called "high steaming coals" including "gas coal." These coals usually contain a large per cent of volatile constituents with a modicum of oxygen, and the tendency to ignition is greater when lignite or sulphur in any form, and especially when iron pyrites is present. Dirty or mine-run coals, wherein fine particles sift to the bottom and are compressed are dangerous. He claims that no satisfactory explanation of spontaneous ignition of bituminous coal has been made. The best authorities say it is due to chemical changes in the substance of the coal resulting from the absorptive powers of carbon increasing with the rise of temperature. Rise in temperature may be due to the chemical action in the form of the slow oxidation or to mechanical force or pressure and these conditions may be stimulated by pyrite or moisture. Some claim that over 2½ per cent of sulphur in the form of pyrites is dangerous. Various tests have been proposed to determine the liability of a coal to heat, such as determining the gain in weight of a sample at 250° Fahr. and also by noting the absorption of bromine, but these are not valuable as it is difficult to tell how much oxygen the coal has already absorbed. He recommends that no wood be used in the construction of bins, that all iron work be covered with concrete, that there should be no steam pipes or flues in bins, the coal should not be above a depth of twelve feet, bins should be roofed, permanent pipes should be provided, if possible, containing thermostats through the bin, and when 140 Fahr. is reached something should be done to stop rise in temperature.

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[1] Engineering News 27, 1902, July 21, 1904, Aug. 18, 1904 and Nov. 10, 1904.

Professor Vivian B. Lewis, Royal Naval College, 1906[1], is authority for saying that increase of mass leads to spontaneous combustion. Substances, especially those of vegetable origin, undergo slow oxidation at temperatures below ignition point. A certain increment of temperature is generally needed to start slow combustion, but when once the required rise takes place, the operation commences and the ignition point is reached. Initial increase may be brought about in several ways: first, by physical action as in the absorption of a large volume of gas and its compression within the pores of the substance; second, by a rise in atmospheric temperature; third, by a direct chemical reaction taking place at ordinary temperature and by the action of ferments on moist organic matter. To the first class belongs the spontaneous ignition of a mass of powdered charcoal or lamp black. Coal may be considered as consisting of carbon, hydrocarbons and inorganic constituents. Among the latter is iron pyrites. If piled in heaps and exposed to air and moisture, it rapidly heats and often inflames, owing to the oxidizing action of the air and moisture upon the sulphur; many think this is the cause of spontaneous ignition. Careful study of phenomena occurring during the heating of coal leads to the conviction that pyrites plays but a subsidiary part, and that it is really the absorption of oxygen by the freshly won coal and the activity of the condensed gas in contact with the hydrocarbons of the coal that are the active factors in causing ignition. In the coal seam, coal pores are filled with methane or methane and carbon dioxide.

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[1] Engineering and Mining Journal July 14, 1906.

When coal is brought to the surface, it exudes these gases and absorbs oxygen from the air. As long as the pieces are fairly large, no heat is perceptible, but as coal becomes broken up, the surface increases and the absorption of oxygen is increased. Mere absorption of oxygen is insufficient to bring about serious consequences unless there is an initial rise in temperature. Hence, spontaneous ignition is found to occur when cargoes go through the tropics and when coal is stored close to boilers, steam pipes, fire boxes, etc. Water aids the action of the occluded oxygen, and hence rain, when coal is being loaded, causes danger. Ventilation of coal on land may aid in preventing ignition, but this is hard to obtain on shipboard. Steam and water have failed to quench fires successfully. Sulphur dioxide and carbon dioxide will extinguish the fires, but not cool the coal and prevent another fire. He suggests that liquid carbon dioxide be placed in vessels whose nozzles are made of an alloy of lead, tin, bismuth and cadmium melting at 93° C. and that these vessels be placed in the coal bins. When the melting point of the alloy is reached, the carbon dioxide will quench the fire, cool the coal, be absorbed by the coal and prevent any further occlusion of oxygen.

A. E. Dixon, 1906[1], says that bituminous and semi-bituminous coals are losing constantly in heating value. Gas is being liberated, and the loss is greater in a warm climate and in warm weather. Bituminous coals undergo a slacking process, the lumps shrink and the percentage of dust increases. In winter the contained moisture freezes and breaks up lump coal. Spontaneous ignition occurs with bituminous, friable coal, and particularly with those grades containing brasses or iron pyrites, and when the coal is damp, the trouble is augmented. The cause is probably due to the absorption of oxygen by carbonaceous material, just as is the case with oily cotton waste.

H. R. King, 1905[2], claims that the carefully executed tests in Europe show that 30 per cent of the fuel value of coal is lost in six weeks when coal is stored out of doors.

The Naval and Military Record of England[3] gives an instance of where the British ship, Spartiate, required 3000 tons of coal stored in England in running to China and 4400 tons of practically the same grade of coal that had been stored in China for the return trip.

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[1] Engineering Magazine, Sept., 1906.

[2] Journal Western Society of Eng., Aug. 1906.

[3] Engineer, July 24, 1903.

Lord Charles Beresford, 1903[1], stated that in his experience a vessel would have to consume more than twice the normal amount of coal per indicated horse-power, if the coal had been kept too long in store.

Churchyard, locomotive engineer of the Great Western Railway, England, 1902[1], stated that judging from his personal experience and observation, the loss of stacked coal in steam raising power is about 10 per cent.

Mines and Minerals, 1901[2], says that for many years there has been in vogue in New South Wales a custom of taking certain percentages from the gross weight of coal cargoes, giving bills for the net quantity only. The idea was to allow for the wastage which it was thought took place in various ways between the time of weighing at port and delivery. The practice has been to deduct two per cent from the gross weight of foreign exportations and one per cent from that intended for intercolonial markets. On account of the dissatisfaction with this method, on January 1, 1901, the deduction on foreign cargoes was reduced to one per cent and no allowance is now made on intercolonial cargoes.

The Journal of the Society of Chemical Industry, 1894[3], says that various kinds of coal were exposed freely to the air, immersed for twelve months in water both running and stagnant and changes produced in their composition and heat of combustion determined. The three kinds of coal used were, (1) from Frankenholz mine, Bavaria; (2) from Drocourt; (3) from Arsean, Prele. These coals were broken and passed through 10 mm. mesh, but not 3 mm. mesh. Measurements show that exposure to the air or immersion in water for the time indicated produces changes in the composition and heat of combustion which are so small as to be neglected for practical purposes.

John Macaulay, General Manager of the Alexandra Docks and Railway, Newport, Monmouthshire, 1903[1], estimates that in case of coal stacked by the Mersey Railway the loss was between 10 and 12 per cent and, if kept over a year, the greatest loss is in the first twelve months. In hot climates the loss is greater. Mr.

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[1] Practical Engineer, Oct. 2, 1903.

[2] Mines and Minerals, 1901, p. 6.

[3] Jour. Soc. Chem. Ind., 1894, p. 1182.

Macaulay says that the mud men along the Usk River gathered parts of submerged cargoes and found that the coal gave a hotter fire than did fresh coal. In North Pembrokeshire, they refloated a vessel which had been sunken for two years and found that the coal was the best they had ever used. Mr. Macaulay's own experiments include the placing of a sample of the best Monmouthshire coal under sea water for two months and comparing the calorific value before and after immersion. He found the loss in heating value was less than one per cent. In his further experiments he made a practical test of fresh coal and coals known to have been submerged various lengthy periods of time, by using them in a locomotive hauling a known load a certain distance under similar conditions. His first sample was the best Monmouthshire coal procurable, the second sample had been under water three years, the third had been submerged ten years, the fourth had been recovered by mud men outside of the mouth of the River Usk. This latter was driftage from the wrecks in the Bristol Channel, and had probably been under water considerably longer than ten years; this sample he called "river coal." The order of value in steam raising and actual working results, in which the coals came out in the tests, was: (1) the river coal; (2) coal that had been under water ten years; (3) fresh coal; (4) coal that had been under water three years. Comparing values with the fresh or test coal, the river coal was four per cent better, and that which had been under water ten years 1.8 per cent better. That which had been under water three years had lost 1.6 per cent of working power. The high value of the older river coal, he says, may probably be accounted for by the fact that in traveling through the mud and sand that gave it its rounded form, the harder and better kernels, as it were, had been preserved, and the looser textured, less valuable outside portions were worn away. As a result of his experiments Mr. Macaulay concludes that steam coal loses very little of its power by submersion under water for the length of time that it would reasonably be kept in naval store, and that as it is so important to naval vessels to gain the benefit of their full working power, and so much of this is due to the coal, the subaqueous storage of coal is advisable in place of the present methods of storing with access to air, by which so high a percentage of working power is lost.

SUMMATION OF OPINIONS

Judging from these opinions of practical engineers and scientists, the present methods of coal storage without doubt often result in much loss from fires of spontaneous origin and more or less loss by a deterioration in fuel value of the coal itself. The leading factors entering into the cause of these losses have been pointed out as being; (1) the kind of coal as to its volatile combustible content; (2) the presence of occluded inflammable gases in the coal both before and after mining; (3) the presence of pyrites or other sulphur compounds; (4) the size of the coal: (5) the presence of moisture: (6) the temperature; and (7) the accessibility of oxygen to the coal.

From the evidence at hand there seems to be very little doubt that the coals of the lignitic, bituminous and semi-bituminous character with their relatively high amounts of volatile combustible matter have a much greater tendency to weather than the anthracites where the volatile matter is low. There is considerable evidence that methane and other inflammable gases formed during the decomposition of vegetable matter which produces the coal are contained in the crevices of the coal as it lies in the earth, and are liberated both during and after mining. This exudation of inflammable gaseous matter may be a prime element in mine explosions, and its continuance after storage may be a large factor in the deterioration processes.

Opinions differ as to just what part sulphur compounds, the most important of which is pyrites, play in the deterioration of coal. Some assign the leading part in cases of spontaneous ignition to pyrites, while others think that its action in this connection is of only minor importance and that absorbed oxygen has most to do with this phenomenon. Observations on the effect of the air upon pyrites, however, seem to have pretty generally established the notion that pyritic oxidation tends to raise the temperature of the coal as well as to increase the tendency of the coal to break up, and that this oxidizing action is quite appreciably increased by the presence of moisture.

That slack is much more liable to spontaneous ignition and the deteriorating influence of weathering agents seems to be the general opinion. Having more surface the finer particles absorb oxygen much more rapidly and this rapidity of absorption causes an increase in temperature which in turn produces better conditions for absorption and chemical action between the carbon, hydrogen and pyrites of the coal and the absorbed oxygen. It would seem that the finer coal would hold the moisture longer, resulting in a greater use being made of its catalytic qualities.

It is thought by some authorities that the only part moisture plays in the deterioration of coal is to materially assist the pyritic oxidation or by alternate freezing and thawing in the crevices of the coal to expose more surface to weathering agents. There are many, however, who believe that aside from increasing the oxidation of pyrites, water has to do with other chemical activities, which result in the decomposition of the coal. These believe that oxidation of the carbon and hydrogen of the coal is hastened by the action of the water present. This latter view seems to be based on the fact that moisture has seemingly, in some instances, greatly increased the deterioration of practically non-pyritic coal.

That an increase of temperature has much to do with increasing the activity of the other deteriorating agents is the general belief. This rise of temperature whether coming from outside sources or physical or chemical action within the coal tends to accelerate the absorption of oxygen and thereby increases the oxidation going on and also evaporates the gases which may still be occluded in the coal. Thus heat assists in decreasing the fuel value of the coal and at the same time increases its liability to ignition. That the exclusion of oxygen from coal will decrease its loss in heating value is a growing belief.

From the evidence at hand, therefore, it would seem that not only do observers differ widely as to the causes and extent of weathering, but no very exact study of the problem has been made in all of its phases on which could be based very much either of theory or fact concerning the deterioration of coal in storage.

Any discussion of the matter from our own standpoint will be reserved until after presenting the results of our experiments as outlined in the following pages.

Experimental Work

It seems necessary to concede at the outset that coals will differ as to the extent of their deterioration, because of their individual peculiarities of either a chemical or physical nature. It should be said at the outset, therefore, that in the present studies no attempt has been made to include all types of bituminous coals, but only those of the Illinois field. In this series an effort has been made to cover a number of localities furnishing a fair representation of the different coals of the state. Briefly outlined the conditions under which the coals were studied were as follows. The starting point was the coal in its normal state, that is, as nearly as possible corresponding to the condition existing when broken out of the vein. The period of time between the mining of the coal and the initial analysis varied somewhat, but the first series of tests was made as soon as possible after the coal was mined. In the light of subsequent developments greater stress should be put on the early examination of samples to determine the initial condition. Even under the most careful disposition of samples in laboratory containers, a deterioration takes place which, while not exactly a "weathering" process, is still a large element in any study of the case and must be considered if exact conclusions are to be available. This feature will be more fully discussed under another heading[1].

There were nine initial samples taken of approximately 100 pounds respectively. The coal was of small lump or nut size and each sample was subdivided in order to subject the same kind of coal to various conditions. These conditions were to be continued through nine months and in general were:

(a) Outdoor exposure.

(b) Exposure to a dry atmosphere at a somewhat elevated temperature, ranging between 85° and 120° Fahr.

(c) Under the same conditions as (b) so far as temperature was concerned, but to be drenched with water two or three times per week.

(d) Submerged in ordinary water at a temperature approximately 70°.

The periods for examination were divided as nearly as the work would permit into

1. The initial analysis of the fresh coal.
2. After exposure for five months.
3. After exposure for seven months.
4. After exposure for nine months.

For the sake of comparison also the calorific values were determined under uniform conditions throughout by means of the Parr calorimeter and the results calculated to the ash and water free basis to eliminate any variations in the process of sampling and to make as far as possible the different samples as well as the different lots comparable among themselves. The results for each sample are charted in the following diagrams, a discussion of which is taken up after the presentation of the charts.

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[1] The Deterioration of Coal Samples.

Concerning these results as exhibited in the diagrams it should be noted that while experiments are of a preliminary nature largely devised to gain information for carrying out more elaborate and comprehensive tests, they are sufficiently consistent among themselves to justify certain tentative propositions as follows. There is evidence first of all of a distinct difference between the submerged coal and that which was exposed to the air. The values found for the submerged coal throughout the nine months did not vary, with possibly one or two exceptions, by greater amounts than would occur in tests made on succeeding days by the same operator. Values obtained so far apart as to time with the inevitable modifications due to temperature, atmospheric conditions, etc., may fairly be considered as checking if they agree within 100 or 150 units. There may indeed seem to be a uniform tendency to fall slightly in values, but from the showing the submerged coal may very fairly be said to remain constant. There may seem to be a slight inconsistency in this statement so far as chart No. 1 is concerned, there being a considerable drop in the value for the submerged coal. This may be the true state of the case, but more likely an explanation lies in the fact that initial values are too high. Some variables may have entered into the work at the outset that would not appear under the more settled conditions of routine and better control later on.

If we consider next the outdoor exposure, it should be said that these samples were contained in shallow boxes placed on the nearly flat roof of a building and subjected to the changes of temperature and moisture common to the months from October to July. The coals varied somewhat in their tendency to crumble, but all showed more or less of the "slaking" process. The remarkable fact in this series is the wide variation in the amount lost, ranging from approximately 2 to 10 per cent. The question naturally arises as to whether this is a natural characteristic of the different coals or whether the same variations would be found under different conditions, as for example, the storing in large masses instead of the small lots worked with. Attention should also be called to the fact that during the progress of this work, as will be shown under another topic, proof was obtained of the positive loss of values in samples stored under supposedly the best laboratory conditions. The rate of progress of this loss has not been determined though it is presumably slow. The effort was made to obtain the initial values on all the samples to be subjected to weathering at the earliest possible date after being mined, but variations as to time were inevitable and the importance of guarding this point was not so fully appreciated at the outset as it was at the close of the work. In further studies along this line, this particular element in the case will be guarded with due care, but there is no evidence so far that the results would be materially affected or would be different from the general indication of the charts.

Not the least striking of all the results are those obtained from coal stored in a thoroughly dry atmosphere. The fact that these samples lost in the aggregate quite as seriously as those exposed to outdoor conditions, and, if anything, even to a greater extent, would seem to indicate that moisture has little to do with the processes of deterioration. It must be conceded, of course, that so far as relates to the weathering out of pyritic sulphur, moisture is an essential condition, but the losses are so much greater than would be represented by the leaching out of sulphur that this element is practically obliterated as a factor in the results. If anything is proved indeed concerning the effect of moisture, it is that it retards rather than accelerates the loss of heat.

Other facts than these just cited substantiate the above feature of the case, tending to show that, after all, weathering is not a leaching process, or one which primarily results from the direct action of water and attendant weather conditions, but seems to be a direct loss of volatile hydrocarbons. To how great an extent there is a reabsorption of atmospheric gases and to what extent oxidation accompanies such absorption can not be stated from this series of experiments. That water indirectly is a factor can not be doubted, for anything which promotes disintegration facilitates the escape of combustible gases. Disintegration results from handling, from freezing and thawing, and from the decomposition of pyrites. In general, we would expect greater persistency of values in the dense and less friable coals, and in those with less of iron pyrites throughout their texture. In submerged coal, the decomposition of the pyrites is checked and without special reference to fineness of division. The loss of volatile matter seems also to cease. These processes which are active in the air and cease under water where the element of pressure or lack of it can hardly be a factor, suggest the idea of displacement of hydrocarbon gases by oxygen, by some process akin to osmosis or catalysis whereby a certain amount of oxidation of the carbon or hydrogen occurs. Altogether, the results are of value not only as touching the facts relating to the storage of coal, but especially as to their suggestions for further studies looking to fuller information of practical value on this very important topic.

SUMMARY

(a) Submerged coal does not lose appreciably in heat value.

(b) Outdoor exposure results in a loss of heating value varying from 2 to 10 per cent.

(c) Dry storage has no advantage over storage in the open except with high sulphur coals, where the disintegrating effect of sulphur in the process of oxidation facilitates the escape of hydrocarbons or the oxidation of the same.

(d) In most cases the losses in storage appear to be practically complete at the end of five months. From the seventh to the ninth month, the loss is inappreciable.

(e) The results obtained in small samples are to be considered as an index of the changes affecting large masses in kind rather than in degree, but since the losses here shown are not beyond what seems to conform in a general way to the experience of users of coal from large storage heaps, it may not be without value as an indication of weathering effects in actual practice.

Further studies are to be continued, having reference to actual storage conditions.

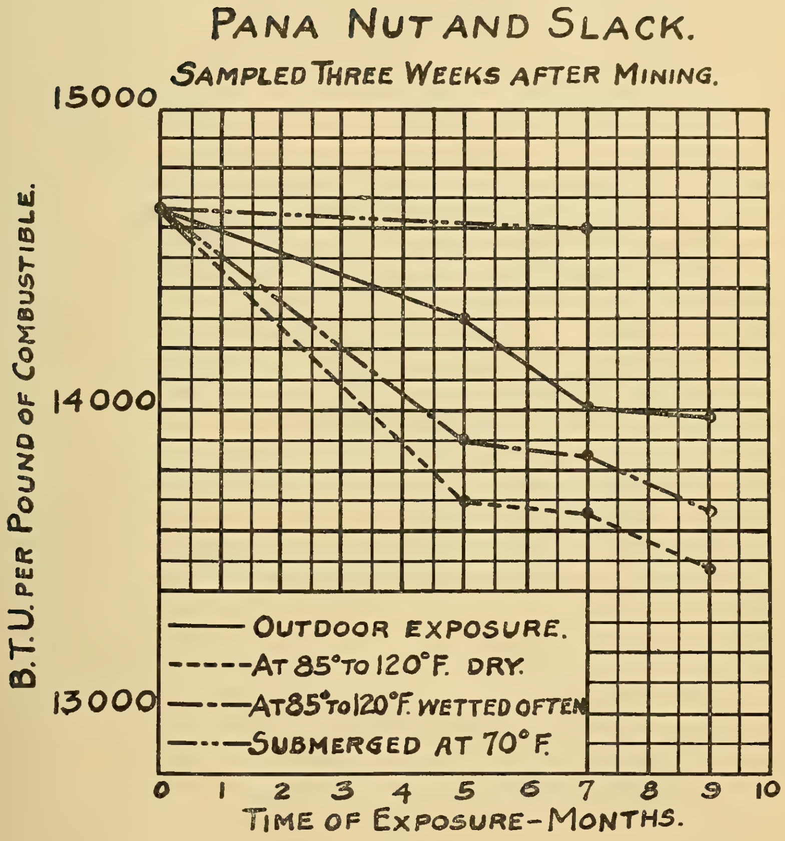
PANA NUT AND SLACK.
SAMPLED THREE WEEKS AFTER MINING.
15000
14000
13000
B.T.U. PER POUND OF COMBUSTIBLE.
OUTDOOR EXPOSURE.
AT 85° TO 120° F. DRY.
AT 85° TO 120° F. WETTED OFTEN
SUBMERGED AT 70° F.
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE-MONTHS.

FIG. 1

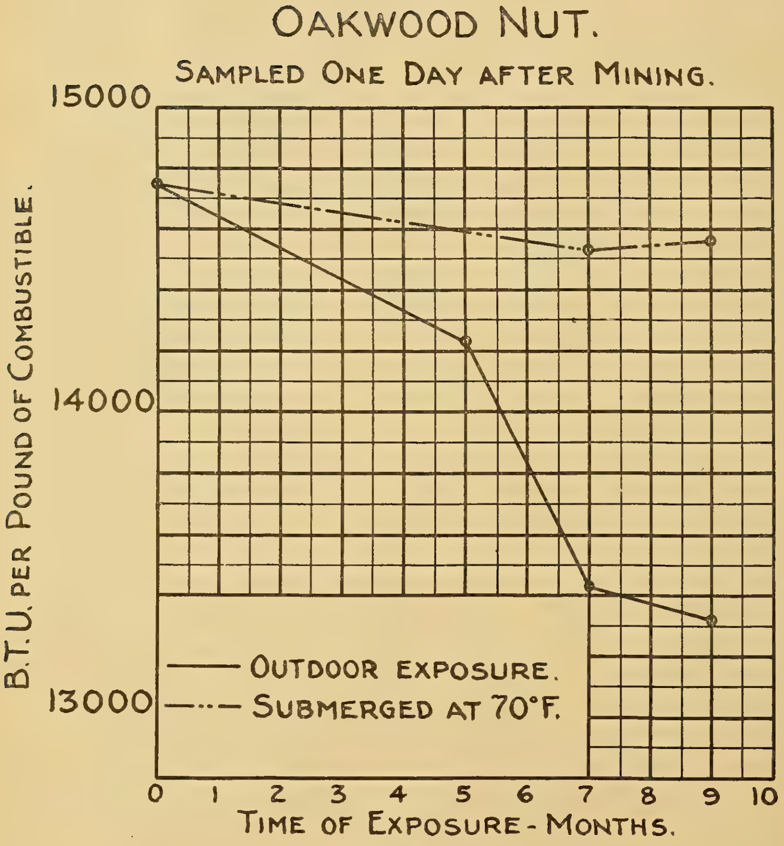
OAKWOOD NUT.
SAMPLED ONE DAY AFTER MINING.
15000
14000
13000
B.T.U. PER POUND OF COMBUSTIBLE.
OUTDOOR EXPOSURE.
SUBMERGED AT 70°F.
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.

FIG. 2

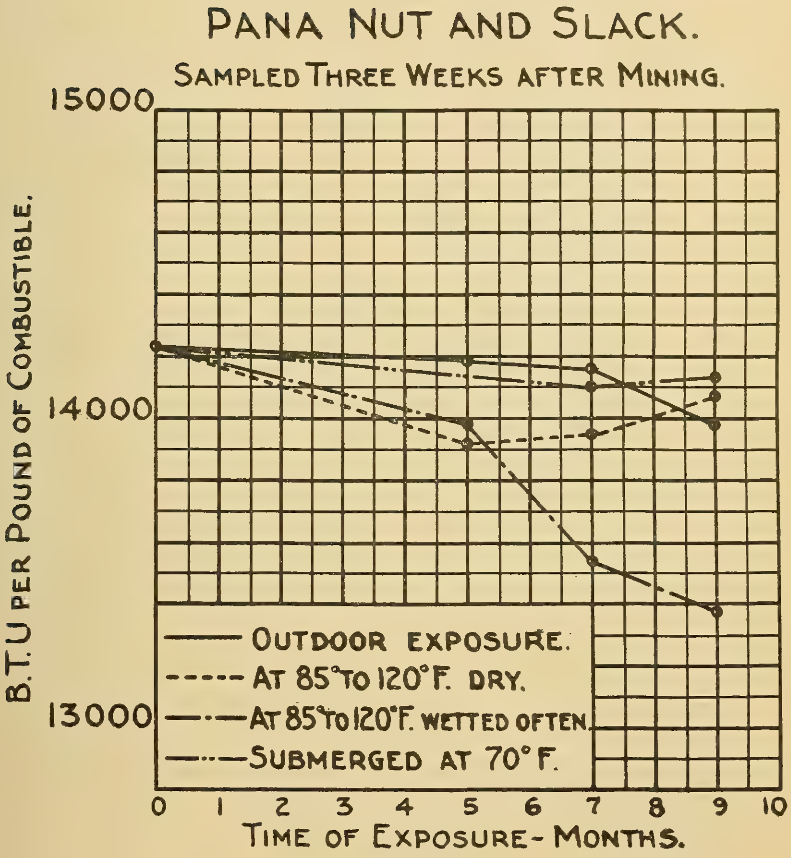
PANA NUT AND SLACK.
SAMPLED THREE WEEKS AFTER MINING.
B.T.U PER POUND OF COMBUSTIBLE.
15000
14000
13000
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.
OUTDOOR EXPOSURE.
AT 85° TO 120° F. DRY.
AT 85° TO 120° F. WETTED OFTEN.
SUBMERGED AT 70° F.

FIG. 3

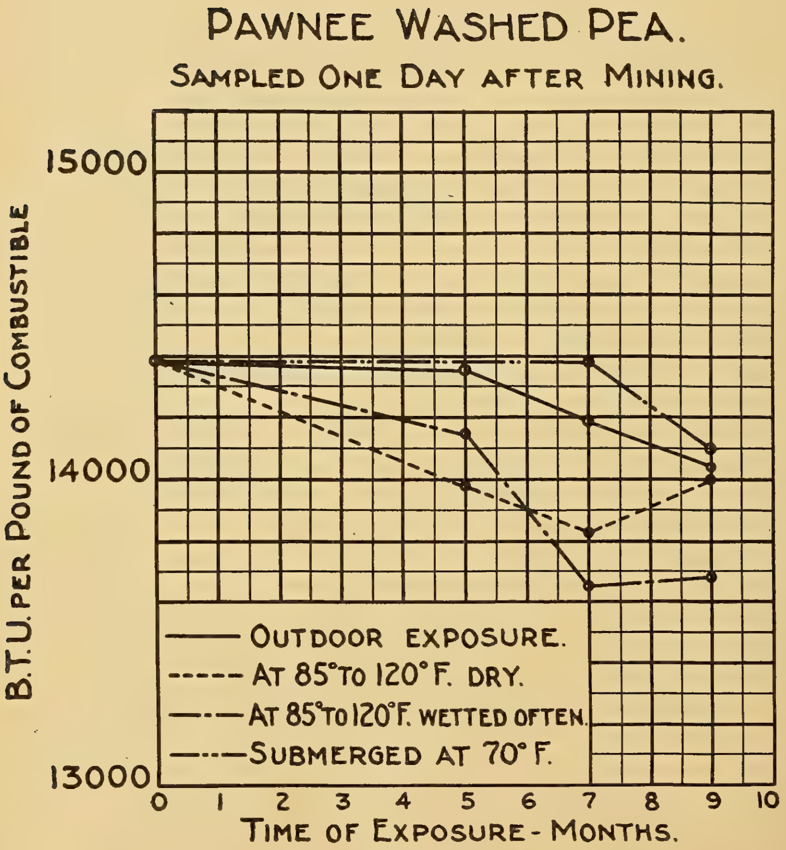
PAWNEE WASHED PEA.
SAMPLED ONE DAY AFTER MINING.
B.T.U. PER POUND OF COMBUSTIBLE
15000
14000
13000
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.
—— OUTDOOR EXPOSURE.
----- AT 85° TO 120° F. DRY.
—·— AT 85° TO 120° F. WETTED OFTEN.
—··— SUBMERGED AT 70° F.

FIG. 4

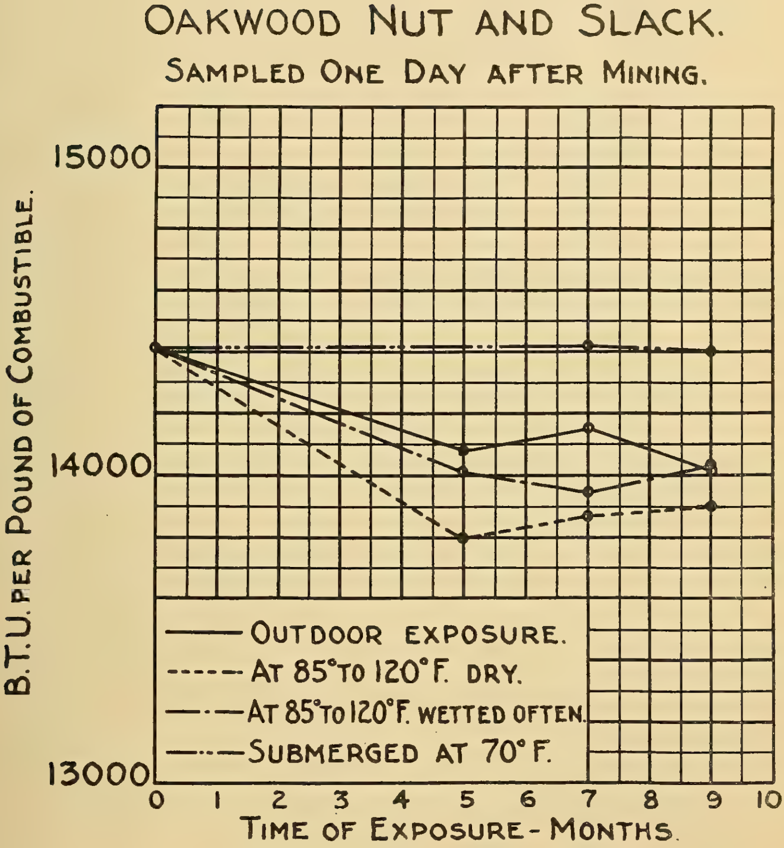
OAKWOOD NUT AND SLACK.
SAMPLED ONE DAY AFTER MINING.
15000
14000
13000
B.T.U. PER POUND OF COMBUSTIBLE.
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.
—— OUTDOOR EXPOSURE.
------ AT 85° TO 120° F. DRY.
—·— AT 85° TO 120° F. WETTED OFTEN.
—··— SUBMERGED AT 70° F.

FIG.

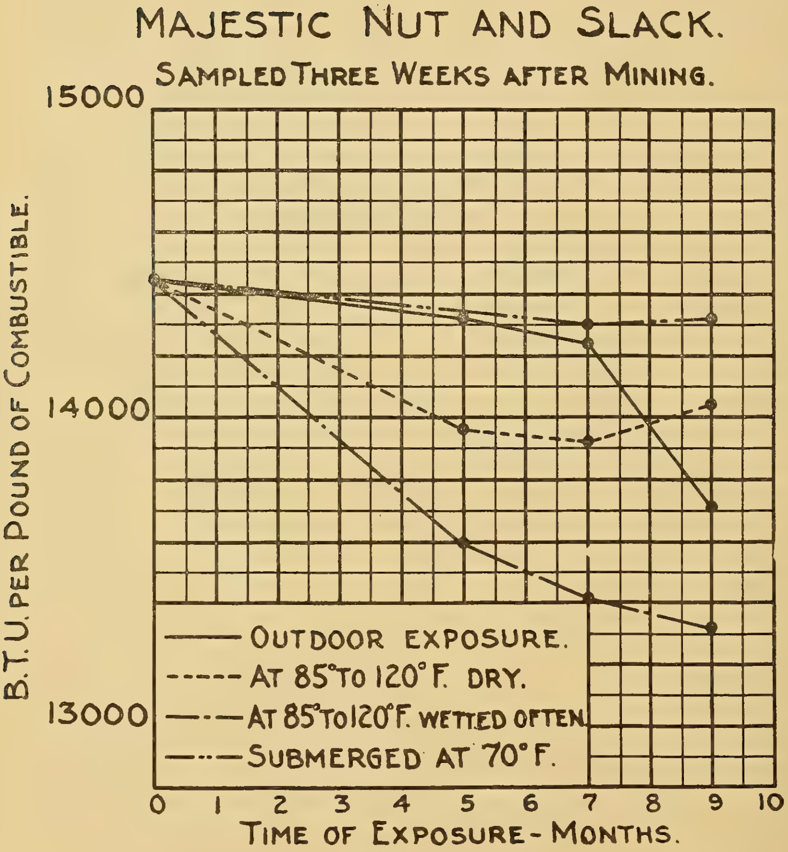
MAJESTIC NUT AND SLACK.
SAMPLED THREE WEEKS AFTER MINING.
15000
14000
13000
B.T.U. PER POUND OF COMBUSTIBLE.
OUTDOOR EXPOSURE.
AT 85° TO 120° F. DRY.
AT 85° TO 120° F. WETTED OFTEN
SUBMERGED AT 70° F.
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.

FIG. 6

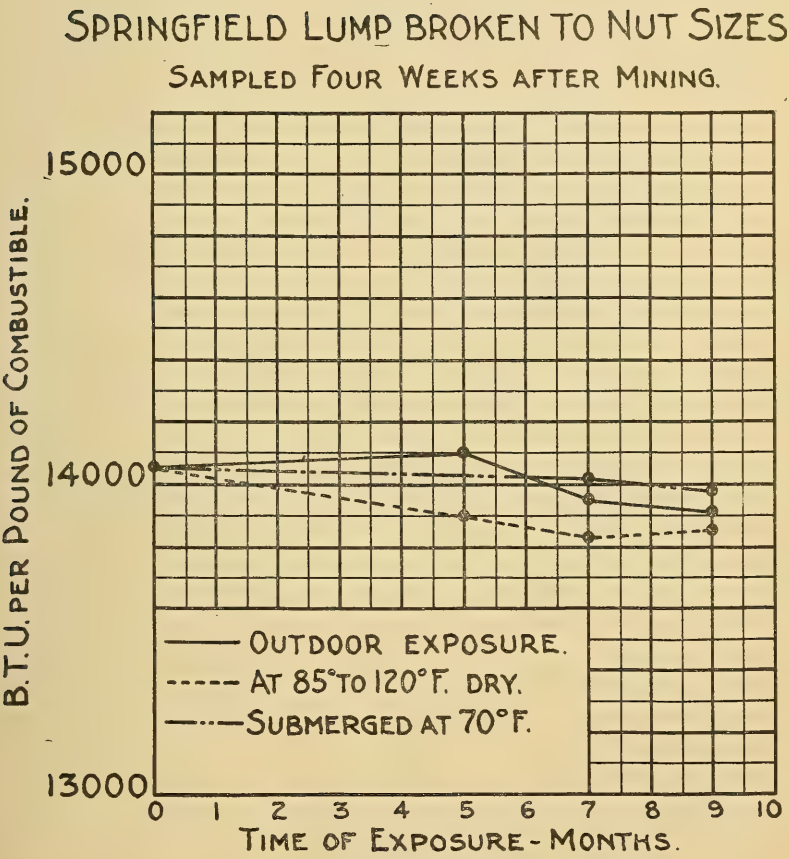
SPRINGFIELD LUMP BROKEN TO NUT SIZES
SAMPLED FOUR WEEKS AFTER MINING.
B.T.U. PER POUND OF COMBUSTIBLE.
15000
14000
13000
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.
OUTDOOR EXPOSURE.
AT 85° TO 120° F. DRY.
SUBMERGED AT 70° F.

FIG. 7

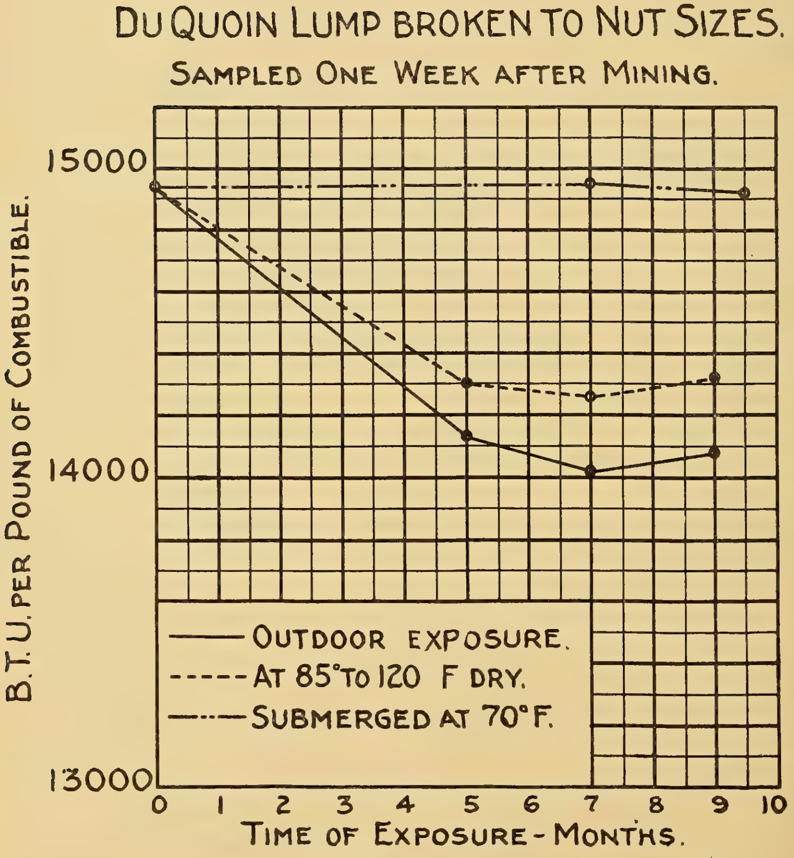
DU QUOIN LUMP BROKEN TO NUT SIZES.
SAMPLED ONE WEEK AFTER MINING.
B.T.U. PER POUND OF COMBUSTIBLE.
15000
14000
13000
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.
OUTDOOR EXPOSURE.
AT 85° TO 120 F DRY.
SUBMERGED AT 70° F.

FIG. 8

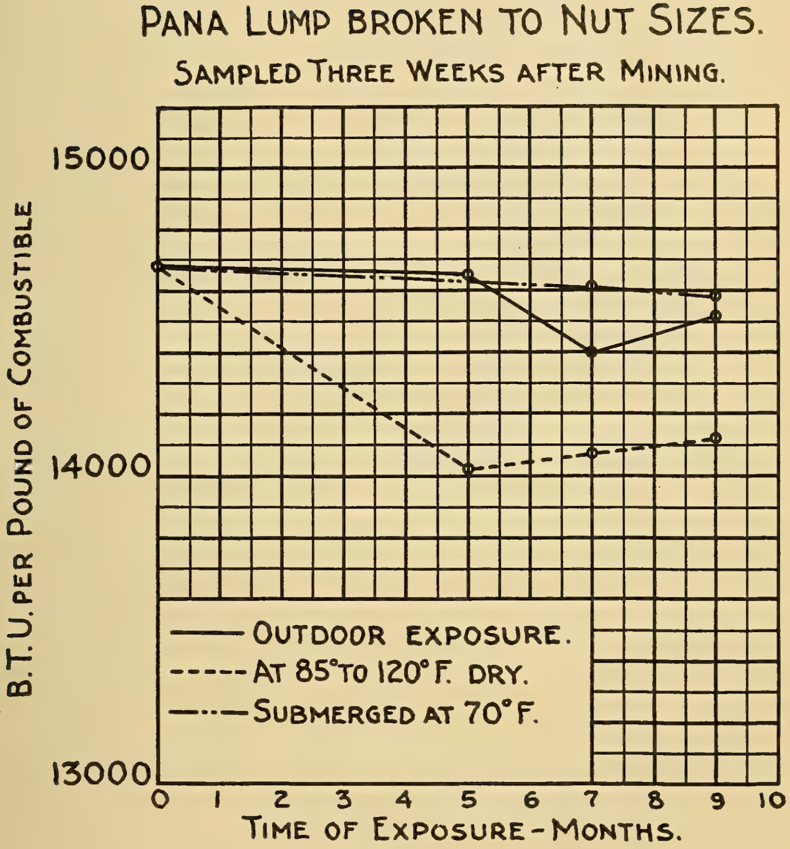
PANA LUMP BROKEN TO NUT SIZES.
SAMPLED THREE WEEKS AFTER MINING.
15000
14000
13000
B.T.U. PER POUND OF COMBUSTIBLE
0
1
2
3
4
5
6
7
8
9
10
TIME OF EXPOSURE - MONTHS.
OUTDOOR EXPOSURE.
AT 85° TO 120° F. DRY.
SUBMERGED AT 70° F.

FIG. 9

THE DETERIORATION OF COAL SAMPLES

By S. W. Parr, Professor of Applied Chemistry, and
W. F. Wheeler, Assistant Chemist, Illinois State
Geological Survey

(In Cooperation with the Illinois State Geological Survey)

Closely related to the weathering of coals is the subject of the deterioration of samples in storage. It would be assumed as a general proposition, that carefully sealed samples, kept at normal temperature, in glass containers, would remain constant as to their composition. Many facts have accumulated which seem to disprove this proposition. In an article by one of the writers[1], reference is made to the necessity of making calorific determinations where comparisons between different instruments are involved, at approximately the same date. To quote from that article —"A comparison of calorimeters should be made at approximately the same time. A series of calorific determinations made on finely ground samples on May 12, 1900, was found to give a reading 2.4 per cent less on July 12, 1900. It was necessary to repeat practically all of the above determinations on this account, all of the results showing a deterioration in the finely ground samples. This subject will receive further attention later."

In correspondence and conference with other workers, this fact has been questioned, as, for example, Dr. Bunte, of the Karlsrhue Polyteknikum, says that coal samples kept for analytical purposes, and which are determined by students year after year, give evidence of a constancy as to their calorific values. However this may be, the coals of the Mississippi Valley, being of a different type from the German coals, may not necessarily follow the same behavior in storage. Indeed, it may be doubted if the coals of Western Pennsylvania and Virginia would show the same behavior in this respect as the coals of the Mississippi Valley.

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[1] A New Coal Calorimeter, by S. W. Parr, Jour. Am. Chem. Soc., Oct., 1900, p. 650.

At any rate, there have accumulated a number of facts which point to the deterioration of Illinois coal samples in laboratory storage and it is the purpose of this paper to give the evidence which has come to hand up to the present time.

The matter is of importance, not only from its bearing upon an understanding of the matter of the weathering of coals, but in connection with all matters pertaining to the comparison of values as between different samples. It is to be taken into account also in considering the value which is to be placed upon the "pure coal" idea as frequently set forth, to the effect that the ash and water free basis is common ground for comparison under all conditions. It is exceedingly helpful and indeed essential, in the scientific study of coals, to have a unit of reference which may be used as a basis of comparison, but it is essential also to know the variations which may enter into such a unit, in order to avoid errors in the ultimate conclusions.

In comparing the values of Illinois coals, as obtained by the United States fuel testing plant at St. Louis, with the values determined in this laboratory by the Illinois State Geological Survey on coals from the same districts, it was found that considerable discrepancy existed upon referring the results in both cases to unit basis, as, for example, the ash and water free condition. Perhaps the most striking fact in this connection was the uniformly lower calorific values obtained by the State Geological Survey. The possibility of this difference being due to variations in methods and to different operators was duly considered. However, the method of sampling at the mine was the same, the sample being taken from the face of the vein, reduced by quartering in the usual manner to about two pounds in weight, sealed in tins with screw cap and insulating tape, exactly as followed by the fuel testing plant at St. Louis[1]. The type of calorimeter was also duplicated, the instrument in this laboratory used in this comparison, being of the Mahler-Atwater type with platinum lining and operated in a room under temperature control. A careful standardization of the instrument and a redetermination of its water equivalent were also made. The results here were so uniformly lower than those obtained at St. Louis as to call for a special study to ascertain the cause. An examination of the accompanying table will make evident the difference in calorific values as above described.

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[1] U. S. Geol. Surv. Coal Testing Plant, Bul. 261, p. 20.

It is not a sufficient explanation to say that the samples in the two cases were not identical. Indeed, frequently they were not taken from the same mine. It is a well established fact, however, that samples taken from the same locality, when referred to a unit basis, as the ash and water free conditions, will show relatively small variations, at least, within a limited area. It is true that variations in the same region do occur often, but evidence of these variations is largely dependent upon the accuracy of the unit adopted for reference, and when properly compared, these variations are not of sufficient size nor of sufficient uniformity to explain either the magnitude or the constancy in direction of the differences shown in the accompanying table. Concerning the unit of reference, it should be said that the ash, moisture and pyrite-free basis is used as approaching the nearest possible to the actual material under consideration, thereby eliminating such variables as would obviously result if no notice were taken of the presence or absence of sulphur. The weight of ash, therefore, is corrected by adding $\frac{5}{8}$ of the weight of sulphur present, as representing the original pyritic condition.

In taking up the study of the conditions under which the work was carried on in the two laboratories, so far as can be determined, the only point of divergence seemed to reside in the time elapsing between the date of sampling at the mine and the date at which the determinations were made in the laboratory. In the case of the fuel testing plant at St. Louis, this difference in time was relatively short, being presumably, as a rule, not more than two or three weeks. In the case of the State Geological Survey, because of certain exigencies, an unavoidable delay occurred, hence the time elapsing varied from six months to a year.

Two methods of caring for the samples were followed. First, where time permitted, the coal as received in the tins was at once emptied into shallow pans and the amount of moisture lost upon air drying was determined by allowing the pans to stand exposed to the air over night. The sample was then reduced to buckwheat size, one-half was sealed in glass jars, of the so called Lightning or Putnam pattern, and the other half was ground in the Ball mill with porcelain jars of the Abbe type. This part of the sample was also sealed in a similar jar and set aside for the analytical work later. The analyses used in Table 1 for comparison with the St. Louis results were made on these finely ground portions.

The other method of caring for the samples consisted in simply transferring the coal as received in the tins to glass containers without air drying. Fifty samples were thus disposed of, about half being stored in the Putnam jars and half in jars of the common Mason type.

TABLE 1

COMPARISON OF NEW AND OLD SAMPLES OF ILLINOIS COALS

In Laboratory Storage

Sample Numbers	Locality	Mine Sample U. S. G. S. Ash, Moisture and Pyrite Free	Mine Sample Ill. Geol. Surv. Ash, Moisture and Pyrite Free	Difference	Per cent of Variation
U. S. G. S., Ill. No. 1	O'Fallon	14567			
Ill. Geol. Surv. No. 95	O'Fallon *a*		14084	483	-3.31
U. S. G. S., Ill. No. 3	Marion	14561			
Ill. Geol. Surv. No. 330	Marion *a*		14335	226	–1.55
U. S. G. S., Ill. No. 9	Staunton	14615			
Ill. Geol. Surv. No. 94	Staunton *a*		13923	692	–4.72
U. S. G. S., Ill. No. 10	W. Frankfort	14647			
Ill. Geol. Surv. No. 364	W. Frankfort *b*		14332	315	–2.15
U. S. G. S., Ill. No. 11	Carterville	14731			
Ill. Geol. Surv. No. 325	Carterville *a*		14213	518	–2.15
U. S. G. S., Ill. No. 15	Centralia	14587			
Ill. Geol. Surv. No. 167	Centralia *b*		14200	387	–2.65
Ill. Geol. Surv. No. 169	Centralia *a*		14376	211	–1.44
U. S. G. S., Ill. No. 16	Herron	14558			
Ill. Geol. Surv. No. 323	Herron *a*		14321	237	–1.62
U. S. G. S., Ill. No. 18	LaSalle		14722		
Ill. Geol. Surv. No. 393	LaSalle *b*	14440		282	–1.91

(a) not same mine
(b) same mine

As already noted, it has been impossible in this table of comparisons to select cases where the samples were exact duplicates from the same mine in each instance, but it may fairly be claimed that the uniformity with which the lower values are indicated for the older samples precludes the possibility of ascribing the difference to the character of the coal. The same thing may also be said with reference to variations in results which may be expected from different operators. If this were the reason for the discrepancy, one would hardly expect the uniformity as to direction of the results here indicated in the tables. To determine whether deterioration might result from the finely ground state of the samples, the reserve sample in the coarse condition in a number of cases was taken and calorific determinations made as upon the fine samples. These results were found practically to duplicate those obtained upon the ground samples. Hence, it was concluded that if deterioration were the explanation, it had affected both the coarse and fine samples alike. It was then decided to obtain new samples from the same localities and so far as possible from the same mines, collecting and preparing the samples in exactly the same way as before. The determinations on these samples were made within a period not to exceed ten days from the date of collection. The results arranged for comparison with the old samples are shown in Table 2. The striking uniformity with Table 1 is to be noted in that practically the same difference in values is shown between the new samples and the old as exists between the St. Louis results, presumably, also on fresh samples, and our own results known to be on samples of from six months' to a year's standing.

TABLE 2
Comparison of Coal Samples for Varying Lengths of Time In Laboratory Storage

Lab. No. Ill. State Geol. Survey	Source of Sample	Time of Storage	B. t. u. of Ash, Moisture and Pyrite free	Loss in B. t. u.	Percentage of Loss
421	Majestic Mine, DuQuoin, Ill. Apr. 17, 1907	10 days	14386		
307	Paradise Coal & Coke Co., DuQuoin, Ill.	1 year	14116	207	–1.9

TABLE 2 (*Continued*)

Lab. No. Ill. State Geol. Survey	Source of sample	Time of storage	B. t. u. of Ash, Moisture and Pyrite free	Loss in B. t. u.	Percentage of Loss
459	Big Muddy C. & I. No. 7 Apr. 18, 1907	10 days	14615		
323	Squirrel Ridge Mine Herrin, Ill.	1 year	14321	294	−2.0
460	Big Muddy Coal Co. No. 8, Clifford, Apr. 18, 1907	10 days	14615		
325	Clifford, Carterville	1 year	14213	402	−2.7
462	Peabody Coal Mine No. 3 three miles west of Marion, Ill. Apr. 18, 1907	10 days	14781		
330	Peabody Coal Co., Marion, Ill.	1 year	14335	446	−2.6
540	Sangamon Mine, Springfield	1 week	14567		
82	Sangamon Mine, Springfield	7 months	14100	467	−3.2
81	Sangamon Mine, Springfield	7 months	13940	841	−4.3
557	Kelly No. 4, now Dearing No. 44	5 days	14450		
332	Kelly Coal Co., Westfield, Ill.	1 year	14054	396	−2.8
558	Kelly Coal Co., Himrod Mine, Himrod, Ill.	5 days	14564		
333	Himrod Mine, Himrod, Ill.	1 year	14087	477	−3.3

Still a third series of results has been arranged in Table 3. Here the comparison is made between the St. Louis values and our own, obtained on relatively fresh samples from the same or near-by mines. There is further confirmation of the proposition here in that the results on our own samples are sometimes higher and sometimes lower than the St. Louis values. The relative dates are not at hand in each case to indicate whether or not the plus and minus values conform to a greater or less transpiration of time before analysis but it is worthy of noting that the discrepancies are not so wide as in Tables 1 and 2, where a positive and wide variation of time existed. Even in the third sample from Stanton (Table 3) where a difference of 2.14 per cent is indicated, in Table 1 for the same mine, a difference of 4.72 per cent is given.

TABLE 3

Comparison of Illinois Coal Samples with Relatively Short Period In Laboratory Storage

Sample Numbers	Locality	Mine Sample U. S. G. S. Ash, Moisture and Pyrite Free	Mine Sample Ill. Geol. Surv. Ash, Moisture, and Pyrite Free	Difference	Per cent of Variation
U. S. G. S. Ill. No. 3	Marion	14561			
Ill. Geol. Surv. No. 462	Marion *a*		14781	220	+ 1.51
U. S. G. S. Ill. No. 7	Collinsville	14373			
Ill. Geol. Surv. No. 725	Collinsville *b*		14659	286	+ 1.98
Ill. Geol. Surv. No. 723	Collinsville *a*		14640	267	+ 1.85
Ill. Geol. Surv. No. 724	Collinsville *a*		14564	191	+ 1.32
U. S. G. S. Ill. No. 9	Stanton	14615			
Ill. Geol. Surv. No. 737	Stanton *a*		14301	314	− 2.14
U. S. G. S. Ill. No. 11	Carterville	14731			
Ill. Geol. Surv. No. 460	Carterville *b*		14615	116	− .787
U. S. G. S. Ill. No. 14	E. Springfield	14464			
Ill. Geol. Surv. No. 540	E. Springfield *a*		14567	103	+ .711
Ill. Geol. Surv. No. 740	E. Springfield *a*		14408	56	− .387
Ill. Geol. Surv. No. 741	E. Springfield *a*		14429	35	− .243

TABLE 3 (*Continued.*)

Sample Numbers	Locality	Mine Sample U. S. G. S. Ash, Moisture and Pyrite Free	Mine Sample Ill. Geol. Surv. Ash, Moisture and Pyrite Free	Difference	Per cent of Variation
U. S. G. S. Ill. No. 16	Herron	14558			+ .391
Ill. Geol. Surv. No. 459	Herron *b*		14615	57	
U. S. G. S. Ill. No. 19	Ziegler	14601			
Ill. Geol. Surv. No. 419	Ziegler *b*		14480	121	− .829
Ill. Geol. Surv. No. 420	Ziegler *b*		14445	156	−1.06

(*a*) Not same mine.
(*b*) Same mine.

It is now in order to refer to certain other facts which still further confirm the proposition in hand. The fifty samples above referred to (p. 29) as having been transferred without air drying to glass containers upon their arrival at the laboratory, were examined after about ten months' standing. Twenty-nine of the samples had been placed in the type of jar shown herewith and known as the Lightning or Putnam jar. Extended experience with this jar as a container for sodium peroxide, a chemical with unusual avidity for moisture from the atmosphere, has proved it to be possessed of an absolute seal. The remaining twenty-one samples had been placed in the common Mason fruit jar with metal screw cap and very indifferent seal. After the ten months of storage, upon opening the Lightning jars a slight pressure of gas was noted which suggested the testing of the same with a lighted match. In twenty-six of these jars the gas ignited with a strong blue flame, burning up from one-half to six inches above the top of the jar. Upon covering with the cap and testing again with a match, these jars would reignite for two or three successive times. Two of the jars had been previously opened without attention to the contents and it is not known whether they contained inflammable gas or not. In one other of these jars, the gas was carbon dioxide, judging from the fact that instead of igniting the match was extinguished. Not one of the Mason jars with the zinc cover contained any pressure of gas and no tendency to ignite was manifested. It should be noted that all of these jars were in diffused light, but not in direct sunlight, and that only the Lightning jars possessed perfect seal. By reference to Fig. 10, it will be seen that, with the exception of the rubber gasket, the entire inclosure of the material is of glass. The gasket, however, is an exception to this, but it is held with a very positive pressure by reason of the lever device for clamping on the top. The conditions in the ordinary Mason jar are different in that a metal screw cap is employed and the positiveness of the seal of the rubber gasket is questionable. We have herein evidence of the tendency of the coal to give off combustible gas after being broken out of the vein, and if we accept the theory proposed by Richter and others, that this exudation of combustible gas is accompanied by a corresponding absorption of oxygen from the air, we may readily understand some of the processes which go on in the deterioration of samples as well as in the weathering of coal.

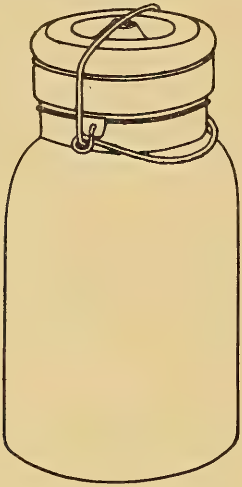

FIG. 10

One further incident is worthy of notice in this connection. Certain samples used in Coal Bulletin 2, published as the University of Illinois Studies on the Composition of Coal, 1904, were opened after three years of storage in jars of the Lightning type as above described. These samples had not been opened during the three years. It was noted that much of the iron pyrites disseminated throughout the coal had become oxidized to ferric sulphate. This suggested that by leaching out this compound, and estimating the sulphur thus transformed to sulphate, we could calculate the amount of oxygen necessary to bring about the reaction from the form of iron pyrites, (FeS_2) to ferric sulphate, $Fe_2(SO_4)_3$. It was found that the amount of oxygen required to oxidize the weight of sulphur found was 1.99 grams and calculated to the equivalent volume of pure oxygen would be 1.39 liters, or, if calculated to the equivalent of atmospheric air, it would require 7 liters of such air to furnish the necessary oxygen for the reaction. When it is remembered that the jars in which these samples were kept had a volume all told of only one pint and that this space was occupied at least to the extent of three quarters of the total with coal of buckwheat size; and when we further remember that these jars were possessed of absolute seal, without opportunity for transference of oxygen from without, there is furnished evidence of the fact that occluded oxygen or absorbed air must have been present in sufficient amount to accomplish the work indicated by the transformation of the pyrites to ferric sulphate.

It would seem from the above experiences that there is not only sufficient evidence to establish the fact of the deterioration of coal samples but a fairly well established explanation as to how this deterioration takes place. Other tests along this same line are being carried out from still other standpoints, and while they are not complete, the evidence is all in the same direction as that adduced above.

SUMMARY

(a) An exudation of combustible gases from coal occurs from the time of breaking out of the sample from the vein.

(b) An absorption of oxygen accompanies the exudation of hydrocarbons.

(c) Samples of coal in most carefully sealed containers are subject to deterioration.

(d) The process of deterioration is probably due to oxidation of hydrogen or hydrocarbons by means of the absorbed oxygen. It may also be due to a simple loss of combustible gases and the replacement of the same by non-combustible gases such as oxygen.

(e) The rapidity or extent of this deterioration varies with different coals but is probably most active during the first two or three weeks from the taking of the sample, but does not seem to reach a normal state till after a few months have elapsed. Further data on this point especially are necessary.

It is interesting, also, to bring together the averages of the results in the three tables for further comparison. There is thus afforded further evidence suggesting the fact of deterioration.

TABLE 4

Averages from Tables 1, 2 and 3

Illinois State Geological Survey Eight old in comparison with eight new samples— from Table 1.	Average 2.85 per cent lower
Illinois State Geological Survey Nine old samples in comparison with nine results by U. S. G. S.— from Table 2.	Average 2.40 per cent lower
Illinois State Geological Survey Twelve fresh samples in comparison with results by U. S. G. S.— from Table 3.	Average 0.20 per cent higher

Publications of The Engineering Experiment Station

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N Talbot. 1904. (*Out of print*).

Circular No. 1. High Speed Tool Steels, by L. P. Breckenridge. 1905.

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905.

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Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. (*Out of print*).

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Bulletin No. 8. Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. (*Out of print*).

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906.

Bulletin No. 10. Tests of Plain and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907.

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July 16, 1894]

BULLETIN NO. 18

THE STRENGTH OF CHAIN LINKS

BY

G. A. GOODENOUGH

AND

L. E. MOORE

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS
PUBLISHED BY THE UNIVERSITY

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The results of these investigations will be published in the form of bulletins, and will record mostly the experiments of the Station's own staff of investigators. There will also be issued from time to time in the form of circulars, compilations giving the results of the experiments of engineers, industrial works, technical institutions and governmental testing departments.

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BULLETIN NO. 18

THE STRENGTH OF CHAIN LINKS

BY

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 18 SEPTEMBER 1907

THE STRENGTH OF CHAIN LINKS

By G. A. Goodenough[1], *Associate Professor of Mechanical Engineering, University of Illinois and L. E. Moore, Assistant Professor of Civil Engineering, Massachusetts Institute of Technology, formerly Associate in Theoretical and Applied Mechanics, University of Illinois.*

The chain is one of the most familiar as well as one of the most useful of mechanical devices. It is universally employed in hoisting and transmission, and for attaching and securing movable bodies, as, for example, in anchoring ships. As a rule, a chain is subjected to heavy loads and must transmit large forces, and upon its ability to withstand the stresses to which it is subjected by its loading may depend the success of a great mechanical operation, or even the safety of lives.

In view of these facts, it is surprising that the chain has received scant attention from investigators in the field of elasticity and strength of materials. Aside from two or three scattered memoirs, the theory of the stresses in chain links has been untouched. Experiments have been made, it is true, but these have been for the purpose of determining the ultimate strength of the chain, not for the purpose of testing a theory. Formulas for the loading of chains have been based upon the ultimate strength of the chain when tested to destruction and are thus purely empirical. No attempt seems to have been made to place such formulas on a rational basis supported by theory. It may be urged that the present empirical rules are satisfactory, inasmuch as they lead to satisfactory results. As a matter of fact, the results are not satisfactory; chains break, often with disastrous consequences, and the only reason that more do not break is that a chain is seldom subjected to its rated load. Further arguments to show the importance of a rational analysis seem hardly necessary.

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[1] For the theoretical analyses contained in the appendices and for the discussion of the experimental results Professor Goodenough is responsible. The experimental work was conducted under the direction and supervision of Professor Moore, and he is responsible for the methods employed in making the tests and for the accuracy of the experimental results.

In undertaking the work described in this paper, three things were held in view.

(1) The development of the theory of the stresses induced in chain links with given conditions as regards loading.

(2) Experimental tests of the validity of the theory employed and also of the validity of the assumptions made as to the distribution of pressure between adjacent links.

(3) The deduction from theoretical considerations alone of rational formulas for the loading of chains.

The beginning of this work dates back to 1900, when the analytical investigations were largely worked out. In 1906, Mr. R. M. Evans of the class of 1906 undertook the experimental verification of the theory, and presented in his graduating thesis certain of the results contained herein. The following year the experimental work was continued by Messrs. M. L. Millspaugh and R. L. Baker. The data obtained have been worked over carefully, all calculations have been repeatedly checked, and it is believed that the results derived are worthy of confidence, whatever may be the conclusions that are drawn from them.

METHOD OF ANALYSIS

The analytical investigation was first suggested by Bach's analysis of the stresses in a hollow cylindrical roller.[1] It seemed evident that the general method there used could be employed to determine the stresses in links with circular or elliptical center lines. The fundamental equations may be found in Bach's work, but for the sake of completeness they are given in condensed form in this paper. (See Appendix A.) Grashof[2] gives an analysis using the same fundamental equations, but owing to untenable assumptions, the analysis gives results wide of the truth. The only other analysis is that made by Winkler in a memoir published in Der Civilingenieur.[3] A discussion of this memoir in which some of the results have been corrected is given by Professor Karl Pearson.[1] To show Pearson's estimate of Winkler's work the following paragraphs from the introduction of the discussion are quoted:

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[1] Bach, *Elasticitat und Festigkeit*, p. 458.

[2] Grashof, *Elasticitat und Festigkeit*, Sec. 178–180, pp. 273–277.

[3] *Formandering und Festigkeit Gekrummter Körper in besondere der ringe.* Der Civilingenieur, Bd. IV; S. 232–246.

"This is an important memoir both from the theoretical and practical standpoint; although many of its results require correction and modification. Some of these corrections have been made in Kapitel XL (*Ringformige Körper*) of the author's well known treatise: *Die Lehre von der Elasticität und Festigkeit*, Prag, 1867, but this treatise does not cover anything like the same area as the memoir. I propose therefore to indicate the correct analysis and compare its results with those of Winkler.

"The importance of the subject will be sufficiently grasped when I remind the reader that it is the only existing theory of the strength of the links of chains. To investigate the strength of such links by the complete theory of elasticity would involve even for the case of anchor rings an appalling investigation in toroidal and allied functions; while for the oval chain links with studs in ordinary use, any successful attempt at a general investigation seems inconceivable. We shall have the less hesitation, however, in applying the Bernoulli-Eulerian theory, if we remember how close an approximation Saint-Venant's researches on flexure have shown it to be in the case of *straight* bars. At the same time we are certainly going to put it to the very limit of its application, namely, to *curved* bars in which the dimensions of the cross sections are not very small as compared with either the length or the radius of curvature of the central axis." . . .

"Remembering that we need not assume adjacent cross sections of our link to remain undistorted, if we only suppose them to be approximately equally distorted, we can easily investigate an expression for the stretch at any point by a method akin to that which results from the Bernoulli-Eulerian theory."

The method here referred to is that given by Bach and Grashof for the analysis of bars with curved axes. An outline of it, as already stated, is given in Appendix A.

While the method employed in the investigations herein described is essentially the same as that of Winkler and Pearson, there are one or two important points of difference in the assumptions made. Professor Pearson considers only two cases, the link with elliptical center line, and the link made up of two circular arcs and two straight lines. The analysis here given is extended to links of four and six circular arcs so as to approximate as closely as possible to the forms actually occurring; it is also extended to links with studs. Furthermore, it appears that in all cases Winkler assumed the pressure between adjacent links to be concentrated at a point at the end of the link. The present analysis assumes a distribution of pressure over a definite area. As will be shown later, this question of distribution has an important bearing upon the results obtained.

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[1] Todhunter and Pearson, *History of the Elasticity and Strength of Materials.* Vol. II, Part I, p. 423 et seq.

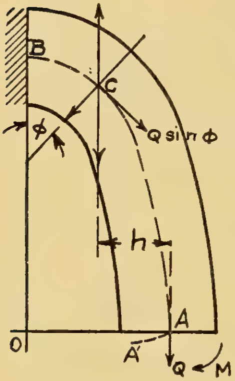


FIG. 1.

The complete analysis of the open link is given in Appendix B. The following is merely a brief outline of the method of attacking the problem. Consider one quadrant of the link as shown in Fig. 1. Denoting by $2Q$ the load on the link, the section at A lying along the minor axis will be subjected to a normal force Q. There will also be at this section a bending moment M, which can be determined from the conditions of the problem. Now assume any other normal section, as C, and consider the part of the link between sections A and C a free body. At C let two forces, each equal to Q but opposite in sense, be added to the system. One of these forces with the force Q at section A forms a couple whose moment is Qh; the other force is resolved into components, one $Q \cos \phi$ along the section, the other $Q \sin \phi$ normal to the section. The component $Q \cos \phi$ produces shearing stress and is neglected in the subsequent discussion. At the section C we have therefore:

$$\text{a normal force,} \quad P = Q \sin \phi;$$
$$\text{a bending moment,} \quad M_b = Qh + M.$$

The unknown moment M is now found from considerations explained in the analysis; and with P and M_b fully known, the intensity of stress at any fiber is readily determined from the fundamental equation (C), Appendix A. It may be noted that instead of (C), the usual formula

$$S = \frac{P}{f} + \frac{M_y}{I}.$$

may be used, though the results may not be quite exact.

Pressure Between Adjacent Links

At the very beginning of the analysis arises a question as to the way in which the pressure between adjacent links is distributed. The analysis is somewhat simplified by assuming that two links have contact at one point only and that in consequence the pressure between them is concentrated at this point. [See Fig. 2(a)]. As a matter of fact, however, the links after a little wear have contact over a considerable surface and the pressure between them must be distributed in some way or other over this surface.

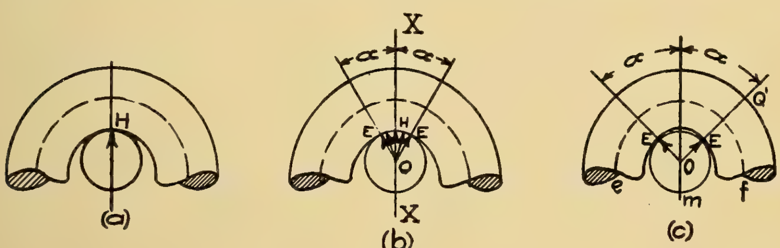


FIG. 2.

Referring to Fig. 2(b), suppose that contact exists over the arc *EE*, which subtends the angle 2α at the center O. Though the parts of the link in contact are curved, the action of one link on another may be likened to that of a journal and bearing. We may assume (1) that the pressure is uniformly distributed along the arc *EE*, or if we make use of the more exact analysis of journal and bearing, we may assume (2) that the intensity is greatest at H and decreases towards E, being at any point proportional to the cosine of the angle made with the axis XX. Because angle α is small, the second assumption changes but little the results obtained by using the first; hence we shall consider only the assumption of uniform distribution.

A third possible distribution is represented in Fig. 2(c). Under heavy load the link suffers a considerable distortion and the sides *e* and *f* approach each other. Now if the distributed pressure along *EE*, Fig. 2(b), were in the nature of a fluid pressure so that the points of application of the forces could move as the points *EE* moved, the law of distribution would be unchanged by the distortion of the link. But the part *m* of the adjacent link lying between the sides *e* and *f* is practically unyielding; hence when *e* and *f* approach each other the part *m* is pinched and there ensues a new distribution of pressure. Evidently the result of this pinching action is to increase the intensity of pressure near *E*, *E* and to decrease it at *H*. We cannot, of course, know the precise effect of the action just described. For the sake of comparison with the other cases, we may assume, however, that the effect is equivalent to concentrating the pressure at the two points *E*, *E*.

In the subsequent analysis we shall make the three assumptions just stated, namely:

(1) Pressure concentrated at single point *H*, Fig. 2(a).
(2) Pressure uniformly distributed over arc *EE*, Fig. 2(b).
(3) Pressure concentrated at points *E*, *E*, Fig. 2(c).

As a matter of interest, we may in passing call attention to Grashof's analysis. The links are supposed to be in contact along an arc *EE* subtending the angle 2α, as in Fig. 2(c). It is then assumed that the part of the link lying between the sections *E*, *E* takes no part in the straining action, but acts as a rigid base or foundation to which are attached the sides *e* and *f*. As will be shown later, this neglected part of the link plays a most important rôle, and Grashof's assumption is anything but justified.

EXPERIMENTAL VERIFICATION OF ANALYSIS

Referring to Fig. 1, *OA* and *OB* denote respectively the semi-minor and semi-major axes of the link. Under a load these axes change, *OA* becomes shorter and *OB* longer, and these changes can be measured with reasonable accuracy. Now the theoretical analysis here employed furnishes a means of calculating the change of position of any point, as, for example, the point A, on the center line of the link. Thus for a given load, the new position A' to which A will move can be found. Evidently the component of AA' in the direction of AO is the change in OA, that is, one-half the change in the length of the minor axis; likewise, the component of AA' in the direction of OB is one-half the change in the major axis.

We have here a means of verifying theory by experiment. The changes of length of the axes of the link for given loads can be calculated from purely theoretical considerations. The actual changes for those loads can be measured. A comparison of the calculated and actual values of the changes of length affords therefore a delicate test of the theoretical analysis.

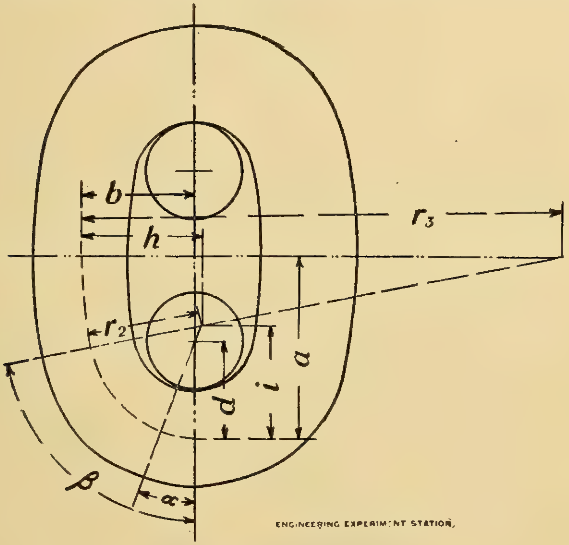


FIG. 3.—DREDGE CHAIN.

Dimensions.

$d = 1.000$ in. $b = 1.178$ in. $i = 1.161$ in. $r_2 = 1.173$ in. $\alpha = 21°$
$a = 1.875$ in. $h = 1.240$ in. $e = 0.000$ in. $r_3 = 5.000$ in. $\beta = 79°15'$

Because of the doubt regarding the distribution of pressure between adjacent links it was considered advisable to use circular rings of rectangular cross section. With these rings a true knife-edge bearing was possible, and the general theory (Appendix A) could be tested without danger of introducing unknown factors resulting from the pressure distribution. The experiments on the rings are to be considered, therefore, as more reliable than the link tests in establishing the truth or falsity of the analysis. The tests of the actual chain links are, however, valuable in two ways: (1) They may be used to establish more firmly the analysis when applied to oval links; (2) Assuming that the ring experiments sufficiently establish the analysis, the link experiments may be used to test the assumptions made as to the distribution between adjacent links.

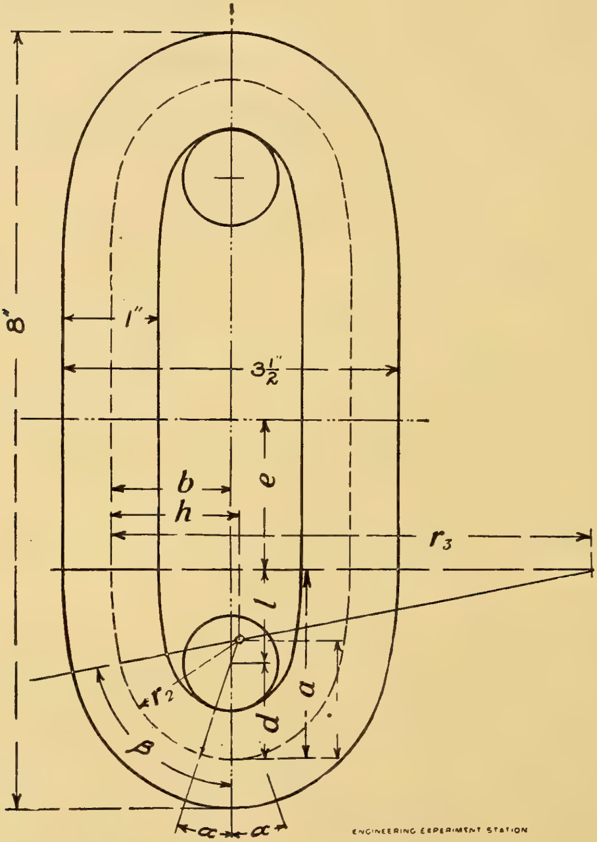
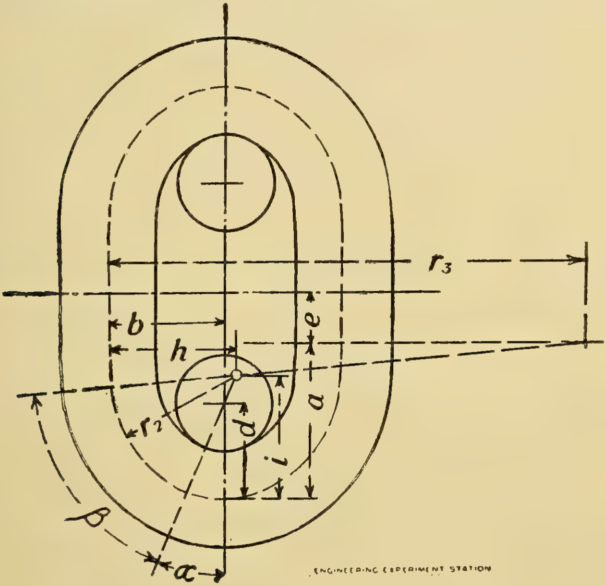


Fig. 4. — Conveyor Chain.

Dimensions.

$d = 1.00$ in. $b = 1.23$ in. $i = 1.294$ in. $r_2 = 1.340$ in. $\alpha = 30°0'$
$a = 2.00$ in. $h = 1.40$ in. $e = 1.500$ in. $r_3 = 5.000$ in. $\beta = 78°53'$

As already stated, the experiments were extended over a period of two years and were performed as thesis work in the Laboratory of Applied Mechanics of the University of Illinois by senior students in the College of Engineering. The experiments of the first year (1906) made by R. M. Evans were wholly on chain links. Those of the second year (1907) made by Messrs. Baker and Millspaugh were partly on heavy chain links and partly on finished steel rings. These three men deserve great credit for the amount and the character of the work done and for their untiring efforts to do the work as well and accurately as possible.



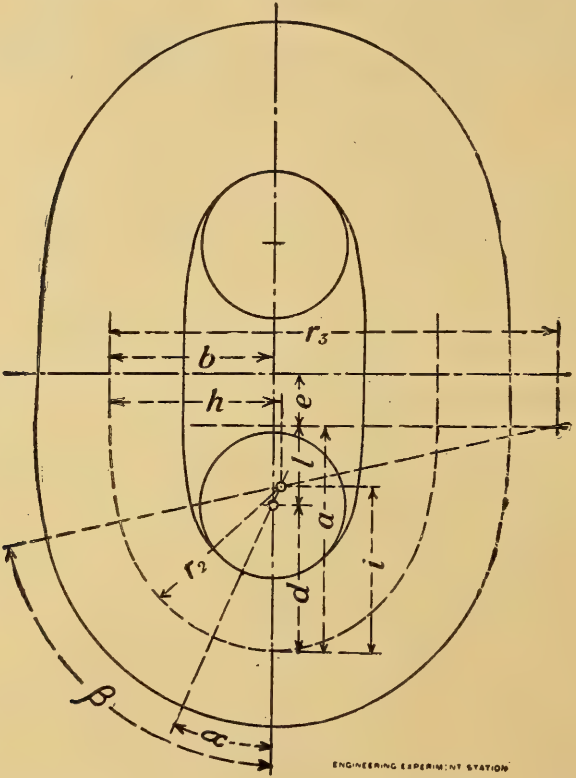
ENGINEERING EXPERIMENT STATION

FIG. 5. — PROOF COIL CHAIN.

Dimensions.

$d = 1.00$ in. $b = 1.214$ in. $i = 1.316$ in. $r_2 = 1.35$ in. $\alpha = 25°18\frac{2}{3}'$
$a = 1.625$ in. $h = 1.364$ in. $e = 0.500$ in. $r3 = 5.00$ in. $\beta = 85°9'$

It has not seemed necessary or desirable to differentiate in these pages between the tests made in the different years, as the objects of the tests and methods used were the same. The chain links tested were ordinary commercial links bought in the market. The test pieces for determining the modulus of elasticity of the material were ordered cut from the same bar from which the chains were made. The links with dimensions are shown in Fig. 3, 4, 5 and 6. In making the tests a short piece of chain, consisting of either three or five links, five links being used whenever the dimensions of the machine would permit, was held in the jaws of the testing machine by a clevis at each end. These clevises were flattened to afford a better grip for the jaws of the machine. This is illustrated in Fig. 7. Deformations were measured by micrometers reading directly to .001 inch, and by interpolation to .0001 inch.



ENGINEERING EXPERIMENT STATION

FIG. 6. — TWO-INCH DREDGE CHAIN.

Dimensions.

$d = 2$ in. $b = 2.214$ in. $i = 2.305$ in. $r_2 = 2.333$ in. $\alpha = 24°0'$
$a = 3.125$ in. $h = 2.42$ in. $e = 0.7$ in. $r_3 = 6.25$ in. $\beta = 77°54\frac{1}{3}'$

A micrometer having a "rachet contact" which insured practically the same pressure on the points in all measurements was used for measuring the deformations of the transverse or minor axis. To insure measuring between the same points each time small brass buttons were soldered to all the links except the 2-in. dredge chain

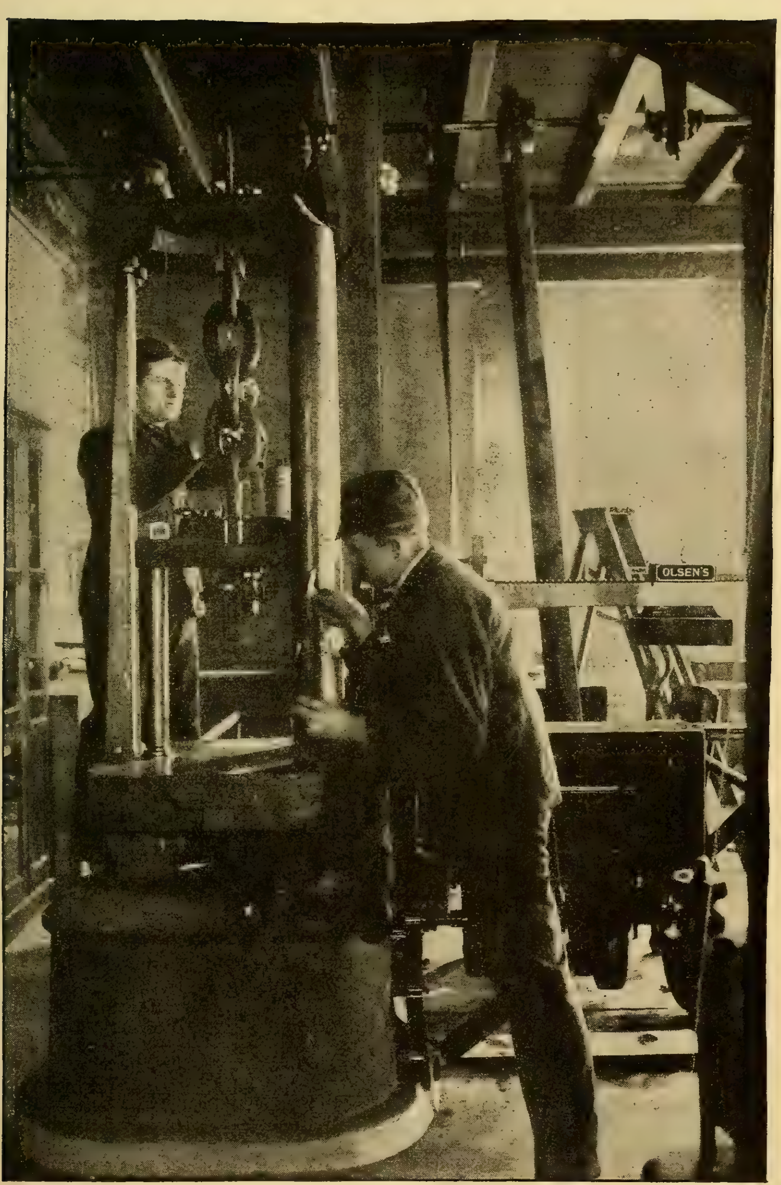


Fig. 7.

link at the ends of the minor axis. The measurements were taken over these buttons. On the dredge chain a small spot was polished on the link itself at each end of the axis. The longitudinal deformations were taken between brass contact points screwed to the inside of two transverse bars of $\frac{3}{4}$-in. square iron. One of these bars was soldered to the link at each end of the long diameter or major axis. The distance between the contact points was then readily measured by means of an inside micrometer. An electric bell and battery were used in this connection, the bell ringing as soon as contact was established between the points. This device is shown in Fig. 8 and 9.

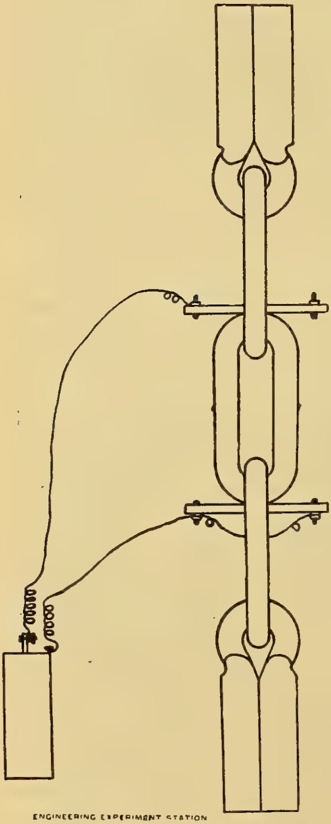


FIG. 8.

The dimensions of the rings were 12 in. outside and 9 in. inside diameter by 1 in. thick, as shown in Fig. 10. Two of these rings were cast steel and the third was wrought steel with a perfect weld. The rings were finished all over in a lathe. The method of holding the rings in the machine and applying the load is clearly shown in Fig. 11 and 12. The load was applied to the ring through knife edges. This was done to remove the uncertainty as to the distribution of pressure between the links. By referring to Fig. 11 and 12 it will be noted that the method of loading the rings gives flexibility in all directions and prevents any eccentricity of loading.

The moduli of elasticity of the different materials were determined from test specimens cut, except in the case of the cast steel rings, from the same bars from which the chains and ring were made. The modulus of elasticity of the cast steel was determined from test pieces poured from the same heat as the rings.

The plan followed in testing was to increase the load by such nearly equal increments that from fifteen to twenty readings would have been obtained when the estimated elastic limit was reached. In testing the links the load was increased to a point just beyond the elastic limit, which was indicated by the change in the increment of the deformation. The links were allowed to rest at least twenty-four hours and the second test was then run up to the same maximum load as in the first case, no attention being paid to the possible raising of the elastic limit by this treatment. It may be readily seen that the exact elastic limit is of little importance in this work compared to the modulus of elasticity. So long as the material was not injured to such an extent as to render values of the modulus of elasticity doubtful, the slight exceeding of the elastic limit was of no consequence. In testing the rings care was taken not to exceed the elastic limit.

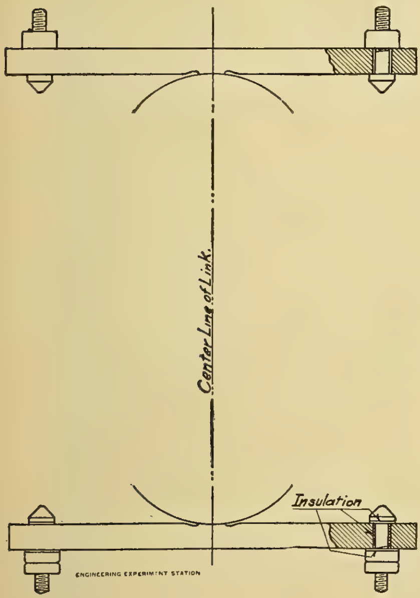


FIG. 9.

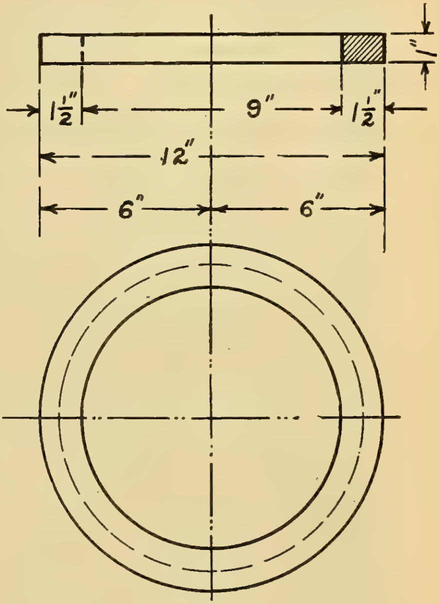


FIG. 10.

In all cases save one, two tests of each specimen will be found recorded, and called the "first test" and the "second test" respec-

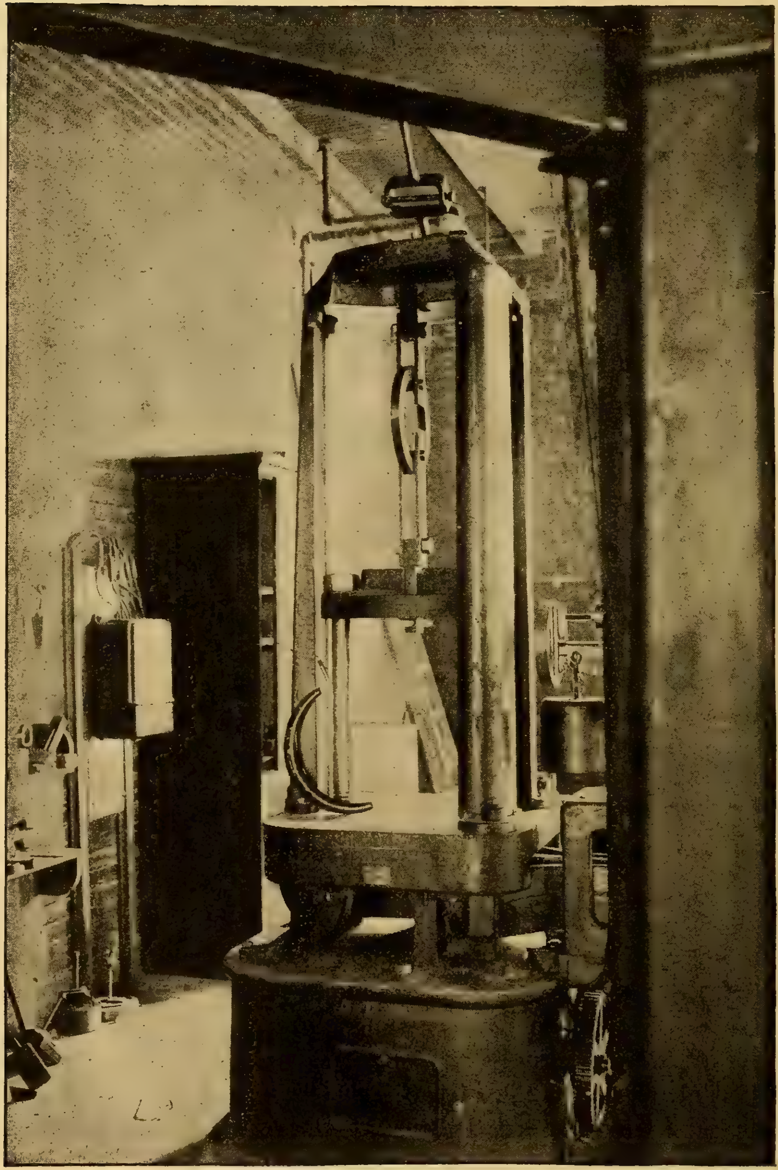


FIG. 11.

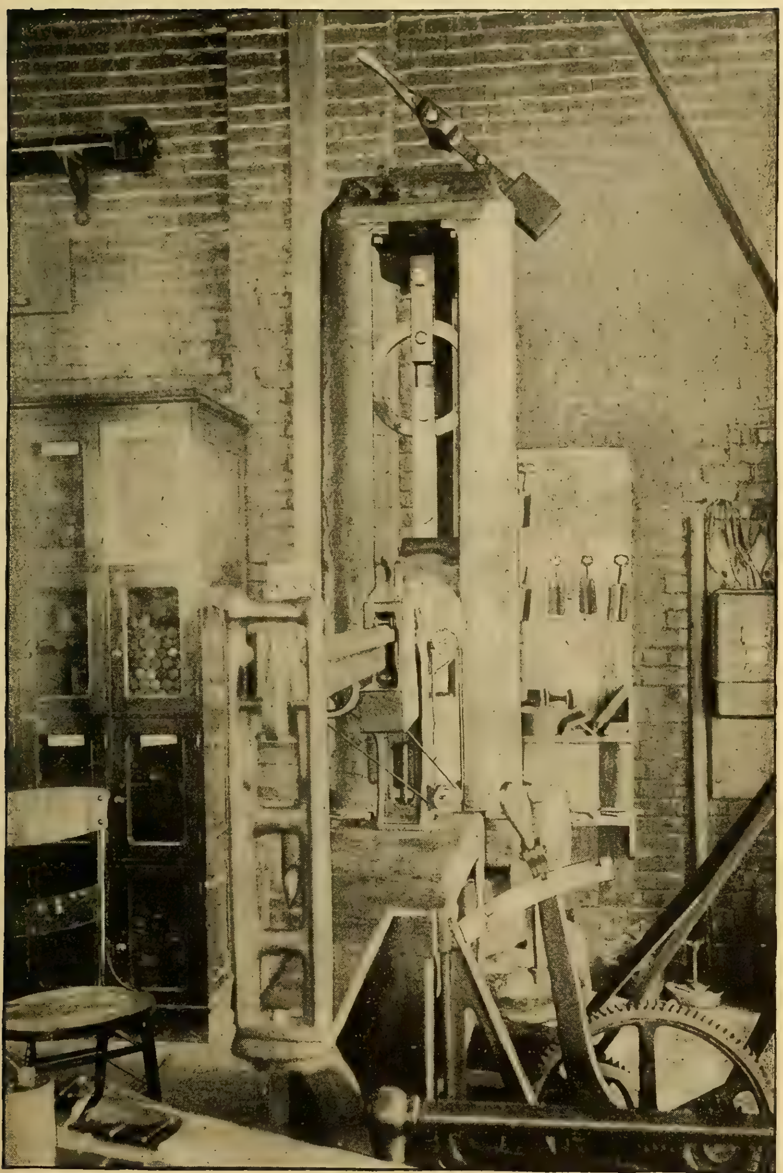


Fig. 12.

tively. These were not the only tests run. In some cases it was found desirable to run one or two preliminary tests to accustom the students to taking their observations and to get an idea of the behavior of the piece under load.

RESULTS

The results of the tests are given in tabular form on pages 37-44. Tables 6 to 11 inclusive apply to the tests of the circular rings; tables 12 to 18 to the tests of the four chain links shown in Fig. 3, 4, 5 and 6. The headings of the various columns render detailed explanation of the tables unnecessary.

The tabular values plotted to scale are shown in Fig. 13 to 25 inclusive. Fig. 13 to 18 show the results obtained from the circular rings; Fig. 19 to 25 those from the chain links. The first test in each case is denoted by a small circle o, the second test by a filled circle, ●. For the sake of convenience in comparison, the two tests are given in the same figure, but for distinctness, different origins have been used.

It is to be emphasized that the lines appearing in these figures are in all cases theoretical lines calculated from the known dimensions of the ring or link and from the modulus of elasticity experimentally determined. These lines in fact could have been drawn before the deflection tests were made.

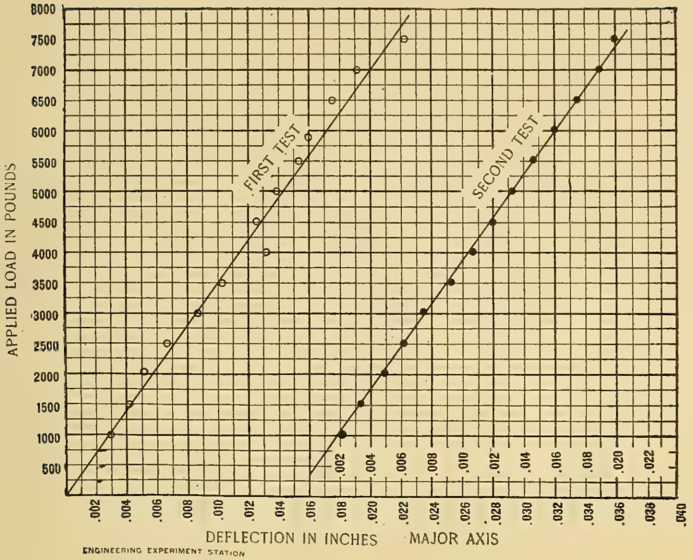
DISCUSSION OF EXPERIMENTAL RESULTS

1. *Circular Rings.* Table 1 gives the calculated deformation for the three circular rings tested.

TABLE 1

DEFLECTION OF CIRCULAR RINGS

	Change of Length of Diameter per 1000-lb. Load	
	Vertical Diameter Inches	Horizontal Diameter Inches
No. 1	.00286	.00247
No. 2	.00294	.00254
No. 3	.00263	.00228


8000
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
APPLIED LOAD IN POUNDS
FIRST TEST
SECOND TEST
.002
.004
.006
.008
.010
.012
.014
.016
.018
.020
.022
.002
.004
.006
.008
.010
.012
.014
.016
.018
.020
.022
.024
.026
.028
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.036
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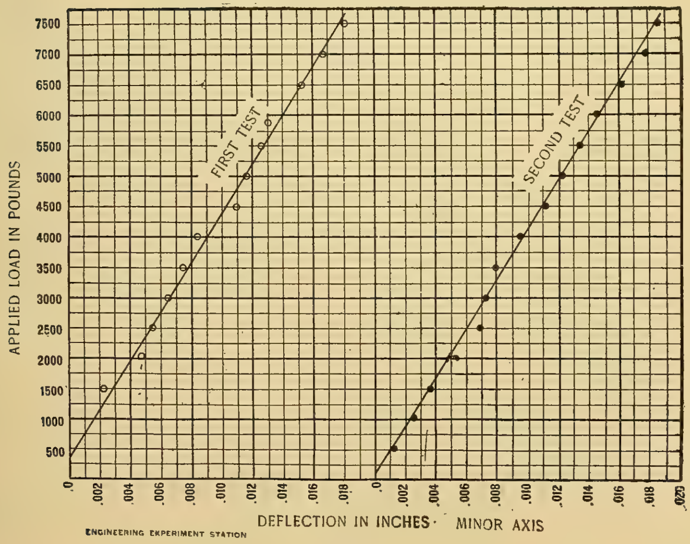
.040
DEFLECTION IN INCHES
MAJOR AXIS
ENGINEERING EXPERIMENT STATION

Fig. 13.


7500
7000
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3500
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2000
1500
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APPLIED LOAD IN POUNDS
FIRST TEST
SECOND TEST
0
.002
.004
.006
.008
.010
.012
.014
.016
.018
0
.002
.004
.006
.008
.010
.012
.014
.016
.018
.020
DEFLECTION IN INCHES
MINOR AXIS
ENGINEERING EXPERIMENT STATION

Fig. 14.

The values in this table were obtained from equations (H′) Appendix C. The following is the calculation for ring No. 1:

Mean radius $r = 5.25$ in.
Area of cross section $f = 1.5605$ sq. in.

Modulus of elasticity (by experiment), 26,200,000.

$z = .006887$; (See Appendix A).

$$\frac{1}{z} = 145.197;$$

$$\frac{1}{1+z} = .99315$$

Substituting in formulas (H′), p. 65,

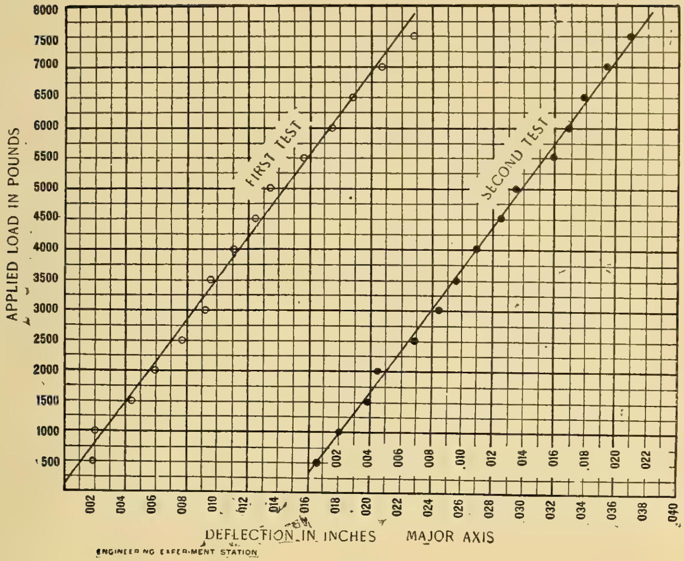
$$\Delta x_a = -\left[\frac{5.25 \times 145.197}{26{,}200{,}000 \times 1.5605}\left(\frac{2 \times .99315}{3.1416} - 0.7854\right)\right]Q = .00000286Q,$$

$$\Delta y_a = -\left[\frac{5.25 \times 145.197}{26{,}200{,}000 \times 1.5605}\left(\frac{2 \times .99315}{3.1416} - 0.5\right)\right]Q = -.00000247\,Q.$$

As is evident from (H′), the curve giving the relation between the change of length of the axis and the applied load is a straight line through the origin. The value in the table gives the slope of this line. In Fig. 13 to 18 these lines have been drawn through the plotted points, and a comparison may be made between the line determined by calculation based on analysis and the points found by experiment.

The lines representing the mean of the experimental values will not, in general, pass through the origin, because of unavoidable errors at the beginning of the test. Hence the theoretical lines are not drawn through the origin, but are drawn with the proper slope in such a position as to permit the comparison to be made most easily. This course is entirely justified by the fact that the slope of the line, rather than its absolute position, is the important factor.

In ring No. 1, it will be seen that the agreement is remarkable; in fact, the calculated line is about as near the mean line of the points as could be drawn. It will be noticed that the points of the second test lie a little more regular in all cases than those of the first test.


APPLIED LOAD IN POUNDS
8000
7500
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FIRST TEST
SECOND TEST
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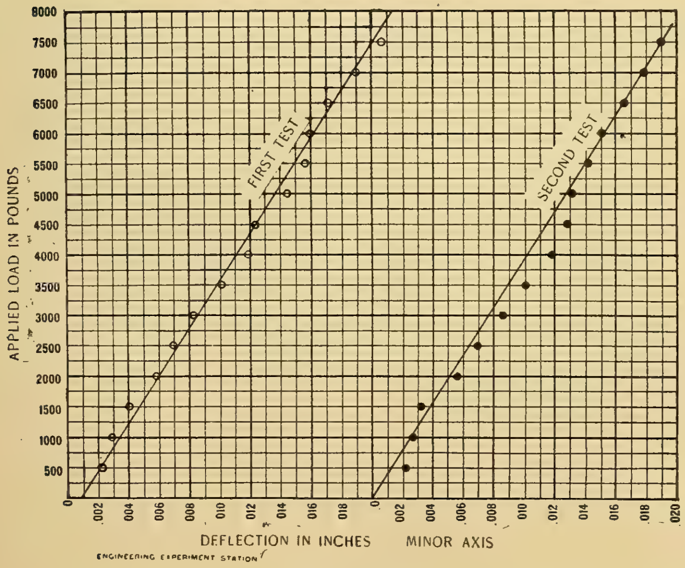
DEFLECTION IN INCHES MAJOR AXIS
ENGINEERING EXPERIMENT STATION

Fig. 15.


APPLIED LOAD IN POUNDS
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FIRST TEST
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002
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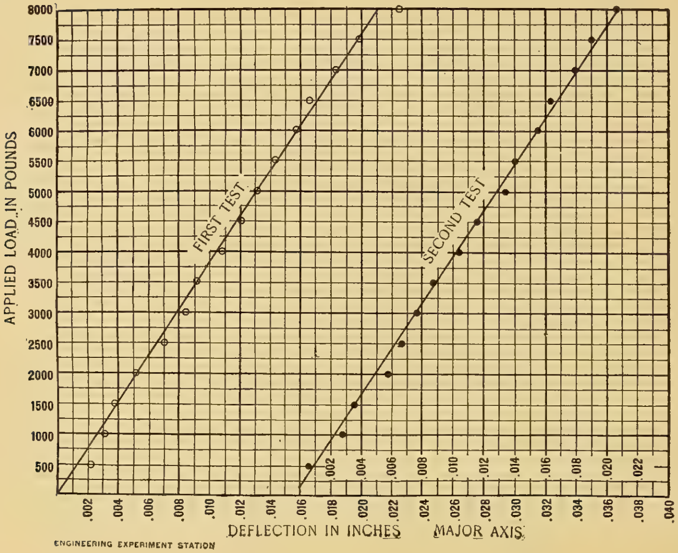
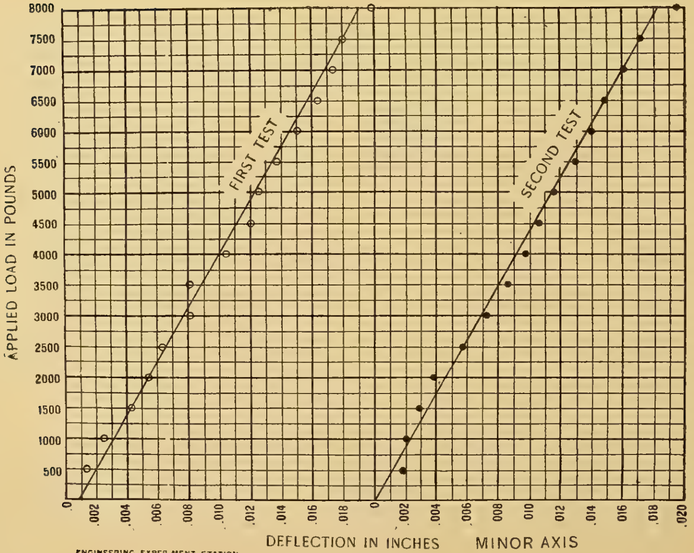
006
008
010
012
014
016
018
020
DEFLECTION IN INCHES MINOR AXIS
ENGINEERING EXPERIMENT STATION

Fig. 16.



ENGINEERING EXPERIMENT STATION

Fig. 17.



ENGINEERING EXPERIMENT STATION.

Fig. 18.

In ring No. 2, the agreement is not quite so close as in No. 1, but still is fairly satisfactory. In the case of ring No. 3, vertical axis, the agreement is good, and it is also good in the second test for the horizontal diameter. In the first test the slope of the actual line seems slightly less than that of the calculated lines. It is possible that the modulus of elasticity as determined for rings 1 and 2 is a

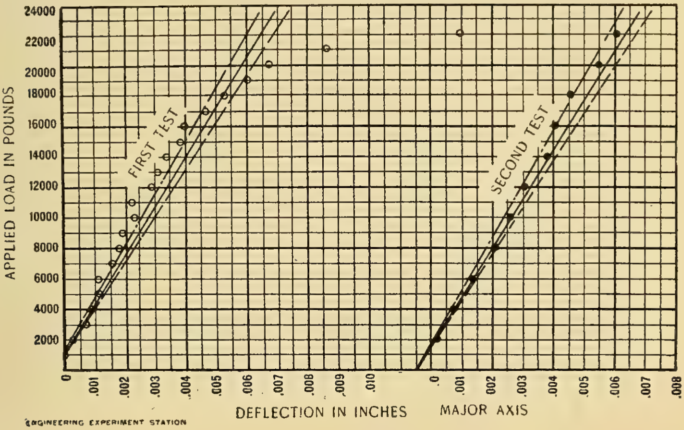


FIG. 19.

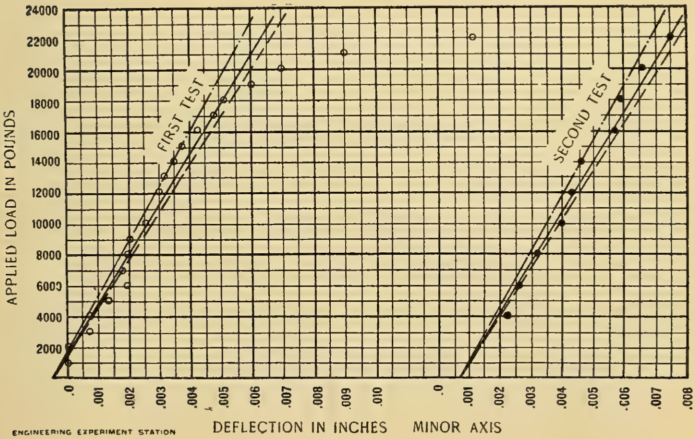


FIG. 20.

little low. A higher value would make the theoretical lines slightly steeper.

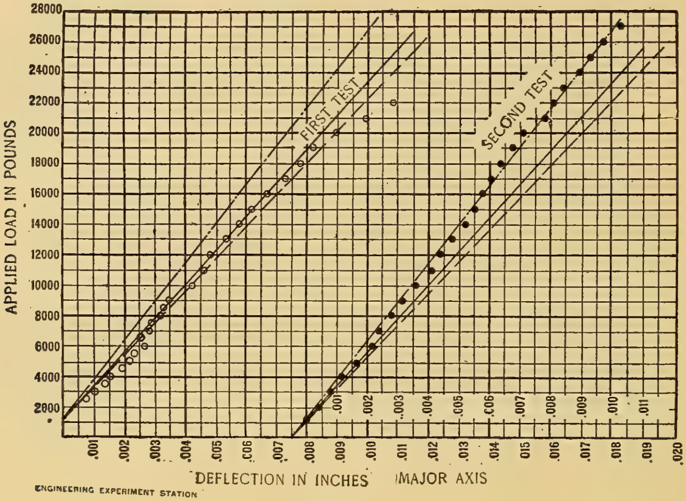


FIG. 21.

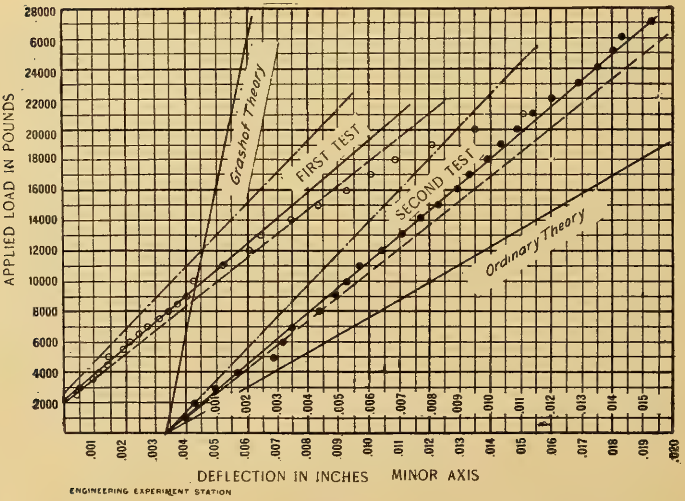


FIG. 22.

The experiments on the rings, on the whole, seem to confirm in a satisfactory manner the theoretical analysis. We may therefore conclude that the fundamental equations employed will give very closely the true stresses in rings, and that if proper assumptions regarding the distribution of pressure between links be made, the same equations will give the stresses in chain links. Assuming, therefore, the correctness of the analysis, we may use the results of the experiments on the links to throw some light on the question of distribution of pressure.

2. *Chain Links.* Fig. 19 and 20 show the experiments on the link shown in Fig. 3. In this, as in all the chain links, three calculations for the change of length of the axis were made. These correspond to the three assumptions as to the pressure noted in a previous section. See p. 6 and Fig. 2(a), (b), (c). The line of least inclination, indicated thus — — — —, corresponds to case (a), concentration at the end of the link; the intermediate full line corresponds to case (b), distributed pressure; while the line of greatest slope, indicated thus —· —· —· —, corresponds to case (c), concentration at two points due to the possible wedging action. If we direct our attention to the second test, Fig. 19, we observe that the experimental points lie well within the region of these three lines; the same may be said of the test showing the change of length of the minor axis, Fig. 20.

In Fig. 21 and 22 are shown the experiments upon the long link of the conveyor chain, Fig. 4. The coincidence between the points of the second test and the theoretical line, Fig. 22, is striking. This test is perhaps of more weight than any other of the link tests because the length of the link caused large deflections. It will be observed that the points for the first test indicate in each case a line of smaller slope than the points for the second test. This fact may be explained possibly as follows: In the second test the links have become accommodated to each other, so to speak, and the action is more nearly that of a journal and bearing: hence condition (b) is approximated to rather than condition (a).

In Fig. 23 is shown the one test made on the link shown in Fig. 5. For some reason, a second test of this link was not made. The results are about as shown for the other links. Probably the points for the second test would have followed more closely the theoretical lines, as in the other cases.

The experiments upon the two-inch link, Fig. 6, are shown in Fig. 24 and 25. The results of the two tests are practically the same, and the agreement between the experimental points and theoretical lines is satisfactory.

The theoretical lines shown in Fig. 19–25 were obtained by calculation from formulas (J) and (K), Appendix C. The following table gives the results of the calculations thus made:—

TABLE 2

BENDING MOMENTS AND DEFLECTIONS OF CHAIN LINKS

	Case	Dredge Link, Fig. 3 Curves, Fig. 19 and 20	Conveyor Link, Fig. 4 Curves, Fig. 21 and 22	Proof Coil Link, Fig. 6 Curves, Fig. 23	Two-Inch Link, Fig. 6 Curves, Fig. 24 and 25
Bending Moment M at End of Minor Axis	(a)	$-0.353\ Qd$	$-0.233\ Qd$	$-0.329\ Qd$	$-0.326\ Qd$
	(b)	-0.345 "	-0.223 "	-0.318 "	-0.315 "
	(c)	-0.328 "	-0.200 "	-0.294 "	-0.292 "
Increase of Length of Major Axis per 1000 lb. Load.	(a)	0.000324	0.000475	0.000364	0.000144
	(b)	0.000312	0.000449	0.000345	0.000139
	(c)	0.000289	0.000394	0.000312	0.000121
Decrease of Length of Minor Axis per 1000 lb. Load.	(a)	0.000353	0.000628	0.000404	0.000133
	(b)	0.000345	0.000583	0.000384	0.000126
	(c)	0.000328	0.000478	0.000341	0.000111

It is self-evident that the results obtained from the rough chain links would not be as concordant as those obtained from the finished rings. However, a comparison of the experimental values with the theoretical lines, Fig. 19 to 25, indicates that the theory is confirmed fairly well. The links tested exhibited some variety in form and size; and the results of the calculations show that the agreement of theory and experiment was equally good whether the link was long or short, of 1-in. or 2-in. iron. Tests of more links would have been desirable if sufficient time had been available. It may be stated that the computations are somewhat laborious and time-consuming. It is felt, however, that these four tests are sufficient to establish the validity of the analysis given in Appendix B.

In Fig. 22, additional lines have been drawn to give a comparison of the theory here developed with other theories. If we adopt the analysis usually given in our text-books for hooks and eccentrically loaded bars, in other words, if we neglect the curvature of the link, the theoretical line for the deflection of the minor axis is the line marked "ordinary theory." On the other hand, if we adopt Grashof's assumption (see page 6) we get the steep line marked "Grashof's theory."

The question of the probable distribution of pressure between adjacent links is not definitely settled. In most cases the experimental points follow most closely the line corresponding to case (a), concentration at the end of the link, for the smaller loads. As the load is increased, however, the line through the points becomes steeper and its slope is about that of the theoretical line for case (b), distributed pressure. In a few of the experiments the points approached more closely the line for case (c). It is probable that the distribution depends somewhat upon the length of time a chain has been used. After the links have been fitted to each other and have worn slightly so as to make a bearing, the distribution will be that indicated by a line lying between the lines for cases (a) and (b). In this connection we may repeat the observation before made that in all cases the second test gave a line of greater slope than the first test.

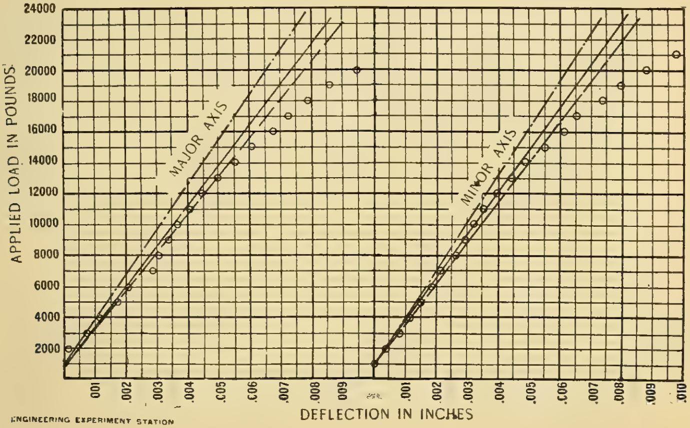


Fig. 23.

By reference to Table 2 it will be seen that the assumed distribution influences in some measure the moment M at the end of the minor axis, and through this the calculated stresses at various sections. The variation of M between cases (a) and (c) is about 7 per cent for the dredge link and 14 per cent for the conveyor link. If it is assumed that case (b) coincides most nearly with the actual distribution, the calculated stresses, taking the value of M from case (b), are not likely to vary more than 3 or 4 per cent from the actual

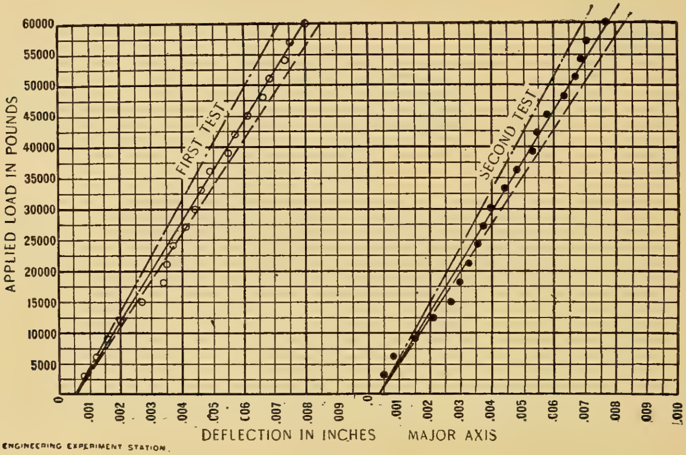


Fig. 24.

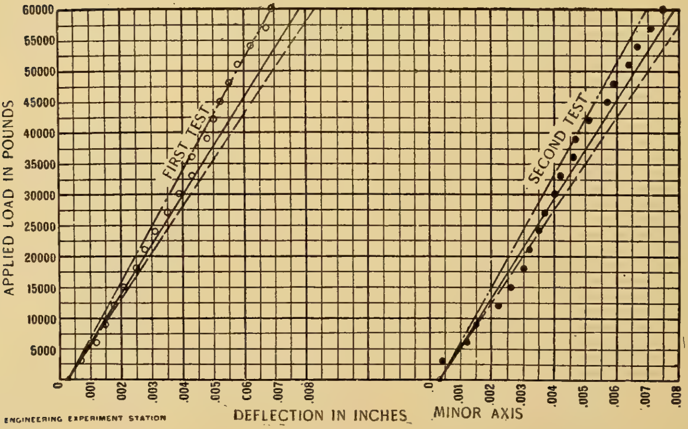


Fig. 25.

stresses even in the most extreme cases. Hence, in the subsequent calculations, we shall assume that the distribution is according to case (b).

Distribution of Stresses in Links

By means of the formulas developed in the Appendices, the intensity of stress can be calculated at any point of any cross section of the link. Thus for an open link, the moment M at the end of the short axis is found from formula (F), and from this the moment at any other section is readily obtained. Now having M_b and the normal force P at the section in question, the stress at different points in the section is found by using different values of y in formula (C). For the outer fiber $y = \frac{1}{2}d$, for the inner fiber $y = -\frac{1}{2}d$, at the axis $y = 0$, and so on.

These calculations have been made for the link shown in Fig. 5, and the results are exhibited in the following table:

TABLE 3

DISTRIBUTION OF STRESS IN OPEN LINK

ϕ	Normal Force P	Moment M_b	Stress at Center of Section $y=0$	Stress in Outer Fiber $y=+\frac{1}{2}d$	Stress in Inner Fiber $y=-\frac{1}{2}d$
0°	0.225 Q	+0.671 Q	+0.896 Q/f	+4.012 Q/f	−8.453 Q/f
10°	0.257	+0.639	0.896	+3.863	−8.006
20°	0.352	+0.544	0.896	+3.420	−6.677
30°	0.500	+0.371	0.774	+2.785	−3.601
40°	0.643	+0.178	0.774	+1.739	−1.325
50°	0.766	+0.011	0.774	+0.836	+0.640
60°	0.866	−0.124	0.774	+0.104	+2.234
70°	0.940	−0.223	0.808	−0.504	+3.225
80°	0.985	−0.284	0.843	−0.917	+3.777
90°	1.000	−0.318	0.936	−1.366	+3.751
End of short axis	1.000	−0.318	1.000	−1.546	+3.546

A better idea of the distribution of stress through the link is shown in Fig. 26. At section a, lying along the minor axis, the inner fiber is subjected to a tensile stress of 3.55 $\frac{Q}{f}$, while the outer fiber is under a compression 1.55 $\frac{Q}{f}$. At section b, the tensile stress at the inner fiber is a little greater, due entirely to the curvature at that section, and at section c this tensile stress is still greater because of the sharper curvature, notwithstanding the fact that the moment M_b is smaller. From here on, however, the tensile stress on the inside of the line rapidly decreases and reaches zero at the point L. At section e the moment M_b changes sign by passing through the value zero; hence at this section the stress is uniformly distributed and equal to $\frac{P}{f}$. From L to C the minor fiber of the link is in compression, the intensity of the compression reaching its maximum value $8.453 \frac{Q}{f}$ at the point C. From A to K the outer fiber of the link is compressed, but from K to D it is in tension, the maximum intensity of the tension reaching the value $4.012 \frac{Q}{f}$ at the point D. The lines HK and LM indicate the points of the link at which the stress is zero.

It will be observed that there are two points of maximum tensile stress; one at D, the other at E on the inside of the link. The compressive stress in the outer fibers is small; but at the point C it is very large.

The following table gives the stresses in the same link when provided with a stud; and Fig. 27 shows the distribution of stress in such a link.

TABLE 4

DISTRIBUTION OF STRESS IN STUD LINK

ϕ	Normal Force P	Moment M_b	Stress at Axis $y=0$	Stress in Outer Fiber $y=+\frac{1}{2}d$	Stress in Inner Fiber $y=-\frac{1}{2}d$
0°	+0.555 Q	+0.401 Q	0.955 Q/f	+2.814 Q/f	−4.623 Q/f
10°	+0.582	+0.373	0.955	+2.689	−4.246
20°	+0.662	+0.293	0.955	+2.314	−3.123
30°	+0.782	+0.152	0.895	+1.722	−0.905
40°	+0.892	+0.003	0.895	+0.913	+0.855
50	+0.975	−0.109	0.895	+0.304	+2.180
60°	+1.029	−0.181	0.895	−0.089	+3.034
70°	+1.051	−0.211	0.895	−0.251	+3.216
80°	+1.041	−0.198	0.895	−0.180	+3.186
90°	+1.000	−0.056	0.989	+0.587	+1.480
End of short axis	+1.000	+0.107	1.000	+1.858	+1.424

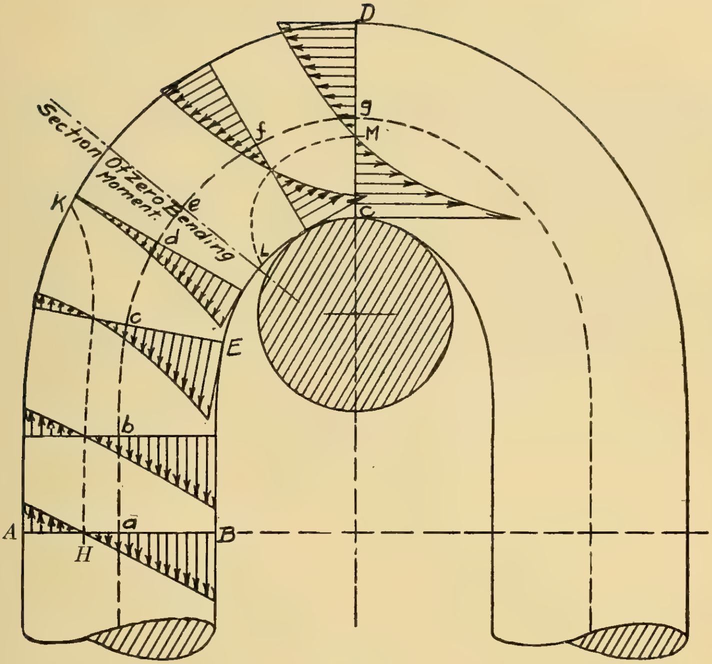

D
g
M
f
Section Of Zero Bending Moment.
e
K
d
L
C
c
E
b
A
a
B
H

FIG. 26.

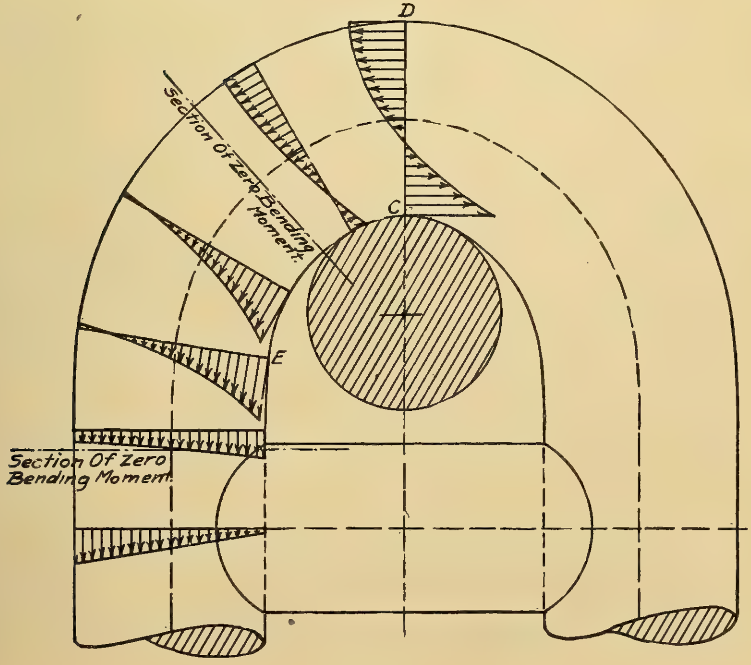

D
Section Of Zero Bending Moment.
C
E
Section Of Zero Bending Moment.

FIG. 27.

It will be observed that in this case there are two sections at which the bending moment is zero. The tensile stress reaches a maximum for the outer fiber at *D*, and for the inner fiber at about the point *E*. The compression is greatest at point *C*, but is only a little over one-half that at *C* in the case of the open link. The tensile stresses are also somewhat smaller than for the open link.

The following table gives the maximum tensile stresses (at points *D* and *E*) and the maximum compressive stress (at point *C*) for each of the four links subjected to analysis.

TABLE 5

MAXIMUM STRESSES

Link	Open Link			Stud Link		
	Tensile Stress		Compressive Stress at *C*	Tensile Stress		Compressive Stress at *C*
	At *E*	At *D*		At *E*	At *D*	
Dredge, Fig. 3	3.98 Q/f	3.66 Q/f	8.38 Q/f	3.18 Q/f	2.81 Q/f	4.62 Q/f
Proof coil, Fig. 5	3.78	4.01	8.45	3.22	2.56	4.02
Two-inch, Fig. 6	3.72	3.47	7.94	3.20	2.38	3.54
Conveyor, Fig. 4	2.78	4.17	9.55	...	...	...

A study of the results presented in the preceding tables leads to some interesting conclusions:

In the first place, it may be observed that the maximum stresses for the different links are not widely different. The first three links may be regarded as typical of the forms ordinarily used in engineering practice, and in these the extreme variation in the maximum tensile stress is a little more than 7 per cent. It is also worthy of remark that the tensile stresses at the two points *D* and *E* are nearly the same. In some cases the greater stress will be at *D*, in others at *E*.

The conveyor link, on account of its relatively great length, presents an exception. As shown by the analysis, the increased length of the side makes the moment *M* at the middle of the side small; consequently the moment at the end of the link is large, and the stress at D is considerably greater than that at E. It may be concluded, therefore, that so far as strength is concerned, the form of this link is not favorable.

The effect of the stud upon the distribution of stress is easily seen. The maximum tensile stresses are reduced about 20 per cent; but what is more essential, the heavy compressive stress at C is reduced 50 per cent or more. We conclude, therefore, that provided the stresses are kept within the elastic limit of the material, the stud is of unquestioned value.

It has been the general opinion of engineers that the stud link chain is stronger than the open link chain; however, the experiments of committee D of the United States board appointed to test iron. steel and other metals (see Executive Document No. 98, House of Representatives, Forty-fifth Congress, Second Session), seem to indicate that the stud actually weakens the chain, causing it to rupture at a load lower than that required to break an open link chain. At first sight these experiments seem to disprove the results given in the preceding pages; however, in this case, fact and theory are easily reconciled. It is quite easy to understand that while the stud link is much stronger than the open link, provided the elastic limit is not reached, the former may rupture with a smaller load than the latter. In the first place, the collapse of the sides of the open link after the elastic limit is passed decreases the effective width of the link, and thus decreases the bending moments and stresses. If the iron of

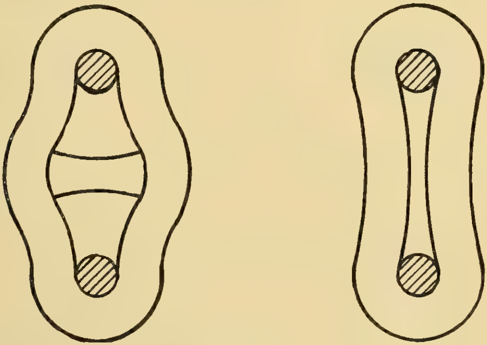


FIG. 28.

which the link is constructed is ductile, the link may collapse until the sides become nearly parallel, and the stresses are lower than in the stud link, the sides of which are prevented from collapsing by the stud. The appearance of the two forms of link under heavy load will be somewhat as shown in Fig. 28. Thus the actual distortion of the open link gives it a form of greater strength, which is not the case with the stud link.

Near the load producing rupture it seems likely, therefore, that the stresses in the open link are less than those in the stud link subjected to the same load. Within the elastic limit, however, the reverse is true, and there can be no doubt that for ordinary working loads the chain made of stud links is materially stronger than the one made of open links.

FORMULAS FOR THE LOADING OF CHAINS

Unwin, *Elements of Machine Design*, Part I, p. 438, gives the following formulas:

$$\begin{aligned} P &= 9\,d^2, \text{ for studded link chain;} \\ &= 6\,d^2, \text{ for unstudded close link chain.} \end{aligned}$$

He says further: "For much used chain, subject frequently to the maximum load, it is better to limit the stress to $3\frac{1}{4}$ tons per sq. in. Then

$$P = 5\,d^2."$$

In these formulas, P denotes the load in tons, and d the diameter in inches of the iron from which the chain is made.

Unwin says that Towne limits the loads in ordinary crane chains to

$$P = 3.3\,d^2,$$

but quotes the following table from Towne's *Treatise on Cranes*.

Diameter of Iron	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$
Load on chain — tons	0.06	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5

This table seems to be obtained from the formula

$$P = 8\,d^2.$$

Weisbach gives the formulas (Kent's *Pocket Book*, p. 339)

$$P = 17800\,d^2,\ \text{stud link,}$$
$$P = 13350\,d^2,\ \text{open link,}$$

where P denotes the load in pounds.

Bach, in his *Maschinenelemente*, p. 513, gives for chains with open links

$P = 1000\,d^2$ for new chains, maximum load seldom applied.
$P = 800\,d^2$ for much used chain.

P and d are taken in kilograms and centimeters, respectively. Using pounds and inches as the units, the formulas become

$$P = 13750\,d^2;$$
$$P = 11000\,d^2.$$

For a stud-link chain, Bach increases the safe load 20 per cent.

If we write the formula for the safe load

$$P = kd^2,$$

the values of k given by the authorities quoted are as follows, P being taken in pounds:

	Open Link	Stud Link
Unwin	13,440 11,200	20,160 ...
Weisbach	13,350	17,800
Bach	13,750 11,000	16,500 13,200

Referring to Table 5, we note that with links of the ordinary form the maximum tensile stresses are about as follows:

$$\left.\begin{aligned} &\text{for open links, } 4\,\frac{Q}{f}\,; \\ &\text{for stud links, } 3.2\,\frac{Q}{f}\,. \end{aligned}\right\} \qquad (1)$$

Of course these values will vary slightly with the form of the link; thus the conveyor link on account of its extreme length shows a maximum tensile stress of $4.17\frac{Q}{f}$. In general, however, the value $4\frac{Q}{f}$ cannot be very far from the truth for an open link of usual dimensions.

Denoting the load on the chain by P, and the maximum permissible unit stress by S, we have, since $P = 2Q$,

$$\left.\begin{aligned} &\text{for open links,} \quad S = \frac{4Q}{f} = \frac{2P}{f}\,; \\ &\text{for stud links,} \quad S = \frac{3.2Q}{f} = \frac{1.6P}{f}\,. \end{aligned}\right\} \tag{2}$$

Now taking $f = \frac{1}{4}\pi d^2$, we readily obtain

$$\left.\begin{aligned} &\text{for open links,} \quad P = \frac{1}{8}\pi d^2 S, \text{ say } 0.4\, d^2 S\,; \\ &\text{for stud links,} \quad P = \frac{1}{6.4}\pi d^2 S, \text{ say } 0.5\, d^2 S\,. \end{aligned}\right\} \tag{3}$$

Comparing these equations with the equation

$$P = kd^2,$$

we see that

$$\left.\begin{aligned} &k = 0.4\,S, \quad \text{for open links,} \\ \text{and} \quad &k = 0.5\,S, \quad \text{for stud links.} \end{aligned}\right\} \tag{4}$$

Now using the values of k just given, we obtain the following values of S when we use the formulas ordinarily given.

	Open Link	Stud Link
Unwin	33,600 28,000	. . . 40,320
Weisbach	33,375	35,600
Bach	34,375 27,500	33,000 26,400

It will probably be agreed that these values are considerably in excess of the values usually regarded as permissible in machine construction.

So far, we have considered only the tensile stresses. Referring to Table 5 it is seen that the compression at the end of the link is more than double the maximum tensile stress; hence when a chain has its full load, if the maximum tensile stress is 30,000 lb., as indicated by the constants above, the compressive stress at the end is something over 60,000 lb. It is probable that when the maximum load is applied, the pinching action heretofore described reduces to a considerable degree this excessive compression. Furthermore, it will be noted that the part of the link subjected to this compression is restrained laterally by the sides of the adjacent link; and this lateral restraint offsets in some measure the compressive stress. In any case, however, this compression is a factor to be seriously considered.

Using the maximum tensile stress as a basis, the formulas

$$P = 0.4\, d^2S \text{ (open)} \qquad (5)$$

$$P = 0.5\, d^2S \text{ (stud)} \qquad (6)$$

for open and stud links respectively, are proposed as substitutes for the formulas now in use. These formulas contain the safe maximum unit stress S, and are in that respect more general than those quoted from Unwin, Bach and others. If desired, the usual form $P = kd^2$ is readily obtained by assuming a proper value of S. Thus if S is taken at 15,000 lb. sq. in., we have

$$P = 6000\, d^2 \text{ (open)},$$

$$P = 7500\, d^2 \text{ (stud)},$$

respectively; if 20,000 lb. sq. in. is considered a permissible value of S, the formulas become

$$P = 8000\, d^2 \text{ (open)},$$

$$P = 10{,}000\, d^2 \text{ (stud)},$$

respectively.

SUMMARY OF RESULTS, AND CONCLUSIONS

The following is a summary of the results obtained from the investigations herein described and the conclusions that may be drawn from them:

1. The experiments on the steel rings confirm the theoretical analysis employed in the calculation of stresses.

2. The experiments on the various chain links further confirm the analysis and show that the distribution of pressure between the links, in general, lies between the extremes (*a*), point contact and (*c*), pressure concentrated at opposite points, as in Fig. 2(c). For purposes of calculation case (*b*), uniform distribution of pressure over an arc 2α may be assumed.

3. The load $2\,Q$ on the link produces an average intensity of stress $\frac{2Q}{2f} = \frac{Q}{f}$ in the cross section of the link containing the minor axis. With an open link of usual proportions the maximum tensile stress is approximately four times this value, i.e., $4\frac{Q}{f}$.

4. The introduction of a stud in the link equalizes the stresses throughout the link, reduces the maximum tensile stresses about 20 per cent, and reduces the excessive compressive stress at the end of the link about 50 per cent.

5. The stud-link chain of equal dimensions will, within the elastic limit, bear from 20 to 25 per cent more load than the open-link chain. The ultimate strength of the stud-link chain is, however, probably less than that of the open-link chain.

6. In the formulas for the safe loading of chains given by the leading authorities on machine design, the maximum stress to which the link is subjected seems to be underestimated and the constants are such as to give maximum stresses of from 30,000 to 40,000 lb. per sq. in. for full load.

7. The following formulas are applicable to chains of the usual form:

$$P = 0.4\, d^2S, \text{ for open links,}$$
$$P = 0.5\, d^2S, \text{ for stud links,}$$

where P denotes the safe load, d the diameter of the stock, and S the maximum permissible tensile stress.

TABLE 6

FIRST TEST OF 12-INCH STEEL RING NO. 1

Outside Diameter	12.000 in.
Inside Diameter	9.000 in.
Width	1.0476 in.
Modulus of Elasticity	26,200,000

Applied Load Pounds	Extensometer Readings		Deformations	
	Horizontal Axis	Vertical Axis	Horizontal Axis	Vertical Axis
500	.2420	.2408	...	...
1,000	.2419	.2379	−.0001	.0029
1,500	.2442	.2367	.0022	.0041
2,020	.2467	.2357	.0047	.0051
2,500	.2474	.2342	.0054	.0066
3,000	.2484	.2322	.0064	.0086
3,500	.2494	.2306	.0074	.0102
4,000	.2503	.2277	.0083	.0131
4,500	.2529	.2284	.0109	.0124
5,000	.2536	.2270	.0116	.0138
5,500	.2546	.2256	.0126	.0152
5,900	.2550	.2249	.0130	.0159
6,500	.2572	.2234	.0152	.0174
7,000	.2583	.2218	.0163	.0190
7,500	.2600	.2187	.0180	.0221

TABLE 7

SECOND TEST OF 12-INCH STEEL RING NO. 1

Applied Load Pounds	Extensometer Readings		Deformations	
	Horizontal Axis	Vertical Axis	Horizontal Axis	Vertical Axis
0	.9985	.0024	...	...
500	.9975	.0048	.0012	.0024
1,000	.9960	.0045	.0025	.0021
1,500	.9948	.0057	.0036	.0033
2,000	.9933	.0073	.0052	.0049
2,500	.9917	.0085	.0068	.0061
3,000	.9913	.0098	.0072	.0074
3,500	.9907	.0116	.0078	.0092
4,000	.9890	.0130	.0095	.0106
4,500	.9874	.0143	.0111	.0119
5,000	.9863	.0156	.0122	.0132
5,500	.9851	.0170	.0134	.0146
6,000	.9840	.0184	.0145	.0160
6,500	.9824	.0199	.0161	.0175
7,000	.9808	.0213	.0177	.0189
7,500	.9800	.0223	.0185	.0199

TABLE 8

FIRST TEST OF STEEL RING NO. 2

Outside Diameter	12.0056 in.
Inside Diameter	9.000 in.
Thickness	1.005 in.
Modulus of Elasticity	26,200,000

Applied Load Pounds	Extensometer Readings		Deformations	
	Horizontal Axis	Vertical Axis	Horizontal Axis	Vertical Axis
0	.0076	.0043	...	...
500	.0054	.0062	.0022	.0019
1,000	.0047	.0063	.0029	.0020
1,500	.0036	.0087	.0040	.0044
2,000	.0018	.0102	.0058	.0059
2,500	.0007	.0120	.0069	.0077
3,000	.9994	.0135	.0082	.0092
3,500	.9975	.0138	.0101	.0095
4,000	.9958	.0153	.0118	.0110
4,500	.9953	.0167	.0123	.0124
5,000	.9932	.0177	.0144	.0134
5,500	.9920	.0199	.0156	.0156
6,000	.9917	.0217	.0159	.0174
6,500	.9905	.0230	.0171	.0187
7,000	.9887	.0249	.0189	.0206
7,500	.9870	.0270	.0206	.0227

TABLE 9

SECOND TEST OF STEEL RING NO. 2

Applied Load Pounds	Extensometer Readings		Deformations	
	Horizontal Axis	Vertical Axis	Horizontal Axis	Vertical Axis
0	1.0050	.0062	...	...
500	1.0028	.0067	.0022	.0005
1,000	1.0023	.0082	.0027	.0020
1,500	1.0018	.0100	.0032	.0038
2,000	.9994	.0106	.0056	.0044
2,500	.9991	.0130	.0059	.0068
3,000	.9964	.0148	.0086	.0084
3,500	.9960	.0158	.0090	.0096
4,000	.9943	.0170	.0107	.0108
4,500	.9932	.0186	.0118	.0124
5,000	.9919	.0196	.0131	.0134
5,500	.9908	.0220	.0142	.0158
6,000	.9899	.0230	.0151	.0168
6,500	.9884	.0240	.0166	.0178
7,000	.9871	.0255	.0179	.0193
7,500	.9860	.0270	.0190	.0208

TABLE 10

FIRST TEST OF FORGED RING NO. 3

Outside Diameter	12.007 in.
Inside Diameter	9.001 in.
Width	1.00 in.
Modulus of Elasticity	30,400,000

Applied Load Pounds	Extensometer Readings		Deformations	
	Horizontal Axis	Vertical Axis	Horizontal Axis	Vertical Axis
0	.0063	.0050	.0	.0
500	.0085	.0036	.0014	.0022
1,000	.0094	.0025	.0025	.0031
1,500	.0101	.0007	.0043	.0038
2,000	.0115	.9996	.0054	.0052
2,500	.0113	.9987	.0063	.0070
3,000	.0147	.9969	.0081	.0084
3,500	.0154	.9969	.0081	.0091
4,000	.0171	.9946	.0104	.0108
4,500	.0184	.9930	.0120	.0121
5,000	.0195	.9925	.0125	.0131
5,500	.0206	.9913	.0137	.0143
6,000	.0220	.9900	.0150	.0157
6,500	.0228	.9887	.0163	.0165
7,000	.0245	.9877	.0173	.0182
7,500	.0261	.9871	.0179	.0198
8,000	.0287	.9852	.0198	.0224

TABLE 11

SECOND TEST OF FORGED STEEL RING NO. 3

Applied Load	Extensometer Readings		Deformations	
	Horizontal Axis	Vertical Axis	Horizontal Axis	Vertical Axis
0	.0032	.0063	.0	.0
500	.0014	.0070	.0018	.0003
1,000	.0012	.0094	.0020	.0027
1,500	.0003	.0102	.0029	.0035
2,000	.9994	.0124	.0038	.0057
3,000	.9960	.0133	.0072	.0076
3,500	.9946	.0143	.0086	.0086
4,000	.9935	.0153	.0097	.0103
4,500	.9926	.0170	.0106	.0115
5,000	.9916	.0182	.0116	.0133
5,500	.9902	.0200	.0130	.0140
6,000	.9992	.0207	.0140	.0155
6,500	.9883	.0222	.0150	.0163
7,000	.9871	.0230	.0161	.0179
7,500	.9860	.0257	.0172	.0190
8,000	.9836	.0273	.0196	.0206

TABLE 12

FIRST TEST OF DREDGE LINK

Applied Load	Vertical Axis					Horizontal Axis	
	Readings		Differences		Deflections	Readings	Deflections
1,000	.0098	.0125	...	...	...	.5100	...
2,000	.0101	.0129	.0003	.0004	.00035	.5099	.0001
3,000	.0103	.0136	.0005	.0011	.0008	.5092	.0008
4,000	.0106	.0136	.0008	.0011	.00095	.5092	.0008
5,000	.0109	.0138	.0011	.0013	.0012	.5086	.0014
6,000	.0109	.0138	.0011	.0013	.0012	.5080	.0020
7,000	.0116	.0140	.0018	.0015	.00165	.5081	.0019
8,000	.0112	.0149	.0014	.0024	.0019	.5080	.0020
9,000	.0117	.0146	.0019	.0021	.0020	.5079	.0021
10,000	.0119	.0152	.0021	.0027	.0024	.5074	.0026
11,000	.0121	.0149	.0023	.0021	.0023	...	...
12,000	.0124	.0158	.0026	.0033	.00295	.5070	.0030
13,000	.0126	.0160	.0028	.0035	.00315	.5068	.0032
14,000	.0129	.0163	.0031	.0038	.00345	.5065	.0035
15,000	.0131	.0170	.0033	.0045	.0039	.5062	.0038
16,000	.0132	.0171	.0034	.0046	.0040	.5057	.0043
17,000	.0138	.0180	.0040	.0055	.00475	.5052	.0048
18,000	.0142	.0188	.0044	.0063	.00535	.5049	.0051
19,000	.0155	.0192	.0055	.0067	.0061	.5040	.0060
20,000	.0161	.0199	.0063	.0074	.00685	.5030	.0070
21,000	.0176	.0221	.0078	.0096	.0087	.5010	.0090
22,000	.0216	.0267	.0118	.0142	.0130	.4969	.0131

TABLE 13

SECOND TEST OF DREDGE LINK

[Modulus of Elasticity 24,600,000]

Applied Load	Vertical Axis					Horizontal Axis	
	Readings		Differences		Deflections	Readings	Deflections
1,000	.0033	.0035	...	...	...	.2036	...
2,000	.0035	.0038	.0002	.0003	.00025	.2035	.0001
4,000	.0038	.0045	.0005	.0010	.00075	.2034	.0002
6,000	.0042	.0054	.0009	.0019	.0014	.2030	.0006
8,000	.0048	.0062	.0015	.0027	.0021	.2024	.0012
10,000	.0048	.0072	.0015	.0037	.0026	.2016	.0020
12,000	.0051	.0078	.0018	.0043	.00305	.2013	.0023
14,000	.0059	.0085	.0028	.0050	.0038	.2010	.0026
16,000	.0061	.0088	.0025	.0053	.00405	.1999	.0037
18,000	.0058	.0101	.0028	.0066	.00455	.0997	.0039
20,000	.0068	.0111	.0033	.0076	.00545	.1990	.0046
22,000	.0069	.0120	.0036	.0085	.00695	.1981	.0055
24,000	.0326	.0248	.0293	.0213	.0253	...	...

TABLE 14

FIRST TEST OF CONVEYOR LINK

Applied Load	Vertical Axis					Horizontal Axis	
	Readings		Differences		Deflections	Readings	Deflections
1,500	.1363	.0400	...	...	...	.4875	...
2,000	.1368	.0399	.0005	.0001	.0002	.4880	.0005
2,500	.1371	.0407	.0008	.0007	.00075	.4870	.0005
3,000	.1383	.0401	.0020	.0001	.00105	.4869	.0006
3,500	.1389	.0401	.0026	.0001	.00135	.4865	.0010
4,000	.1390	.0404	.0027	.0004	.00155	.4863	.0012
4,500	.1495	.0407	.0032	.0007	.00195	.4860	.0015
5,000	.1398	.0408	.0035	.0008	.00215	.4860	.0015
5,500	.1401	.0409	.0038	.0009	.00235	.4855	.0020
6,000	.1401	.0416	.0038	.0016	.0027	.4853	.0022
6,500	.1404	.0410	.0041	.0010	.00255	.4850	.0025
7,000	.1405	.0412	.0042	.0012	.0027	.4847	.0028
7,500	.1408	.0413	.0045	.0013	.0029	.4843	.0032
8,000	.1410	.0417	.0047	.0017	.0032	.4840	.0038
8,500	.1411	.0418	.0048	.0018	.0033	.4837	.0038
9,000	.1413	.0419	.0050	.0019	.00345	.4834	.0041
9,000	.1408	.0422	...	...	...	.4832	...
10,000	.1420	.0425	.0062	.0022	.0042	.4830	.0043
11,000	.1425	.0429	.0067	.0026	.00465	.4822	.0053
12,000	.1427	.0430	.0069	.0027	.0048	.4814	.0061
13,000	.1432	.0436	.0074	.0033	.00535	.4810	.0065
14,000	.1434	.0443	.0076	.0040	.00580	.4800	.0075
15,000	.1437	.0448	.0079	.0045	.00620	.4791	.0084
16,000	.1443	.0452	.0085	.0049	.0067	.4782	.0093
17,000	.1448	.0459	.0090	.0056	.0073	.4774	.0101
18,000	.1458	.0459	.0100	.0056	.0078	.4766	.0109
19,000	.1462	.0463	.0104	.0060	.0082	.4754	.0121
20,000	.1470	.0470	.0112	.0067	.00895	.4740	.0135
21,000	.1485	.0474	.0127	.0071	.0099	.4724	.0151
22,000	.1499	.0480	.0141	.0077	.0109	.4710	.0165
23,000	.1517	.0498	.0159	.0095	.0127	...	...
24,000	.1533	.0512	.0175	.0109	.0142	.4660	.0215
25,000	.1569	.0551	.0211	.0148	.01795	.4620	.0255

TABLE 15

SECOND TEST OF CONVEYOR LINK

[Modulus of Elasticity, 28,400,000]

Applied Load	Vertical Axis					Horizontal Axis	
	Readings		Differences		Deflections	Reading	Deflections
1,120	.1835	.0292	...	...	...	.4013	...
2,040	.1840	.0295	.0005	.0003	.0004	.4010	.0003
3,020	.1843	.0300	.0008	.0008	.0008	.4003	.0010
4,000	.1848	.0302	.0013	.0010	.00115	.3996	.0017
5,110	.1852	.0308	.0017	.0016	.00165	.3984	.0029
6,040	.1858	.0312	.0023	.0020	.00215	.3981	.0032
7,000	.1860	.0315	.0025	.0023	.0024	.3978	.0035
8,030	.1864	.0319	.0029	.0027	.0028	.3969	.0044
9,190	.1867	.0323	.0032	.0031	.00315	.3964	.0049
10,000	.1873	.0326	.0038	.0034	.0036	.3960	.0053
11,000	.1880	.0329	.0045	.0037	.0041	.3956	.0057
12,020	.1882	.0332	.0047	.0040	.00435	.3949	.0064
13,090	.1886	.0336	.0051	.0044	.00475	.3942	.0071
14,090	.1890	.0341	.0055	.0049	.0052	.3936	.0077
15,030	.1892	.0345	.0057	.0053	.0055	.3930	.0083
16,090	.1895	.0347	.0060	.0055	.00575	.3924	.0089
17,020	.1897	.0350	.0062	.0058	.0060	.3920	.0093
18,000	.1898	.0356	.0063	.0064	.00635	.3914	.0099
19,000	.1907	.0355	.0072	.0063	.00675	.3910	.0103
20,000	.1910	.0359	.0075	.0067	.0071	.3904	.0109
21,000	.1918	.0365	.0083	.0073	.0078	.3899	.0114
22,000	.1922	.0367	.0087	.0075	.0081	.3893	.0120
23,000	.1923	.0372	.0088	.0080	.0084	.3884	.0129
24,000	.1928	.0378	.0093	.0086	.00895	.3878	.0135
25,000	.1933	.0380	.0098	.0088	.0093	.3873	.0140
26,000	.1940	.0382	.0105	.0090	.00975	.3870	.0143
27,000	.1944	.0389	.0109	.0097	.0103	.3860	.0153
28,000	.2081	.0562	.0246	.0270	.0258	...	...
29,000	.2185	.0673	.0350	.0381	.03655	...	...

TABLE 16

TEST OF PROOF COIL LINK

[Modulus of Elasticity, 26,100,000]

Applied Load	Vertical Axis					Horizontal Axis	
	Readings		Differences		Average Deflections	Readings	Deflections
1,000	.2006	.2002	...	...	...	.3872	...
2,000	.2010	.1006	.0004	.0004	.0004	.3870	.0002
3,000	.2017	.2008	.0011	.0006	.00085	.3864	.0008
4,000	.2021	.2010	.0015	.0008	.00115	.3860	.0012
5,000	.2022	.2016	.0016	.0014	.0015	.3854	.0018
6,000	.2026	.2019	.0020	.0017	.00185	.3851	.0021
7,000	.2030	.2021	.0024	.0019	.00215	.3843	.0029
8,000	.2034	.2027	.0028	.0025	.00265	.3841	.0031
9,000	.2038	.2929	.0032	.0027	.00295	.3838	.0034
10,000	.2041	.2032	.0035	.0030	.00325	.3835	.0037
11,000	.2045	.2034	.0039	.0032	.00355	.3831	.0041
12,000	.2049	.2038	.0043	.0036	.00395	.3827	.0045
13,000	.2054	.2043	.0048	.0041	.00445	.3822	.0050
14,000	.2060	.1047	.0052	.0045	.00485	.3816	.0056
15,000	.2066	.2052	.0060	.0050	.0055	.3811	.0061
16,000	.2071	.2059	.0065	.0057	.0061	.3804	.0068
17,000	.2078	.2061	.0072	.0059	.00655	.3799	.0073
18,000	.2087	.2068	.0081	.0066	.00735	.3793	.0079
19,000	.2097	.2070	.0091	.0068	.00795	.3786	.0086
20,000	.2108	.2076	.0102	.0074	.0088	.3777	.0095
21,000	.2119	.2088	.0113	.0086	.00975	.3763	.0109

TABLE 17

FIRST TEST OF TWO-INCH DREDGE LINK

[Modulus of Elasticity 29,000,000 (Assumed)]

Applied Load	Vertical Axis					Horizontal Axis	
	Readings		Differences		Deflections	Readings	Deflections
0	.0215	.5488	...	...	...	.5907	...
3,000	.0223	.5496	.0008	.0008	.0008	.5900	.0007
6,000	.0225	.5501	.0010	.0013	.0012	.5895	.0012
9,000	.0230	.5511	.0015	.0017	.0016	.5892	.0015
12,000	.0231	.5518	.0016	.0023	.0020	.5889	.0018
15,000	.0238	.5524	.0023	.0030	.0027	.5886	.0021
18,000	.0240	.5528	.0025	.0036	.0032	.5882	.0025
21,000	.0244	.5531	.0029	.0040	.0035	.5879	.0028
24,000	.0245	.5535	.0031	.0043	.0037	.5876	.0031
27,000	.0249	.5538	.0034	.0047	.0041	.5872	.0035
30,000	.0253	.5540	.0038	.0050	.0044	.5868	.0039
33,000	.0254	.5543	.0039	.0052	.0046	.5864	.0043
36,000	.0258	.5549	.0043	.0055	.0049	.5864	.0043
39,000	.0263	.5551	.0048	.0061	.0055	.5869	.0048
42,000	.0265	.5555	.0050	.0063	.0057	.5857	.0050
45,000	.0269	.5561	.0054	.0067	.0061	.5855	.0052
48,000	.0273	.5563	.0058	.0073	.0066	.5852	.0055
51,000	.0277	.5568	.0062	.0075	.0068	.5849	.0058
54,000	.0280	.5568	.0065	.0080	.0073	.5845	.0062
57,000	.0283	.5570	.0068	.0082	.0075	.5840	.0067
60,000	.0295	.5578	.0070	.0090	.0080	.5838	.0069

TABLE 18

SECOND TEST OF TWO-INCH DREDGE LINK

Applied Load	Vertical Axis					Horizontal Axis	
	Readings		Differences		Deflections	Readings	Deflections
0	.0216	.5495	...	...	...	.5902	...
3,000	.0219	.5502	.0003	.0007	.0005	.5898	.0004
6,000	.0223	.5505	.0007	.0010	.00085	.5890	.0012
9,000	.0228	.5514	.0012	.0019	.00155	.5887	.0015
12,000	.0230	.5523	.0014	.0028	.0021	.5880	.0022
15,000	.0235	.5530	.0019	.0035	.0027	.5876	.0026
18,000	.0238	.5532	.0022	.0037	.00295	.5872	.0030
21,000	.0241	.5535	.0025	.0040	.00325	.5870	.0032
24,000	.0242	.5540	.0026	.0045	.00355	.5867	.0035
27,000	.0246	.5540	.0030	.0045	.00375	.5865	.0037
30,000	.0247	.5543	.0031	.0048	.00395	.5862	.0040
33,000	.0251	.5548	.0035	.0053	.0044	.5860	.0042
36,000	.0255	.5552	.0039	.0057	.0048	.5856	.0046
39,000	.0259	.5558	.0043	.0063	.0053	.5855	.0047
42,000	.0260	.5560	.0044	.0065	.00545	.5851	.0051
45,000	.0265	.5562	.0049	.0067	.0058	.5845	.0057
48,000	.0270	.5568	.0054	.0073	.00635	.5843	.0059
51,000	.0275	.5570	.0059	.0075	.0067	.5838	.0064
54,000	.0278	.5571	.0062	.0076	.0069	.5838	.0067
60,000	.0280	.5573	.0064	.0078	.0071	.5831	.0071
75,000	.0285	.5580	.0068	.0085	.0077	.5827	.0075

TABLE OF SYMBOLS

Q = one-half of load applied to chain;

M_b = bending moment at any chosen section;

M = bending moment in side of link at end of minor axis;

P = normal force on any section of link;

s = intensity of stress at any point of a cross section;

ϕ = angle between any section and major axis of link;

α = one-half of assumed arc of contact between adjacent links;

f = area of cross section of link;

E = modulus of elasticity;

d = diameter of iron in link;

r = general symbol for radius of curvature;

r_2, r_3 = radii of curvature of parts of link;

ε = relative extension of any fiber;

ε_0 = relative extension of center line of link;

ω = ratio $\Delta d\phi : d\phi$;

y = distance of fiber from center line;

$$z = -\frac{1}{f}\int \frac{y}{r+y}\,df;$$

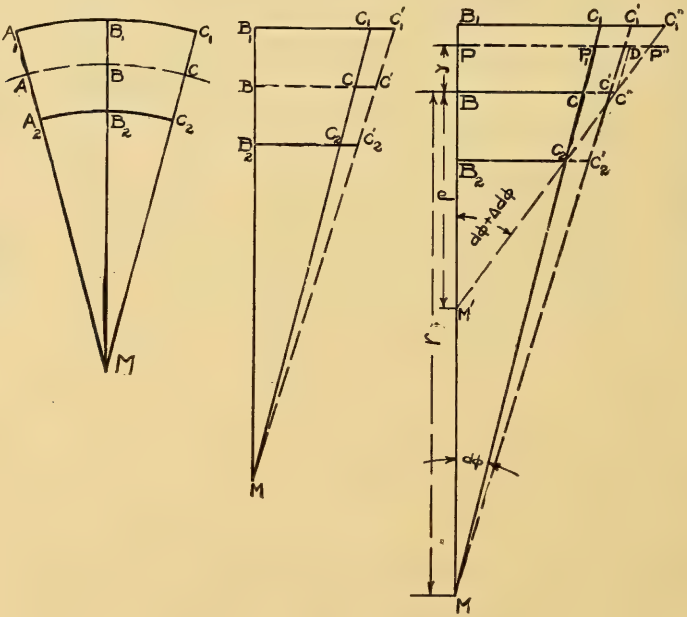
Δx, Δy = deflections of major and minor axes of link, respectively;

S = one-half of pressure between stud and side of link.

APPENDIX A

THEORY OF STRESSES IN CURVED BARS *

Consider an element of the bar included between two cross sections A_1A_2 and C_1C_2, Fig. 29. The planes of these normal cross sections intersect in a line which pierces the plane of the paper at M; this line is the axis of curvature, that is, point M is the center of curvature of the center line AC. For the sake of convenience, we shall make use of only one-half of the element, as shown in Fig. 30, and we shall consider the sides B_1C_1, B_2C_2 to be straight lines, since the sections B_1B_2 and C_1C_2 are taken indefinitely close to each other.



Figs. 29, 30, 31.

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* The theory here given is substantially that laid down by Bach, *Elasticitat und Festigkeit,* § 54.

Suppose now that an external force P acts at right angles to the section C_1C_2. If this force is uniformly distributed over the cross section C_1C_2, each fiber will be elongated (or shortened) by an amount proportional to its original length. Thus, assuming that the stress is tensile, we shall have

$$\frac{C_1C_1'}{B_1C_1} = \frac{CC'}{BC'} = \frac{C_2C_2'}{B_2C_2} = \text{a constant.}$$

It follows that the plane of the cross section will in its new position $C_1'C_2'$ pass through the axis of curvature M.

In addition to the force P normal to the section, let there be a couple of moment M_b acting at the section in question. It is assumed that the sense of the couple is such as to increase the curvature of the bar. The section C_1C_2 of the unloaded bar is brought to $C_1'C_2'$ by the normal force P, as just explained. The couple causes it to assume a new position $C_1''C_2''$, Fig. 31. The plane of the section in this position intersects the plane of the section B_1B_2 in the line M'. The angle between the cross sections is increased from $d\phi$ to $d\phi + \Delta d\phi$, and the radius of curvature is shortened from r to ρ.

Let ds denote the length BC and Δds the elongation CC'' due to the force P and moment M_b. The ratio $\dfrac{\Delta ds}{ds}$ we shall denote by ε_0;

hence
$$\varepsilon_0 = \frac{CC''}{BC}.$$

Consider now a fiber lying along PP_1 at a distance y from the center line BC. The extension of the fiber is P_1P''; hence we have

$$\varepsilon = \frac{P_1P''}{PP_1}.$$

To determine ε, let $C''D$ be drawn parallel to C_1C_2. Then

$$\begin{aligned} P_1P'' &= P_1D + DP'' = CC'' + DP'' \\ &= \varepsilon_0 ds + y \cdot \text{angle } DC''P'' = \varepsilon_0 ds + y \cdot \Delta d\phi, \end{aligned}$$

and
$$PP_1 = (y + r)d\phi.$$

Therefore
$$\varepsilon = \frac{\varepsilon_0 ds + y \cdot \Delta d\phi}{(r + y)\, d\phi} = \frac{\varepsilon_0 \dfrac{ds}{d\phi} + y \dfrac{\Delta d\phi}{d\phi}}{r + y}.$$

Let ω denote the ratio $\frac{\Delta d\phi}{d\phi}$; then since $ds = rd\phi$,

$$\varepsilon = \frac{\varepsilon_0 r + \omega y}{r + y} = \varepsilon_0 + (\omega - \varepsilon_0)\,\frac{y}{r + y}\,. \tag{1}$$

If E is the modulus of elasticity, the stress corresponding to the elongation ε is

$$s = E\varepsilon = E\left[\varepsilon_0 + (\omega - \varepsilon_0)\,\frac{y}{r + y}\right]. \tag{2}$$

The stresses developed over the section must hold in equilibrium the external forces and couples; hence denoting an element of the area of the section by df, we have from ordinary static conditions,

$$P = \int s\,df = \int E\left[\varepsilon_0 + (\omega - \varepsilon_0)\,\frac{y}{r + y}\right] df. \tag{3}$$

$$M_b = \int y\,.\,s\,df = \int Ey\left[\varepsilon_0 + (\omega - \varepsilon_0)\,\frac{y}{r + y}\right] df. \tag{4}$$

The integrals involved, considering E a constant, are

$$\int df,\quad \int y\,df,\quad \int \frac{y}{r + y}\,df,\quad \text{and}\quad \int \frac{y^2}{r + y}\,df.$$

Evidently $\int df = f$, and since y is measured from a gravity axis, $\int ydf = 0$. For the sake of convenience, let

$$\int \frac{y}{r + y}\,df = -\,zf; \tag{5}$$

then

$$\int \frac{y^2}{r + y}\,df = \int \left(y - r\,\frac{y}{r + y}\right) df = -\,r\int \frac{y}{r + y}\,df = zfr. \tag{6}$$

Inserting these values of the integrals in (3) and (4), we get

$$P = Ef\,[\varepsilon_0 - (\omega - \varepsilon_0)\,z],$$
$$M_b = Ef\,(\omega - \varepsilon_0)\,zr.$$

By slight reduction the following important formulas are now obtained:

$$\omega \quad \varepsilon_0 = \frac{1}{Ef} \cdot \frac{M_b}{zr},$$

$$\varepsilon_0 = \frac{1}{Ef}\left(P + \frac{M_b}{r}\right), \tag{A}$$

$$\omega = \frac{1}{Ef}\left(P + \frac{M_b}{r} + \frac{M_b}{zr}\right). \tag{B}$$

Inserting the expressions for ε_0 and ω given by (A) and (B) in (2) we finally obtain for the intensity of stress at any point of the section

$$s = \frac{P}{f} + \frac{M_b}{fr} + \frac{M_b}{zfr} \quad \frac{y}{r + y}. \tag{C}$$

Formula (C) gives the stress in terms of the force P, couple M_b, and other terms which depend solely upon the geometry of the system under consideration. In applying this formula care must be taken to give the quantities their proper signs. Thus:

P is positive when it tends to produce tension, negative when it tends to produce compression;

M_b is positive when it tends to increase the curvature of the bar, negative when it tends to decrease the curvature;

y is positive when measured towards the convex side of the bar, negative when measured towards the concave side, that is, towards the center of curvature.

When the value of s as determined from formula (C) is positive, the stress is tensile; if s is negative, the stress is compressive.

The function z as defined by (5), that is,

$$z = -\frac{1}{f}\int \frac{y}{r + y}\,df,$$

may be obtained by integration in the case of regular sections, circles, rectangles, etc. The following expressions for z are all that are required for present purposes.

For a circular cross section of radius *a*,

$$z = \frac{1}{4}\left(\frac{a}{r}\right)^2 + \frac{1}{8}\left(\frac{a}{r}\right)^4 + \frac{5}{64}\left(\frac{a}{r}\right)^6 + \frac{7}{128}\left(\frac{a}{r}\right) + \ldots$$

$$\frac{1}{z} = 4\left(\frac{r}{e}\right)^2 - 2 - \frac{1}{4}\left(\frac{e}{r}\right)^2 - \frac{1}{128}\left(\frac{e}{r}\right)^4 - \ldots$$

For a rectangular cross section of width *b* and depth 2 *a*,

$$z = \frac{1}{3}\left(\frac{a}{r}\right)^2 + \frac{1}{5}\left(\frac{a}{r}\right)^4 + \frac{1}{7}\left(\frac{a}{r}\right)^6 + \ldots$$

APPENDIX B

ANALYSIS OF OPEN LINK

To give an idea of the general method employed, a simple case is taken first, the more complicated general case later.

It is assumed that a quadrant of the center line of the link is made up of two circular arcs, Fig. 32, one, *BE*, having a radius equal to the diameter *d* of the iron of the link, the other arc, *EA*, having a radius $AC = r$. Denoting by *a* and *b* the major and minor semi-axes, *BO* and *AO*, respectively, the following geometrical relations are easily deduced:

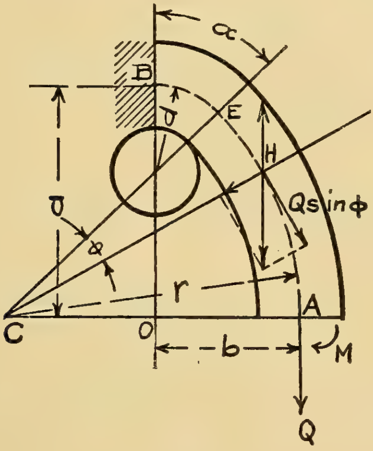

FIG. 32.

$$r = \frac{a^2 + b^2 - 2\,ad}{2\,(b - d)},$$

$$\sin\,\alpha = \frac{r - b}{r - d}, \quad \tan\,\alpha = \frac{r - b}{a - d}.$$

Let it be assumed first that the pressure between two links is concentrated at a point. Denoting this pressure by 2 *Q*, the normal force at section A is Q. The unknown bending moment at this same section may be denoted by M. At any point H between E and A on the center line introduce two equal and opposite forces each equal to Q. One of these forces may be combined with Q giving a moment at H of magnitude

$$Qr\,(1 - \sin\phi).$$

Adding to this moment the moment M at section A, we have for the bending moment at section H,

$$M_b = M + Qr\,(1 - \sin\phi). \tag{1}$$

The other force at H may be resolved into two components, one normal to the section and thus producing tension, the other lying in the plane of the section. The latter component is neglected. The former has the value,

$$P = Q\sin\phi. \tag{2}$$

For sections lying between B and E, that is, for values of ϕ between 0 and α, we have likewise,

$$M_b = M + Q\,(b - d\sin\phi), \tag{3}$$

$$P = Q\sin\phi. \tag{4}$$

The unknown moment M is determined from the following considerations. As shown in Appendix A, the distortion of the link under load changes the angle $d\phi$ between two adjacent cross sections by the amount $\Delta d\phi$, this change being positive at some sections, negative at others. Because of the symmetry of the link, sections A and B originally at right angles remain at right angles; that is, the summation of the changes of angle $\Delta d\phi$ between B and A must be zero.

Hence

$$\sum_{B}^{A} \Delta d\phi = 0,$$

or since $\Delta d\phi = \omega \,.\, d\phi$,

$$\int_0^{\alpha} \omega\, d\phi + \int_{\alpha}^{\frac{\pi}{2}} \omega\, d\phi = 0. \tag{5}$$

The general expression for ω [Eq. (B), Appendix A] is

$$\omega = \frac{1}{Ef}\left(P + \frac{M_b}{r} + \frac{M_b}{zr}\right). \tag{6}$$

For sections between $\phi = 0$ and $\phi = a$, $r = d$; hence,

$$\omega_1 = \frac{1}{Ef}\left(Q \sin \phi + \frac{M_b}{d} + \frac{M_b}{z_1 d}\right), \tag{7}$$

the subscript 1 being used to distinguish the ω and z of this part of the link from those of the other part. For sections lying between $\phi = \alpha$ and $\phi = \frac{\pi}{2}$,

$$\omega_2 = \frac{1}{Ef}\left(Q \sin \phi + \frac{M_b}{r} + \frac{M_b}{z_2 r}\right). \tag{8}$$

Inserting the proper values of the moment M_b from (1) and (3) we have,

$$\left.\begin{aligned} Ef\omega_1 &= \frac{M + Qb}{d}\left(1 + \frac{1}{z_1}\right) - \frac{Q}{z_1} \sin \phi, \\ Ef\omega_2 &= \frac{M + Qr}{r}\left(1 + \frac{1}{z_2}\right) - \frac{Q}{z_2} \sin \phi. \end{aligned}\right\} \tag{9}$$

Inserting these values of ω_1 and ω_2 in (5), integrating and reducing, we get finally,

$$M =$$

$$Qd\left[\frac{\frac{1}{z_1}(1 - \cos \alpha) + \frac{1}{z_2}\cos \alpha - \alpha\frac{b}{d}\left(1 + \frac{1}{z_1}\right) - \left(1 + \frac{1}{z_2}\right)\left(\frac{\pi}{2} - \alpha\right)}{\alpha\left(1 + \frac{1}{z_1}\right) + \frac{d}{r}\left(1 + \frac{1}{z_2}\right)\left(\frac{\pi}{2} - \alpha\right)}\right] \tag{D}$$

The value of M being found by means of formula (D) the bending moment at any section is readily obtained; and with the bending moment and normal force given, the intensity of stress at any fiber is readily determined by means of formula (C).

For a circular ring we have,

$$\left.\begin{aligned} P &= Q \sin \phi, \\ M_b &= M + Qr\,(1 - \sin \phi), \end{aligned}\right\} \tag{10}$$

whence
$$\int_0^{\frac{\pi}{2}} \omega \, d\phi = \int_0^{\frac{\pi}{2}} \left[\left(\frac{M}{r} + Q\right)\left(1 + \frac{1}{z}\right) - \frac{Q \sin \phi}{z}\right] d\phi = 0.$$

Integrating and reducing,

$$M = Qr\left(\frac{2}{\pi\,(1 + z)} - 1\right). \tag{E}$$

We shall now take up a more complicated example which agrees more closely with conditions met in practice. In the first place, the quadrant of the center line of the link cannot usually be represented closely by two circular arcs. Links actually measured show the form shown in Fig. 33. The center line BA is made up of four parts:

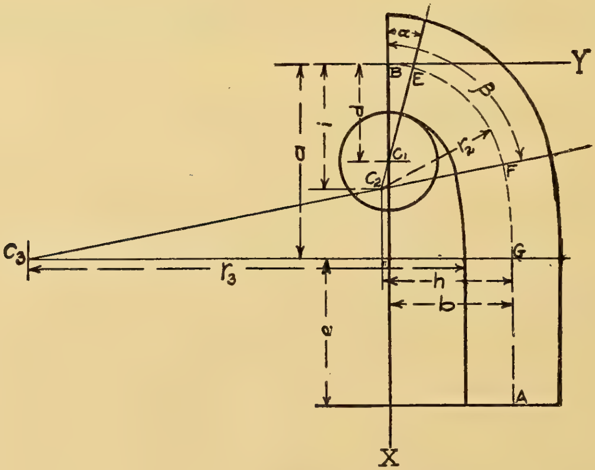

FIG. 33.

(1) the arc BE of radius d struck from C_1 as a center; (2) the arc EF with radius r_2 and C_2 as center; (3) the arc FG with radius r_3 and C_3 as center; (4) in some cases, a straight part of length e.

Secondly, as explained previously, the assumption that the pressure between adjacent links is concentrated at a point is not justified. We shall make the assumption that the pressure is distributed along an arc, as shown in Fig. 2 (b); afterwards the resulting equations will be modified to suit the assumption represented by Fig. 2 (c), namely that the pressure is regarded as concentrated at *two* points E and E.

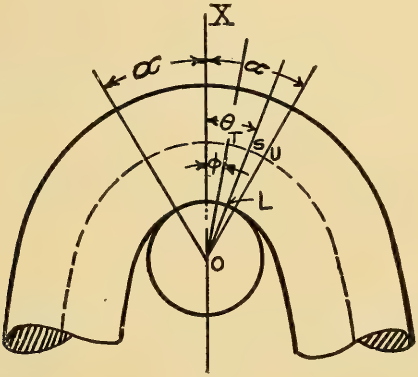

Fig. 34.

The distributed pressure along the angle of contact 2α (Fig. 34) gives rise to a normal force and a bending moment independent of the force Q and moment M at section A. We have now to derive expressions for the force P and moment M_b at any section included within the angle α.

The length of an element of arc of the circumference in contact is $\frac{d}{2}d\phi$; hence the pressure over the element of arc with the assumption of uniform distribution is

$$p\frac{d}{2}d\phi\,\cdot$$

The vertical component of this force is

$$p\,\frac{d}{2}\cos\phi d\phi,$$

and the sum of such vertical components must be in equilibrium with the external force $2\,Q$. That is,

$$\frac{pd}{2}\int_{-\alpha}^{\alpha} \cos\phi\, d\phi = 2\,Q,$$

whence $$pd \sin\alpha = 2\,Q,$$

or $$p = \frac{2\,Q}{d \sin\alpha}.$$

Take any section OS making an angle θ with OX and for the present consider the angle ϕ constant. We are to find the moment and normal force at section T due to the distributed pressure between sections T and S. The intensity of pressure in the direction OS is p and the pressure along an element of arc of the circumference is therefore

$$dF = p\frac{d}{2}\, d\theta.$$

Now at T introduce two equal and opposite forces parallel to OS and of magnitude dF. One of these combines with dF acting through S to form a couple whose moment is,

$$d \sin(\theta - \phi)\, dF = p\,\frac{d^2}{2} \sin(\theta - \phi)\, d\theta.$$

The other force dF is resolved into components, the one perpendicular to section T being

$$p\frac{d}{2} \sin(\theta - \phi)\, d\theta.$$

If now we vary θ from ϕ to α and take the sum of the forces and moments for each element $d\theta$, we get:

$$\text{Normal force at } T = \frac{pd}{2}\int_{\phi}^{\alpha} \sin(\theta - \phi)\, d\theta;$$

$$\text{Moment at } T = \frac{pd^2}{2}\int_{\phi}^{\alpha} \sin(\theta - \phi)\, d\theta.$$

These are the force and moment at section T arising from the distributed pressure between sections T and U.
Taking ϕ constant, we obtain

$$\int_{\phi}^{\alpha} \sin(\theta - \phi)d\theta = 1 - \cos(\alpha - \phi),$$

and making use of the relation $p = \dfrac{2\theta}{d \sin \alpha}$ we get:

$$\text{Normal force at } T = \frac{Q}{\sin \alpha}[1 - \cos(\alpha - \phi)]; \tag{12}$$

$$\text{Moment at } T = \frac{Qd}{\sin \alpha}[1 - \cos(\alpha - \phi)]. \tag{13}$$

The normal force and moment at section T due to the force Q and moment M at section A have been shown to be respectively,

$$Q \sin \phi,$$

and

$$M + Q(b - d \sin \phi).$$

The total normal force at section T is therefore,

$$P = Q \sin \phi + \frac{Q}{\sin \alpha}[1 - \cos(\alpha - \phi)]$$

$$= \frac{Q}{\sin \alpha} - Q \cot \alpha \cos \phi. \tag{14}$$

The moment at T due to the distributed pressure is opposite in sense to the moment due to Q and M at section A; hence the net moment at T is

$$M_b = M + Qb - Qd \sin \phi - \frac{Qd}{\sin \alpha}[1 - \cos(\alpha - \phi)]$$

$$= M + Qb - \frac{Qd}{\sin \alpha} + Qd \cot \alpha \cos \phi. \tag{15}$$

Referring to Fig. 33, we see that (14) and (15) give the values of

P and M_b for cross sections lying between points B and E. Using these values we obtain for ω_1 the following expression:

$$\omega_1 = \frac{1}{Ef}\left[\frac{M + Qb}{\alpha}\left(1 + \frac{1}{z_1}\right) - \frac{Q}{z_1 \sin \alpha} + \frac{Q}{z_1} \cot \alpha \cos \phi\right]. \qquad (16)$$

For the arc EF with radius r_2, the values of M_b and P are found to be,

$$\left.\begin{aligned} M_b &= M + Qh - Qr_2 \sin \phi, \\ P &= Q \sin \phi, \end{aligned}\right\} \qquad (17)$$

whence for this arc, ω_2 is given by the expression

$$\omega_2 = \frac{1}{Ef}\left[\frac{M + Qh}{r_2}\left(1 + \frac{1}{z_2}\right) - \frac{Q}{z_2} \sin \phi\right]. \qquad (18)$$

For the arc FG, likewise

$$\left.\begin{aligned} M_b &= M + Qr_3 - Qr_3 \sin \phi, \\ P &= Q \sin \phi, \end{aligned}\right\} \qquad (19)$$

whence $$\omega_3 = \frac{1}{Ef}\left[\frac{M + Qr_3}{r_3}\left(1 + \frac{1}{z_3}\right) - \frac{Q}{z_3} \sin \phi\right]. \qquad (20)$$

For the straight part GA,

$$M_b = M, \qquad P = Q,$$

whence $$\omega_4 = \frac{1}{Ef}\left[Q + \frac{M}{r_4}\left(1 + \frac{1}{z_4}\right)\right]. \qquad (21)$$

Since the normal sections at B and A must remain at right angles, the summation of $\omega \cdot d\phi$ from B to A must be zero; that is

$$\int_0^{\alpha} \omega_1 d\phi + \int_{\alpha}^{\beta} \omega_2 d\phi + \int_{\beta}^{\frac{\pi}{2}} \omega_3 d\phi + \int_G^A \omega_4 d\phi = 0. \qquad (22)$$

The first three integrals present no difficulties. Care must be taken in evaluating the last integral, however, because of the infinite factors that it contains. The expression for $\frac{1}{z_4}$ is

$$\frac{1}{z_4} = 16\left(\frac{r_4}{d}\right) - 2 - \frac{1}{16}\left(\frac{d}{r_4}\right) \cdot \cdot \cdot$$

Now since for the straight part GA, r_4 is infinite, it appears that $\frac{1}{z_4}$ is also infinite; and neglecting the finite terms, we may write (21) in the form

$$\omega_4 = \frac{1}{Ef} \cdot \frac{M}{r_4} \frac{16 r_4^2}{d^2} = \frac{16}{Ef} \frac{M r_4}{d^2}.$$

Therefore $$\omega_4 \, d\phi = \frac{16\ M}{Efd^2} r_4 \, d\phi.$$

But $r_4 d\phi = ds$; hence

$$\int_G^A \omega_4 \, d\phi = \frac{16\ M}{Efd^2} \int_0^e ds = \frac{1}{Ef} \cdot \frac{16\ Me}{d^2}. \tag{23}$$

If now we substitute in (22) the values of ω_1, ω_2, and ω_3 given by (16), (18) and (20) and for the fourth integral the value just obtained we get the following (neglecting the constant factor Ef):

$$\frac{M + Qb}{d}\left(1 + \frac{1}{z_1}\right)\int_0^\alpha d\phi - \frac{Q}{z_1 \sin\alpha}\int_0^\alpha d\phi + \frac{Q}{z_1}\cot\alpha \int_0^\alpha \cos\phi \, d\phi$$
$$+ \frac{M + Qh}{r_2}\left(1 + \frac{1}{z_2}\right)\int_\alpha^\beta d\phi - \frac{Q}{z_2}\int_\alpha^\beta \sin\phi \, d\phi + \frac{M + Qr_3}{r_3}\left(1 + \frac{1}{z_3}\right)\int_\beta^{\frac{\pi}{2}} d\phi$$
$$- \frac{Q}{z_3}\int_\beta^{\frac{\pi}{2}} \sin\phi \, d\phi + 16\frac{Me}{d^2} = 0.$$

Integrating and reducing, the following is obtained:

$$M = -Qd\left[\frac{\alpha\frac{b}{d}\left(1+\frac{1}{z_1}\right) - \frac{1}{z}\left(\frac{\alpha}{\sin\alpha} - \cos\alpha\right) + \frac{h}{r_2}\left(1+\frac{1}{z_2}\right)(\beta-\alpha) - \frac{1}{z_2}(\cos\alpha - \cos\beta) + \left(1+\frac{1}{z_3}\right)\left(\frac{\pi}{2}-\beta\right) - \frac{\cos\beta}{z_3}}{\alpha\left(1+\frac{1}{z_1}\right) + \frac{d}{r_2}\left(1+\frac{1}{z_2}\right)(\beta-\alpha) + \frac{d}{r_3}\left(1+\frac{1}{z_3}\right)\left(\frac{\pi}{2}-\beta\right) + 16\frac{e}{d}}\right] \tag{F}$$

If the assumption is made that the pressure may be considered as concentrated at two points subtending the angle 2α (see Fig. 2 (c)), we readily obtain instead of (12) and (13):

$$\text{Normal force at } T = Q \sec\alpha \sin(\alpha - \phi). \tag{12'}$$
$$\text{Moment at } T = Qd \sec\alpha \sin(\alpha - \phi). \tag{13'}$$

Equations (14) and (15) then become

$$P = Q \tan \alpha \cos \phi, \tag{14'}$$

$$M_b = M + Qb - Qd \tan \alpha \cos \phi, \tag{15'}$$

whence

$$\omega_1 = \frac{1}{Ef}\left[\frac{M + Qb}{d}\left(1 + \frac{1}{z_1}\right) - \frac{Q}{z_1} \tan \alpha \cos \phi\right]. \tag{16'}$$

Using this value of ω_1 in (22), we get finally

$$M = -Qd\left[\frac{\alpha\frac{b}{d}\left(1+\frac{1}{z_1}\right)-\frac{1}{z_1}\tan\alpha\sin\alpha+\frac{h}{r_2}\left(1+\frac{1}{z_2}\right)\left(\beta-\alpha\right) -\frac{1}{z_2}\left(\cos\alpha-\cos\beta\right)+\left(1+\frac{1}{z_3}\right)\left(\frac{\pi}{2}-\beta\right)-\frac{\cos\beta}{z_3}}{\alpha\left(1+\frac{1}{z_1}\right)+\frac{d}{r_2}\left(1+\frac{1}{z_2}\right)(\beta-\alpha)+\frac{d}{r_3}\left(1+\frac{1}{z_3}\right)\left(\frac{\pi}{2}-\beta\right)+16\frac{e}{d}}\right]. \tag{F'}$$

It will be observed that the change in the assumed law of distribution changes the second term in the numerator of (F) from

$$\frac{1}{z_1}\left(\frac{\alpha}{\sin\alpha} - \cos\alpha\right) \quad \text{to} \quad \frac{1}{z_1}\tan\alpha\sin\alpha.$$

If we assume concentration at the end of the link this term is

$$\frac{1}{z_1}(1 - \cos\alpha).$$

From the value of M as determined from (F) or (F′), the bending moment M_b at any section is obtained, and then the stress at any fiber is found by means of (C).

APPENDIX C

DERIVATION OF THEORETICAL FORMULAS FOR THE CHANGE OF LENGTH IN THE AXES OF THE LINK

THE analysis given in Appendices A and B may be employed to calculate the change in length of either axis of the link due to a given load. As has been stated, the comparison of this calculated change of length with the change actually measured is the basis of the experimental verification of the theory.

The following discussion is substantially that given by Bach: Let $O\ P\ C\ D$, Fig. 35, be the center line of a curved bar before it is subjected to external forces. The point O is chosen as the origin, and the tangent and normal at O are taken as the Y- and X-axes respectively. When the bar is subjected to external forces, it is distorted and the center line changes its form. Any point C is thereby moved to a new position, and if x_c, y_c are the original coördinates

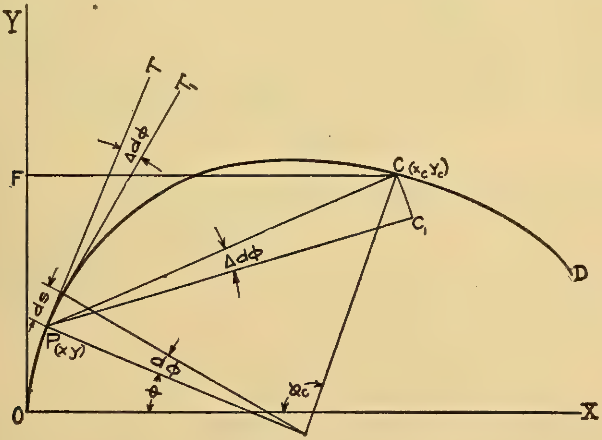

FIG. 35.

of C, these receive increments Δx_c and Δy_c respectively. These increments we now propose to determine by the principles hitherto developed.

Choose any point P whose coördinates are x, y, and let r be the radius of curvature at this point. An element of arc at P has the direction PT and its length is $ds = rd\phi$, where ϕ as usual denotes the angle between r and the x-axis. Because of the action of the external forces and couples, this element of arc will turn about P — which we for the present consider fixed — through the angle $\Delta d\phi$. This rotation causes the point C to move to C_1 on the circular arc $CC_1 = PC \cdot \Delta d\phi$. The components of the displacement CC_1 along the X- and Y-axes are respectively

$$\left.\begin{array}{l} PC \cdot \Delta d\phi \sin PCF = PC \cdot \sin PCF \cdot \Delta d\phi = (y_c - y) \cdot \Delta d\phi \\ \text{and} \\ -PC \cdot \Delta d\phi \cos PCF = -PC \cdot \cos PCF \cdot \Delta d\phi = -(x_c - x) \cdot \Delta d\phi \end{array}\right\} \quad (1)$$

In addition to the coördinate increments due to the change in inclination of the section at P there are increments due to the lengthening (or shortening) of the arc element ds at P. The extension of the element is $\varepsilon_0 ds$; hence because of this extension the point C is moved in the direction of the X-axis a distance,

$$\varepsilon_0 ds \sin \phi = \varepsilon_0 dx$$

and in the direction of the Y-axis a distance

$$\varepsilon_0 ds \cos \phi = \varepsilon_0 dy.$$

Adding together the changes just deduced (and replacing $\Delta d\phi$ by $\omega \cdot d\phi$), we have:

$$\left.\begin{array}{l} \text{change along } X\text{-axis} = (y_c - y)\,\omega d\phi + \varepsilon_0 dx, \\ \text{change along } Y\text{-axis} = -\,(x_c - x)\,\omega d\phi + \varepsilon_0 dy. \end{array}\right\} \quad (2)$$

The total increments of the coördinates x_c and y_c, made up of the changes for all the arc elements lying between O and C, are found by summation. Thus,

$$\left.\begin{array}{l} \Delta x_c = y_c \int_0^{\phi_c} \omega\, d\phi - \int_0^{\phi_c} y\alpha\, d\phi + \int_0^{x_c} \varepsilon_0 dx, \\ \Delta y_c = \int_0^{\phi_c} x\omega\, d\phi - x_c \int_0^{\phi_c} \omega\, d\phi + \int_0^{y_c} \varepsilon_0 dy. \end{array}\right\} \quad (G)$$

We have now to apply these fundamental formulas (a) to the circular ring and (b) to the chain link.

I. *Distortion of Circular Ring*

Referring to Fig. 36, it is evident that the point A at the extremity of the transverse diameter is the point whose coördinate increments

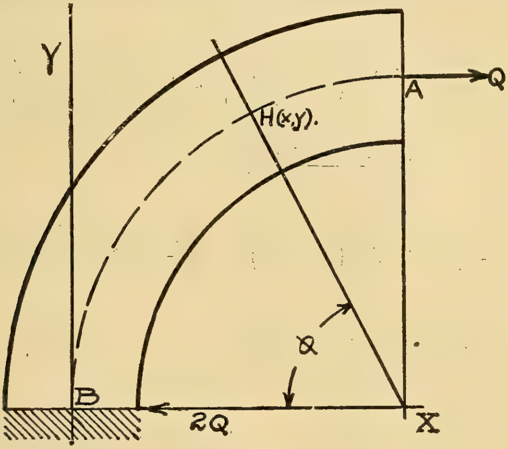

FIG. 36.

are desired. Denoting by x, y the coördinates of any point H on the center line between A and B, we have

$$\left.\begin{aligned} x_a &= y_a = r, \\ x_a - x &= r\cos\phi, \\ y &= r\sin\phi. \end{aligned}\right\} \tag{3}$$

At H, the normal force and bending moment are

$$\left.\begin{aligned} P &= Q\sin\phi, \\ M_b &= M + Qr\,(1 - \sin\phi), \end{aligned}\right\} \tag{4}$$

whence

$$Ef \cdot \varepsilon_0 = \frac{M + Qr}{r}, \tag{5}$$

and

$$Ef \cdot \omega = \frac{M + Qr}{r}\left(1 + \frac{1}{z}\right) - \frac{Q\sin\phi}{z}. \tag{6}$$

Since for the quadrant $BA, \int \omega \, d\phi = 0$, the first of equations (G) reduces to the simpler form

$$\Delta x_a = -\int_0^{\frac{\pi}{2}} y\omega \, d\phi + \int_0^r \varepsilon_0 \, dx. \tag{7}$$

Inserting in (7) proper values from (3), (5) and (6), we get,

$$\begin{aligned} Ef \cdot \Delta x_a &= -(M + Qr)\left(1 + \frac{1}{z}\right)\int_0^{\frac{\pi}{2}} \sin\phi \, d\phi + \frac{Qr}{z}\int_0^{\frac{\pi}{2}} \sin^2\phi \, d\phi \\ &\quad + \frac{M + Qr}{r}\int_0^r dx. \\ &= -(M + Qr)\left(1 + \frac{1}{z}\right) + \frac{Qr\pi}{4z} + M + Qr \\ &= -\frac{1}{z}(M + Qr) + \frac{\pi}{4}\,\frac{Qr}{z}. \end{aligned}$$

In a similar way,

$$\Delta y_a = -\int_0^{\frac{\pi}{2}} (x_a - x)\omega \cdot d\phi + \int_0^r \varepsilon_0 \, dy = -\int_0^{\frac{\pi}{2}} r\cos\phi \cdot \omega \, d\phi + \int_0^r \varepsilon_0 dy.$$

$$\begin{aligned} Ef \cdot \Delta y_a &= -(M + Qr)\left(1 + \frac{1}{z}\right)\int_0^{\frac{\pi}{2}} \cos\phi \, d\phi + \frac{Qr}{z}\int_0^{\frac{\pi}{2}} \sin\phi\cos\phi \, d\phi \\ &\quad + \frac{M + Qr}{r}\int_0^r dy \\ &= -\frac{1}{z}(M + Qr) + \frac{Qr}{2z}. \end{aligned}$$

These results may be written as follows:

$$\left.\begin{aligned} \Delta x_a &= -\frac{1}{Ef} \cdot \frac{1}{z}\left[M + Qr\left(1 - \frac{\pi}{4}\right)\right], \\ \Delta y_a &= -\frac{1}{Ef} \cdot \frac{1}{z}\left[M + \tfrac{1}{2} Qr\right]. \end{aligned}\right\} \tag{H}$$

If finally the expression for M given by equation (E) be introduced, the equations take the form

$$\left.\begin{aligned}\Delta x_a &= -\frac{1}{Ef}\cdot\frac{Qr}{z}\left[\frac{2}{\pi\,(1+z)}-\frac{\pi}{4}\right],\\ \Delta y_a &= -\frac{1}{Ef}\cdot\frac{Qr}{z}\left[\frac{2}{\pi\,(1+z)}-0.5\right].\end{aligned}\right\} \qquad (\mathrm{H}')$$

II. *Distortion of Chain Link*

Referring to Fig. 33, the point A at the end of the minor axis is the point whose movement under load is desired. Since for the quadrant BA, $\int \omega\, d\phi = 0$, the equations (G) when applied to the coördinates of point A become simply,

$$\left.\begin{aligned}\Delta x_a &= -\int_0^{\phi_a} y\omega\, d\phi + \int_0^{x_a} \varepsilon_0\, dx,\\ \Delta y_a &= \int_0^{\phi_a} x\omega\, d\phi + \int_0^{y_a} \varepsilon_0\, dy.\end{aligned}\right\} \qquad (\mathrm{G}')$$

There are four parts in the quadrant BA of the center line; hence separate expressions for x, y, ω and ε_0 must be derived for each of these parts and furthermore the integrations must be separated.

From the geometry of Fig. 33, and from the results obtained previously (Appendix B) we have the following:

For arc BE, subtending angle α

$$\left\{\begin{aligned} x &= d(1-\cos\phi), \quad y = d\sin\phi.\\ P &= \frac{Q}{\sin\alpha} - Q\cot\alpha\cos\phi.\\ M_b &= M + Qb - \frac{Q\,d}{\sin\,\alpha} + Qd\cot\alpha\cos\phi.\\ Ef\cdot\varepsilon_0 &= \frac{M+Qb}{d}.\\ Ef\cdot\omega &= \frac{M+Qb}{d}\left(1+\frac{1}{z_1}\right) - \frac{Q}{z_1}\left(\frac{1}{\sin\alpha}+\cot\alpha\cos\phi\right).\end{aligned}\right.$$

For arc EF, subtending angle $\beta - \alpha$

$$\left\{\begin{aligned} & x = i - r_2 \cos\phi, \quad y = r_2 \sin\phi + b - h. \\ & P = Q \sin\phi. \\ & M_b = M + Qh - Qr_2 \sin\phi. \\ & Ef \cdot \varepsilon_0 = \frac{M + Qh}{r_2}. \\ & Ef \cdot \omega = \frac{M + Qh}{r_2}\left(1 + \frac{1}{z_2}\right) - \frac{Q}{z_2} \sin\phi. \end{aligned}\right.$$

For arc FG, subtending angle $\frac{\pi}{2} - \beta$

$$\left\{\begin{aligned} & x = a - r_3 \cos\phi, \quad y_3 = b - r_3 + r_3 \sin\phi. \\ & P = Q \sin\phi. \\ & M_b = M + Qr_3 - Qr_3 \sin\phi. \\ & Ef \cdot \varepsilon_0 = \frac{M + Qr_3}{r_3}. \\ & Ef \cdot \omega = \frac{M + Qr_3}{r_3}\left(1 + \frac{1}{z_3}\right) - \frac{Q}{z_3} \sin\alpha. \end{aligned}\right.$$

For straight part GA

$$\left\{\begin{aligned} & x = a + u, \quad y = b. \\ & P = Q. \\ & M_b = M. \\ & Ef \cdot \varepsilon_0 = Q. \\ & Ef \cdot \omega = 16\, M \frac{r_4}{d^2}. \end{aligned}\right.$$

The above expressions for x, y, ε_0 and ω are substituted in the equations (G′) and the resulting integrals are evaluated for each arc separately. Then the results are combined.

For the straight part GA, the radius r_4 is infinite, and care must be exercised to get correct results. Letting u denote the distance of a point in GA from G, we have

$$x = a + u,$$

whence

$$x\omega\, d\phi = 16 \frac{M}{d^2} (a + u)\, r_4 d\phi.$$

But

$$r_4 d\phi = du,$$

and therefore

$$\int x\omega\, d\phi = \frac{16\, M}{d^2} \int_0 (a + u)\, du = 16 \frac{M}{d^2} \left(ae + \frac{e^2}{2}\right).$$

Likewise $\displaystyle\int y\omega\, d\phi = \frac{16\, Mb}{d^2} \int_0 du = 16\, \frac{Mbe}{d^2}.$

It is readily seen that for the part GA,

$$\int \varepsilon_0 dx = Qe, \text{ and } \int \varepsilon_0 dy = 0.$$

The results of the substitution and integration are the following equations:

$$\begin{aligned}
Ef\,.\,\Delta x_a = & -\frac{1}{z_1}(1 - \cos\alpha)\,(M + Qb) - \frac{Qd}{2\,z_1}\left[\frac{2\,(1 - \cos\alpha)}{\sin\,\alpha} - \sin\alpha\cos\alpha\right] \\
& - \frac{1}{z_2}(\cos\,\alpha\, -\, \cos\,\beta)\,(M\, +\, Qh) \\
& + \frac{Qr_2}{2\,z_2}\,[\beta\, -\, \alpha\, +\, \sin\,\alpha\,\cos\,\alpha\, -\, \sin\,\beta\,\cos\,\beta] \\
& + \frac{h\, -\, b}{r_2}\left(1\, +\, \frac{1}{z_2}\right)(\beta\, -\, \alpha)\,\,(M\, +\, Qh) \\
& - \frac{1}{z_2}(\cos\,\alpha\, -\, \cos\,\beta)\,(h\, -\, b)\,Q\, -\, \frac{\cos\,\beta}{z_3}(M\, +\, Qr_3) \\
& + \frac{Qr_3}{2\,z_3}\left(\frac{\pi}{2}\, -\, \beta\, +\, \sin\,\beta\,\cos\,\beta\right) \\
& + \left(1\, +\, \frac{1}{z_3}\right)\left(\frac{r_3\, -\, b}{r_3}\right)\left(\frac{\pi}{2}\, -\, \beta\right)(M\, +\, Qr_3) \\
& - (r_3\, -\, b)\,\frac{\cos\,\beta}{z_3}\,Q\, +\, Qe\, -\, \frac{16\,be}{d^2}\,M.
\end{aligned} \tag{J}$$

$$\begin{aligned}
Ef\,.\,\Delta y_a = & \,\alpha\left(1\, +\, \frac{1}{z_1}\right)(M\, +\, Qb)\, +\, \frac{Qd}{z_1}(1 + \cos\alpha) \\
& - \frac{Qd}{2\,z_1}\left(\frac{2\,\alpha -\, \alpha\,\cos\,\alpha}{\sin\,\alpha}\, +\, \cos^2\,\alpha\right) \\
& - \frac{1}{z_1}\sin\,\alpha\,\,(M\, +\, Qb) + \frac{i}{r_2}\left(1\, +\, \frac{1}{z_2}\right)(\beta - \alpha)\,(M\, +\, Qh) \\
& - \frac{1}{z_2}(\sin\,\beta\, -\, \sin\,\alpha)\,(M\, +\, Qh)
\end{aligned}$$

$$
\begin{aligned}
&-\frac{Qi}{z_2}(\cos\alpha - \cos\beta) + \frac{Qr_2}{2\,z_2}(\sin^2\beta - \sin^2\alpha) \\
&+\frac{a}{r_3}\left(1 + \frac{1}{z_3}\right)\left(\frac{\pi}{2} - \beta\right)(M + Qr_3) \\
&-\frac{1}{z_3}(1 - \sin\beta)(M + Qr_3) - \frac{Qa\cos\beta}{z_3} + \frac{Qr_3}{2\,z_3}(1 - \sin^2\beta) \\
&\qquad + \frac{16\,M}{d^2}\left(me + \frac{e^2}{2}\right). \qquad \text{(K)}
\end{aligned}
$$

It will be observed that both (J) and (K) have the general form,

$$Ef \cdot \Delta = c_1M + c_2Q \qquad (8)$$

where c_1 and c_2 are constants depending entirely upon the dimensions and configuration of the link. Since however,

$$M = kQ$$

we have

$$\Delta x_a = c'Q, \quad \Delta y_a = c''Q \qquad (9)$$

where c' and c'' are other constants. Equation (9) shows that the change in the length of either axis is directly proportional to the load.

APPENDIX D

ANALYSIS OF STUD-LINK.

When the link has a transverse stud, as shown in Fig. 37, the analysis of the stresses is much more complicated than in the case of the open link. Hence we shall give here only the general method of attack and the final equations.

Referring to Fig. 37, let 2 S denote the pressure between the end of the stud and the side of the link. Then one-half of this pressure may be considered as acting on the quadrant AB of the link; and to simplify the work, we shall assume the line of action of this force S to lie in the minor axis of the link. The quadrant AB is therefore subjected to the external force Q and moment M, as before, and in addition to the transverse force S.

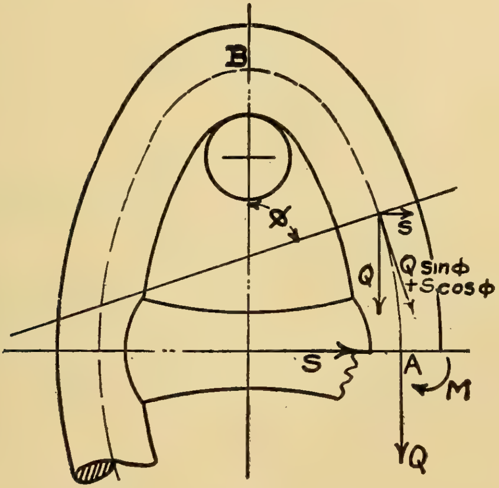

Fig. 37.

The introduction of this force S gives rise to new terms in the expressions for the normal force and bending moment at any section.

Assuming the link quadrant to be made up of three circular arcs and a straight part, as shown in Fig. 33, we readily obtain the following results:

For arc *BE*,

$$\begin{cases} P = \dfrac{Q}{\sin \alpha} - Q \cot \alpha \cos \phi + S \cos \phi, \\ M_b = M + Qb - \dfrac{Qd}{\sin \alpha} + Qd \cot \alpha \cos \phi - S\,(a + e - d + d \cos \phi). \end{cases}$$

For arc *EF*, $\begin{cases} P = Q \sin \phi + S \cos \phi, \\ M_b = M + Qh - Qr_2 \sin \phi - S\,(a + e - i + r_2 \cos \phi). \end{cases}$

For arc *FG*, $\begin{cases} P = Q \sin \phi + S \cos \phi, \\ M_b = M + Qr_3 - Qr_3 \sin \phi - S\,(e + r_3 \cos \phi). \end{cases}$

For straight part *GA*, $\begin{cases} P = Q, \\ M_b = M - S\,(e - u). \end{cases}$

From these expressions for P and M_b we may derive expressions for ε_0 and ω as in Appendix C for open links.

We have in the case of the stud link two unknown quantities to determine, the force S and the moment M at the section A; hence we must obtain two relations between M, S, and Q. As in the analysis of the open link, one equation is found from the relation

$$\int_{\text{Sec. } B}^{\text{Sec. } A} \omega \cdot d\phi = 0. \qquad (1)$$

To obtain a second relation, we make use of the fact that the decrease in the length of the minor axis must be equal to the decrease in the length of the stud. Now an expression for the change of the minor axis may be found by the method of Appendix C, using equations (G′). The length of the stud may be taken as $2b - d$; hence if E' and f' denote respectively the modulus of elasticity of the material of the stud and the average area of cross-section, we have

$$\text{decrease of length} = \frac{2\,S\,(2b - d)}{E'f'}. \qquad (2)$$

One-half of this decrease of length is equal to Δy_a the change of the Y- coördinate of the point A of the link. We have therefore,

$$-\ \Delta y_a = \frac{S\ (2b\ -\ d)}{E'f'}\ . \tag{3}$$

If now we denote by k the ratio $\frac{Ef}{E'f'}$ we obtain finally

$$-\ Ef\ .\ \Delta y_a\ = kS\ (2b\ -\ d). \tag{4}$$

Equation (4) gives a second relation between M, S and Q. The following simultaneous equations are finally obtained:

$$\left.\begin{aligned} A_1M &= B_1Sd - C_1Qd, \\ A_2M &= B_2Sd - C_2Qd, \end{aligned}\right\} \tag{5}$$

in which the coefficients have the following values:

$$A_1 = \alpha\left(1 + \frac{1}{z_1}\right) + \frac{d}{r_2}\left(1 + \frac{1}{z_2}\right)(\beta - \alpha) + \frac{d}{r_3}\left(1 + \frac{1}{z_3}\right)\left(\frac{\pi}{2} - \beta\right) + 16\frac{e}{d}\ \cdot$$

$$\begin{aligned} B_1 = \alpha\left(1 + \frac{1}{z_1}\right)\frac{a + e - d}{d} &+ \frac{1}{z_1}\sin\alpha + \left(1 + \frac{1}{z_2}\right)\left(\frac{a + e - i}{r_2}\right)(\beta - \alpha) \\ &+ \frac{1}{z_2}\left(\sin\beta - \sin\alpha\right) \\ &+ \frac{e}{r_3}\left(1 + \frac{1}{z_3}\right)\left(\frac{\pi}{2} - \beta\right) + \frac{1}{z_3}\left(1 - \sin\beta\right) + 8\frac{e^2}{d^2}\ \cdot \end{aligned}$$

$$\begin{aligned} C_1 = \alpha\frac{b}{d}\left(1 + \frac{1}{z_1}\right) &+ \frac{h}{r_2}\left(1 + \frac{1}{z_2}\right)(\beta - \alpha) + \left(1 + \frac{1}{z_3}\right)\left(\frac{\pi}{2} - \beta\right) \\ &- \frac{1}{z_1}\left(\frac{\alpha}{\sin\alpha} - \cos\alpha\right) - \frac{1}{z_2}\cos\alpha - \cos\beta) - \frac{1}{z_3}\cos\beta. \end{aligned}$$

$$\begin{aligned} A_2 = \alpha\left(1 + \frac{1}{z_1}\right) - \frac{1}{z_1}\sin\alpha &+ \frac{i}{r_2}\left(1 + \frac{1}{z_2}\right)\left(\beta - \alpha\right) - \frac{1}{z_2}\left(\sin\beta - \sin\alpha\right) \\ &- \frac{1}{z_3}\left(1 - \sin\beta\right) \\ &+ \frac{a}{r_3}\left(1 + \frac{1}{z_3}\right)\left(\frac{\pi}{2} - \beta\right) + 8\left(\frac{2\,ae + e^2}{d^2}\right)\cdot \end{aligned}$$

$$\begin{aligned} B_2 = \alpha\left(1 + \frac{1}{z_1}\right)\frac{a + e - d}{d} &- \frac{1}{z_1}\left(\frac{a + e - 2\,d}{d}\right)\sin\alpha \\ &- \frac{1}{2\,z_1}\left(d + \sin\alpha\cos\alpha\right) \end{aligned}$$

$$+\frac{i}{r_2}\left(\frac{a+e-i}{d}\right)\left(1+\frac{1}{z_2}\right)(\beta-d)-\frac{1}{z_2}\left(\frac{a+e-2i}{d}\right)(\sin\beta-\sin\alpha)$$

$$-\frac{1}{2z_2}\frac{r_2}{d}(\beta-\alpha+\sin\beta\cos\beta-\sin\alpha\cos\alpha)$$

$$+\frac{1}{z_3}\left(\frac{a-e}{d}\right)(1-\sin\beta)+\frac{ae}{r_3d}\left(1+\frac{1}{z_3}\right)\left(\frac{\pi}{2}-\beta\right)$$

$$-\frac{1}{2z_3}\frac{r_3}{d}\left(\frac{\pi}{2}-\beta-\sin\beta\cos\beta\right)+\frac{8}{3}\frac{e^2}{d^3}(3a+e)-k\left(\frac{2b-d}{d}\right).$$

$$C_2=\alpha\frac{b}{d}\left(1+\frac{1}{z_1}\right)-\frac{1}{z_1}\left(\frac{\alpha}{\sin\alpha}-\cos\alpha\right)-\frac{1}{z_1}\left(\frac{b}{d}\sin\alpha-1\right)$$

$$-\frac{1}{2z_1}(\alpha\cot\alpha+\cos^2\alpha)+\frac{ih}{r_2d}\left(1+\frac{1}{z_2}\right)(\beta-\alpha)$$

$$-\frac{1}{z_2}\frac{i}{d}(\cos\alpha-\cos\beta)-\frac{1}{z_2}\frac{h}{d}(\sin\beta-\sin\alpha)$$

$$+\frac{1}{2z_2}\frac{r_2}{d}(\sin^2\beta-\sin^2\alpha)+\frac{a}{d}\left(1+\frac{1}{z_3}\right)\left(\frac{\pi}{2}-\beta\right)-\frac{1}{z_3}\frac{a}{d}\cos\beta$$

$$-\frac{1}{z_3}\frac{r_3}{d}(1-\sin\beta)+\frac{1}{2z_3}\frac{r_3}{d}(1-\sin^2\beta).$$

These coefficients are first determined from the known constants z_1, z_2, and z_3, and the known dimensions of the link. The solution of Eqs. (5) then gives the values of M and S, and from these, values of the normal force P and moment M_b for any section are readily found. Having P and M_b, the stress at any point of the cross section is found from the general equation (C).

With the open link the greatest tensile stress is either at the end or at the side of the link, that is, at sections on the major or minor axis. See Fig. 26. In the case of the stud link, the greatest tensile stress is usually at a point on the inside of the link at some distance from the end of the minor axis. See Fig. 27. To determine the exact position of the section of maximum tension, we insert the expressions for P and M_b in (C) and thus obtain

$$S=c+m(Q\sin\phi+S\cos\phi),$$

in which c and m are constant for all sections.

Taking the first derivative, we get,

$$\frac{dS}{d\phi} = m\,(Q \cos \phi - S \sin \phi),$$

and equating this to zero, we find

$$\frac{Q}{S} = \tan \phi.$$

Hence at the section for which

$$\phi = \tan^{-1} \frac{Q}{S},$$

the tensile stress will be a maximum.

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BULLETIN NO. 19

COMPARATIVE TESTS OF CARBON, METALLIZED CARBON AND TANTALUM FILAMENT LAMPS

BY

T. H. AMRINE

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 19 SEPTEMBER 1907

COMPARATIVE TESTS OF CARBON, METALLIZED CARBON AND TANTALUM FILAMENT LAMPS

BY T. H. AMRINE, B. S., FIRST ASSISTANT, DEPARTMENT OF ELECTRICAL ENGINEERING, ENGINEERING EXPERIMENT STATION

At the present time there are only three types of incandescent lamps having a wide enough commercial use to make them important factors in incandescent lighting. The first, and by far he most widely used, is the familiar carbon filament lamp, which n ordinary sizes gives an efficiency seldom exceeding 3.1 watts per candle power with an effective life of approximately 500 hours. The second type is also a carbon filament lamp, but the carbon by he process through which it passes in manufacture is given somewhat the characteristics of a metal, and for this reason is called he metallized filament lamp. The manufacturers have claimed for it an efficiency of about 2.5 watts per mean horizontal candle power. In the third type the filament is made of the metal tantaum and there is claimed for it an efficiency of about 2 watts per candle.

It is the purpose of this bulletin to give the results of tests made upon these lamps in the laboratory of the Electrical Engineering department of the University of Illinois, with the view of bringing out, if possible, the good points of each lamp together with any other facts that will help in the selection of the proper ype for any particular purpose. Especial care was taken

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The writer wishes to acknowledge his indebtedness to J. M. Bryant, Associate in Electricl Engineering, for valuable suggestions and co-operation; also, for aid in making the tests, to R. T. Calloway, L. G. Schumacher, O. M. Ward and W. R. Scott, of the Class of 1907 in Elecrical Engineering.

throughout all the work to make the conditions under which the tests were conducted exactly the same for each type of lamp in order to have a fair basis of comparison between types. Although comparative rather than absolute results have been particularly striven for it is felt that dependable quantitative values have been obtained.

DESCRIPTION OF LAMPS*

The lamps chosen for the tests were selected from a lot of 100 of each type of lamp. These were bought directly from the manufacturer, a well known and reliable incandescent lamp company.

Ratings.—The carbon lamp was rated by the manufacturer at 25 candle power at 110 volts with an efficiency of 3.1 watts per candle. The metallized filament lamps were rated at 50 watts at 110 volts, no mention being made of candle power on label. The tantalum lamps had a rating of 22 candle power at 110 volts.

Filaments.—The filaments of the carbon lamp were of the familiar single-loop type anchored in the middle of the loop. They had a smooth uniform appearance and were of the steel-gray color common to all properly flashed carbon filaments. The filaments of the metallized lamp were not so smooth in appearance as those of the carbon lamp. They had a kinked appearance, which, according to the statement of the manufacturer, was due to the method of treating the filament and in no way affected the efficiency of the lamp. The color was steel gray, slightly darker than the carbon filament. The filaments were double, consisting of two horseshoe loops, the inside ends of which were attached to a common anchor. The tantalum filaments were very fine and long, as was necessitated by the rather low specific resistance of the metal tantalum. They were mounted in the well known zigzag fashion upon supporting spires. The color was a silver-gray with a metallic luster. Fig. 1 is a photograph of the three types of lamps used in the tests. Table 1 gives the approximate effective dimensions of the three types of filaments.

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* At the time these tests were started, tungsten lamps could not be purchased on the market. In the matter of current consumption, life and candle power maintenance these lamps seem much superior to the three kinds tested. The manufacturers claim an efficiency of 1 watt per candle with a life of 1000 hours with a decrease in candle power of only 10 per cent. However, as is the case with the tantalum lamps, they are rather delicate and require even more careful handling.

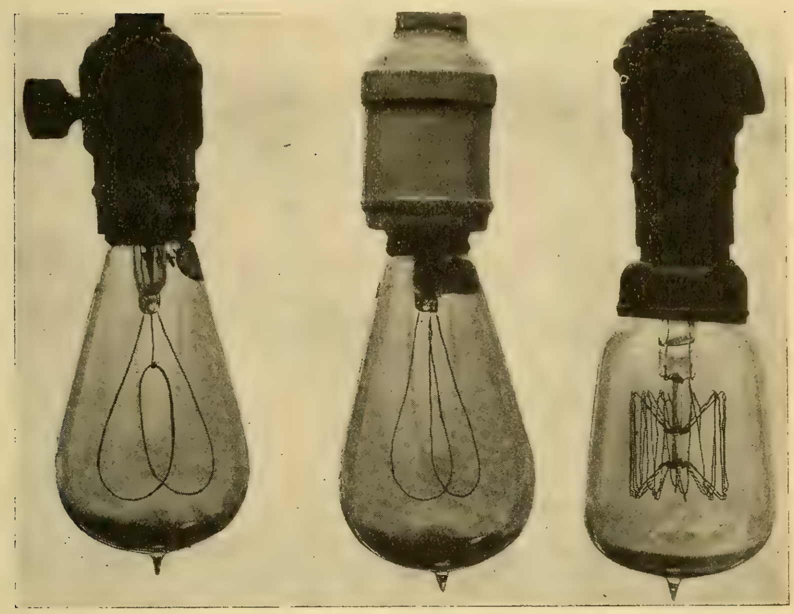
CARBON METALLIZED TANTALUM

FIG. 1

TABLE 1

Filament	Effective Length inches	Mean Diameter	Effective Area square inches
Carbon	9.4	.0060	.1774
Metallized	9.5	.0037	.1108
Tantalum	23.4	.0018	.1324

Bulbs.—The shapes of the bulbs and their comparative sizes are shown in Fig. 1. Those of the carbon and metallized lamps are of the same size and shape, having a length over all of 5 $\frac{1}{16}$ in., a maximum diameter of 2$\frac{3}{8}$ in. and a minimum diameter of 1$\frac{1}{8}$ in. The tantalum lamp is about 5$\frac{1}{8}$ in. in length and has a maximum diameter of 2$\frac{11}{32}$ in. tapering to a minimum diameter of 1$\frac{15}{16}$ in., thus making the sides much more nearly parallel than in the case of the carbon and metallized lamps.

ELECTRICAL CHARACTERISTICS

The three types of lamps differ radically in their temperature characteristics. The carbon filament has a negative temperature coefficient; that is, its resistance decreases as the temperature increases. On the other hand, on account of the treatment which it has undergone in manufacture, the metallized carbon filament has a positive temperature coefficient similar to the metals when in the incandescent stage. The tantalum filament, being of metal, has, of course, a positive coefficient. Fig. 2 plainly shows the increase of resistance with the increase of temperature in the metallized and tantalum filaments and the opposite effect in the carbon after reaching the incandescent stage. At lower temperatures the change is probably greater than after it becomes incandescent, especially in the carbon lamp.

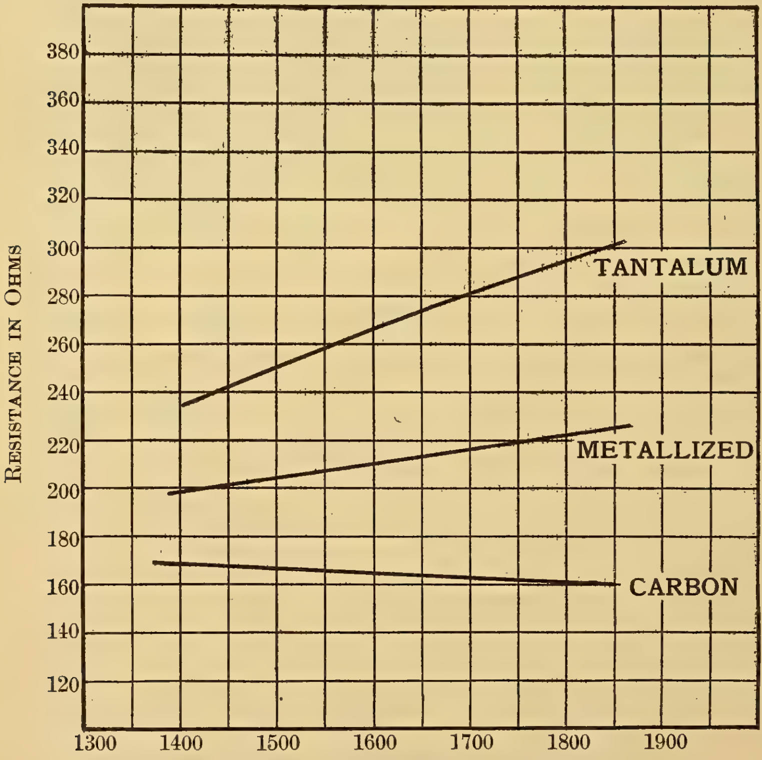

FIG. 2 FILAMENT TEMPERATURE, DEGREES CENT.

As a result of the positive coefficients of the metallized and tantalum filaments, the lamps flash up to full incandescence much more quickly than the carbon lamp. When the current is turned on, the filament, being cold, has a low resistance and there is a rush of current considerably above normal. This excessive current is rapidly cut down as the lamp reaches incandescence on account of the increase in resistance. The carbon lamp, having the greatest resistance when cold, allows but a comparatively small current to pass at first, but gradually allows it to increase as the resistance becomes less. This is beautifully shown by the oscillograms of the rise in current in the three lamps shown in Fig. 3. With the carbon lamp it is seen that the current almost instantly rises to about .35 ampere, and then rises in almost a straight line to the full steady value of current in about .26 sec-

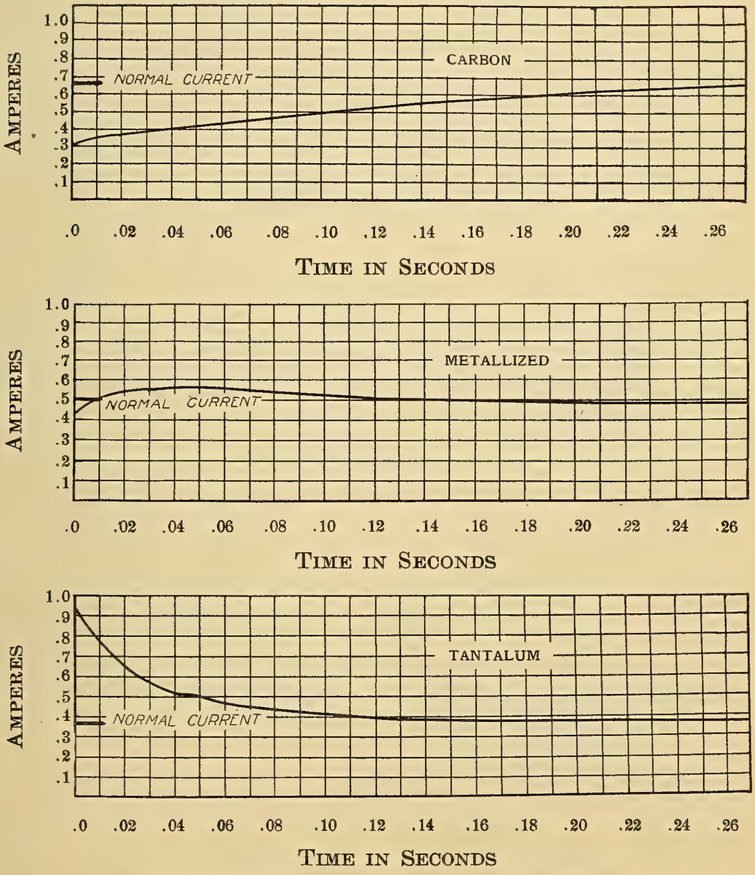

FIG. 3 OSCILLOGRAMS OF INITIAL CURRENTS

ond. With the metallized lamp the current rises at once to about .45 ampere, almost the full steady current, then increases to a maximum value of .55 ampere in approximately .05 second, indicating a negative temperature coefficient at the lower temperatures. It then decreases gradually to the normal steady value in about .16 second. The curve for the tantalum lamp indicates that a rush of current takes place as soon as the circuit is closed, reaching a maximum of about .93 ampere, almost three times the full steady current, practically instantaneously. This rush of current in the tantalum lamp will probably require that some precautions be observed in switching feeders heavily loaded with tantalum lamps on to the generator. The heavy excess current at the first instant might easily be sufficient to damage the machine. The current then rapidly drops to the normal value which is reached in about .14 second. A suggestion of these differences of action of the three lamps can be noticed when they are simultaneously lighted side by side. The carbon lamp appears to come at once to a rather dull incandescence and then gradually increases to its maximum brilliancy. The metallized lamp appears to reach at once its normal incandescence without any later change while the tantalum lamp bursts forth immediately in brilliant incandescence and then subsides gradually, giving the effect of a flash at the first instant.

The candle power voltage characteristic curves in Fig. 4 also show important differences in the three types of filaments. At 80 volts, the carbon filament starts at the lowest value for the three lamps, and rises rapidly until at normal voltage and above, it has the greatest candle power. The tantalum filament takes the highest position at the 80-volt point and increases more gradually until at normal voltage and above, it has the lowest candle power. The metallized filament curve takes an intermediate position. The equations for these curves obtained by the method of least squares from the experimental data are:

For the carbon lamp

$$CP = 143.5 \times 10^{-13} \times E^{5.97}$$

For the metallized lamp

$$CP = 50.7 \times 10^{-11} \times E^{5.20}$$

For the tantalum lamp

$$CP = 166.4 \times 10^{-10} \times E^{4.45}$$

Below is shown a table exhibiting the change in candle power for an increase of 5 per cent in the voltage and for a decrease of 5 per cent in voltage from the normal.

TABLE 2

Lamp	C. P. Increase for Five per cent Increase in Voltage above Normal	C. P. Decrease for Five per cent Decrease in Voltage below Normal
Carbon	7.3 or 33.2 per cent	6.8 or 31.0 per cent
Metallized	5.6 or 25.7 per cent	5.8 or 27.6 per cent
Tantalum	4.4 or 22.0 per cent	4.8 or 24.0 per cent

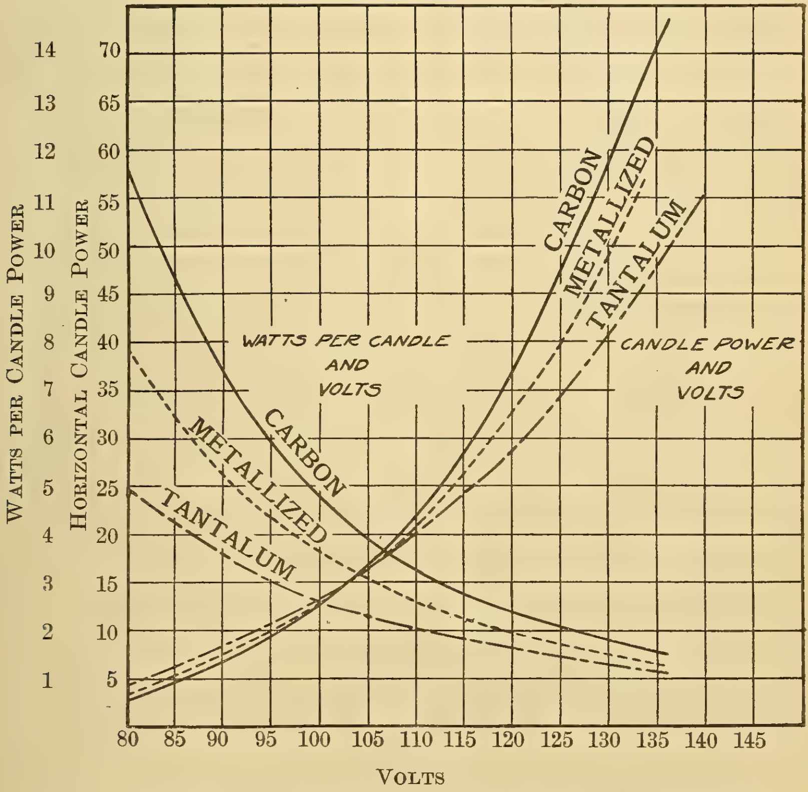

FIG. 4 CHARACTERISTIC CURVES OF CANDLE POWER AND WATTS PER CANDLE

The change in efficiency or watts per candle is also shown in Fig. 4, indicating a wide difference in favor of the tantalum lamp throughout the range of voltage. The curves showing the change of resistance with the voltage (Fig. 5) indicate that the tantalum and the metallized filaments tend to regulate for constant current. In these filaments the resistance rises with the voltage. Hence in a poorly regulated circuit, when there is an increase in pressure, the resistance of the filament becomes greater on account of the increase in temperature. This prevents such a great rise in current and candle power. When the pressure drops the resistance is decreased, thus preventing such a large decrease in current and candle power. With the carbon lamp the change in resistance is such that it tends to aggravate the effects of a fluctuating voltage. When the voltage increases there is a decrease in resistance. This decrease in resistance adds to the change in current naturally brought about by the increase in pressure and the result is a very rapid change in current and candle power. For a decrease in voltage, of course, the reverse is true; a drop in voltage causing a rise in resistance, the change in both of them being in the direction to decrease the current and candle power.

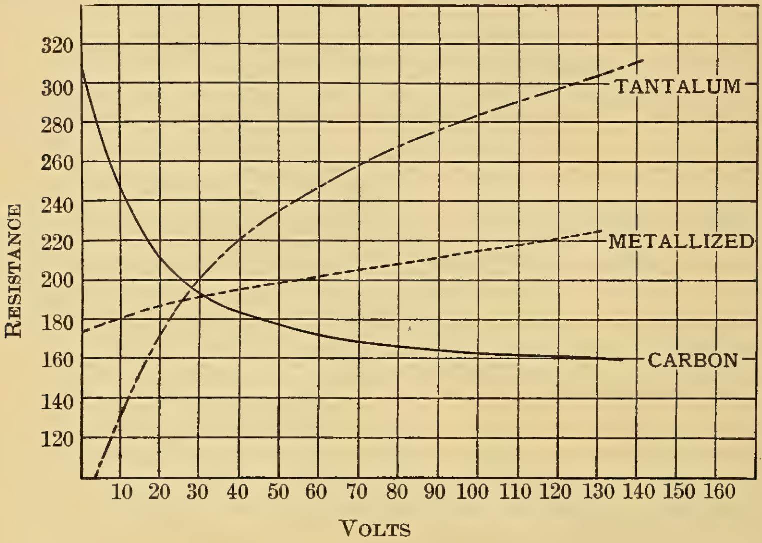

FIG. 5 CURVES SHOWING CHANGE OF RESISTANCE WITH VOLTAGE

DISTRIBUTION

Horizontal and vertical distribution curves for the different lamps when new and after 800 hours of burning are shown in Fig. 6 to 14. The small figure below each set of curves indicates the position of the filament in each case. The horizontal distribution in all three lamps evidently changes equally in all directions after a period of burning. That for the tantalum lamp would be almost a circle except for the influence of the leading-in wires. These cause a minimum point on the side nearest to them. The vertical distribution of the carbon and metallized lamps changes but little after 800 hours of service. That of the tantalum, however, changes to a marked extent; the candle power for the position 30° from the tip of the lamp being greater after burning the 800 hours than when new. The change in the spherical reduction factor (the constant for changing mean horizontal candle power to mean spherical candle power) of the lamps during their life gives a good indication of the way the distribution changes. Following is a table of these values for three periods of life.

TABLE 3

Lamp	Spherical Reduction Factors		
	New	400 Hours	800 Hours
Carbon	.810	.805	.794
Metallized	.803	.805	.801
Tantalum	.787	.811	.865

The causes of this change in the distribution of the intensity in the tantalum lamp must be due principally to the change in the character of the surface of the filament after burning. The microphotographs of the filaments shown in Fig. 30 indicate how roughened and pitted they become after burning for a few hundred hours. The irregularities of the surface cause the light to be radiated and reflected from their surfaces more and more in a direction parallel with the length of the filament as the period of burning increases. This, of course, shifts the maximum of in-

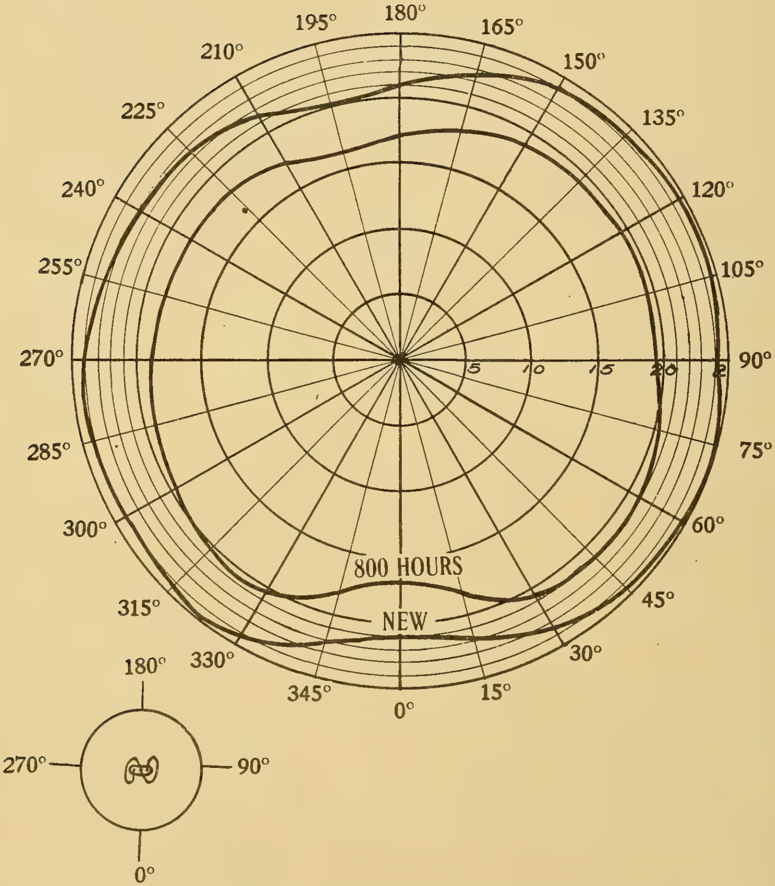

Fig. 6 Horizontal Distribution of Carbon Lamp When New and After 800 Hours of Burning

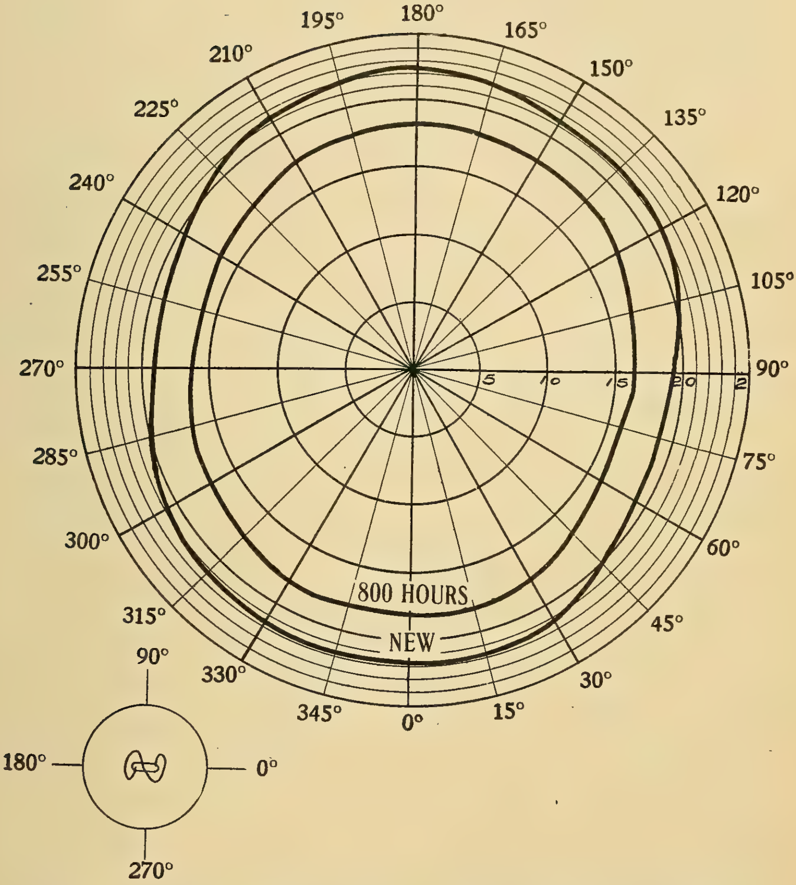

Fig. 7 Horizontal Distribution of Metallized Lamp When New and After 800 Hours of Burning

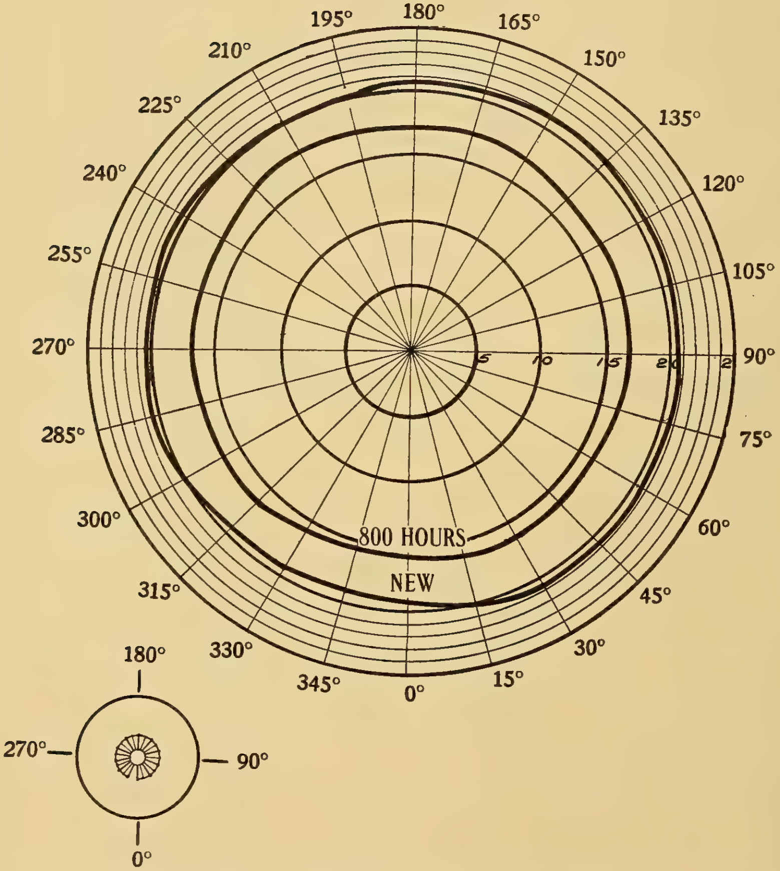

FIG. 8 HORIZONTAL DISTRIBUTION OF TANTALUM LAMP WHEN NEW AND AFTER 800 HOURS OF BURNING

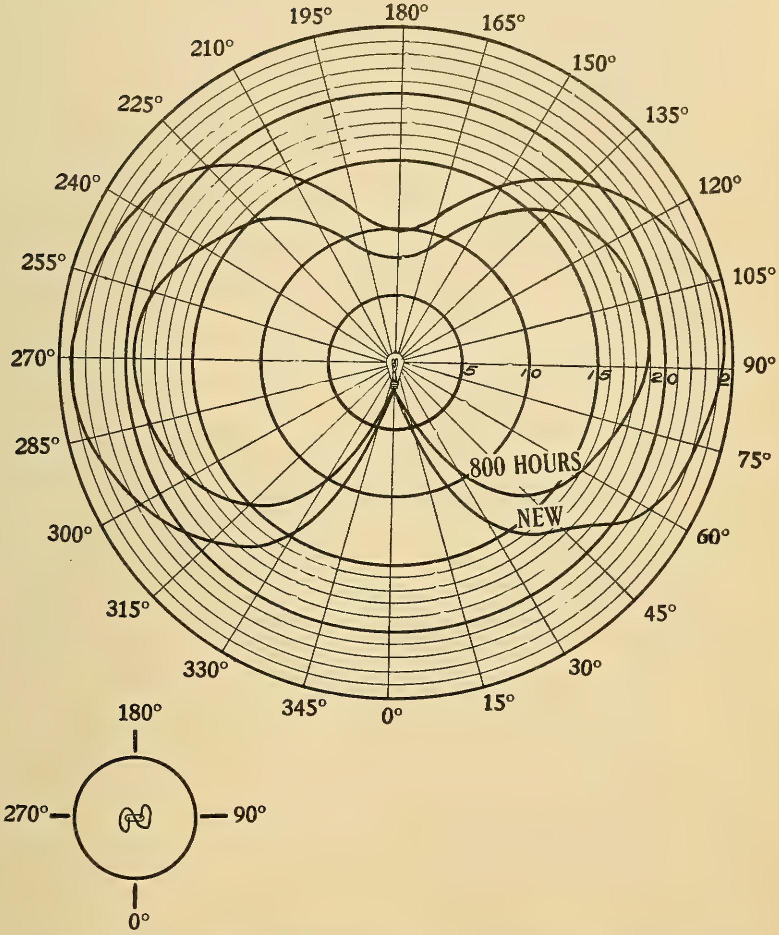

FIG. 9 VERTICAL DISTRIBUTION AT 90° AZIMUTH OF CARBON LAMP WHEN NEW AND AFTER 800 HOURS OF BURNING

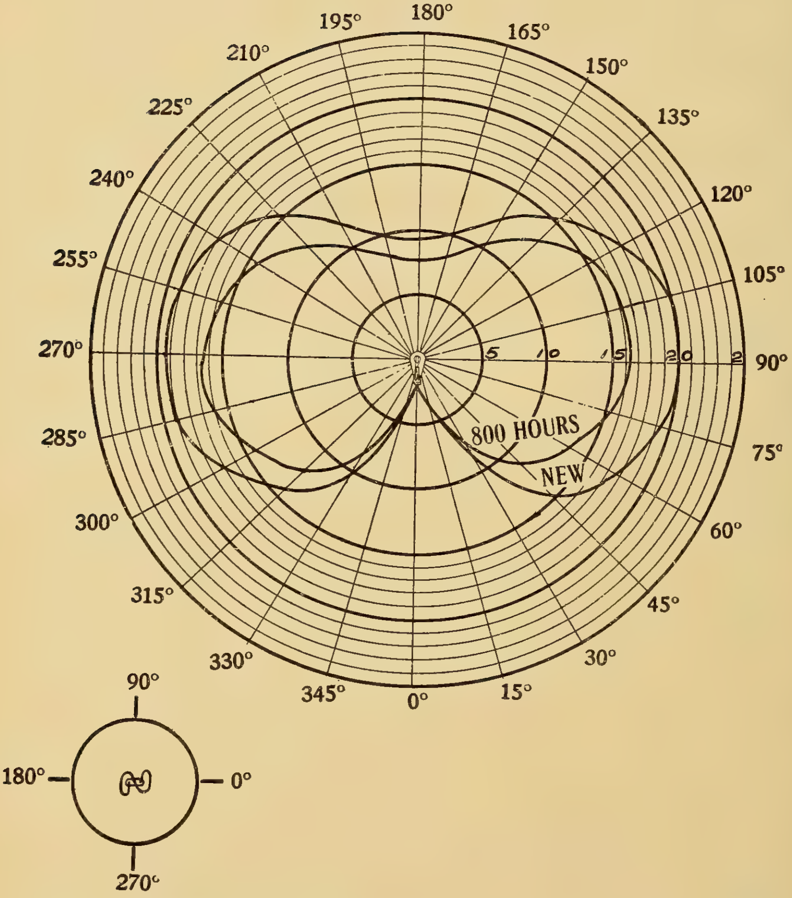

FIG. 10 VERTICAL DISTRIBUTION AT 90° AZIMUTH OF METALLIZED LAMP WHEN NEW AND AFTER 800 HOURS OF BURNING

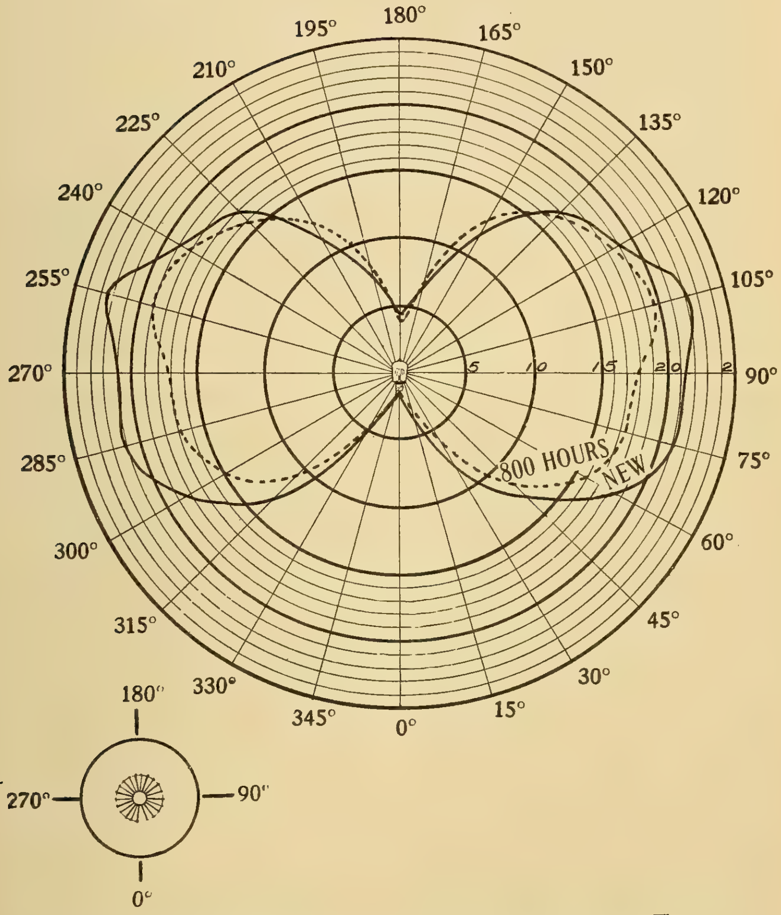

FIG. 11 VERTICAL DISTRIBUTION AT 90° AZIMUTH OF TANTALUM LAMP WHEN NEW AND AFTER 800 HOURS OF BURNING

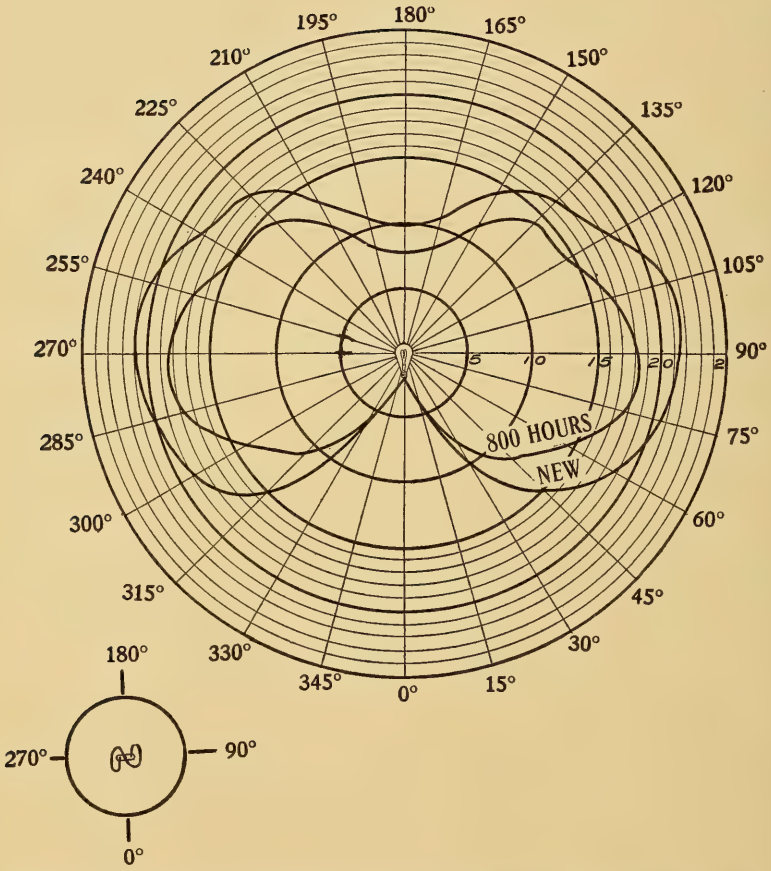

Fig. 12 Vertical Distribution at 0° Azimuth of Carbon Lamp When New and After 800 Hours of Burning

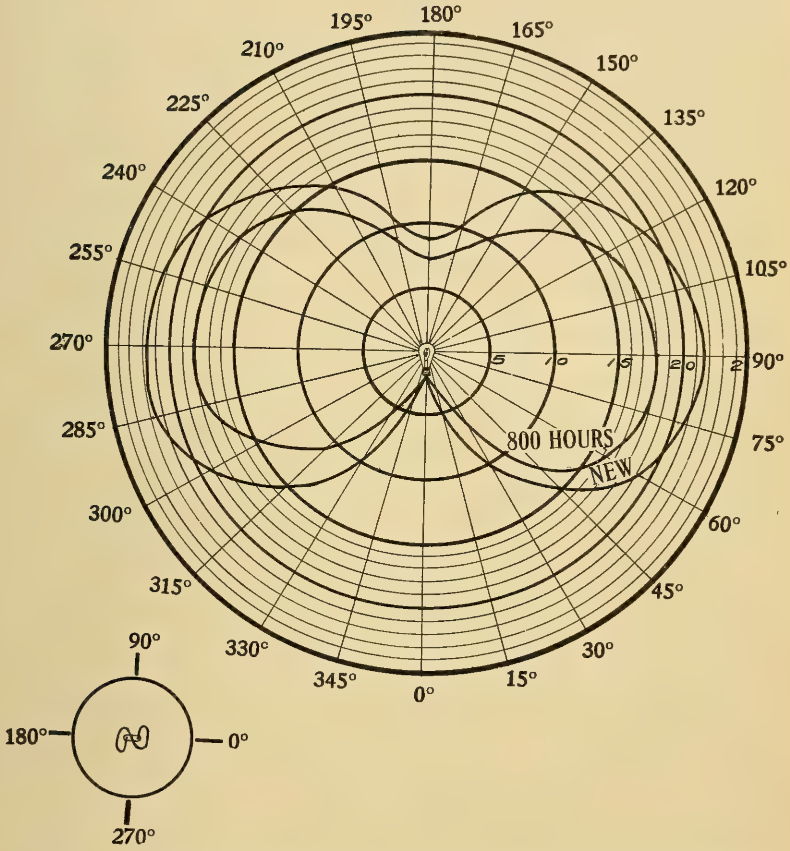

FIG. 13 VERTICAL DISTRIBUTION AT 0° AZIMUTH METALLIZED LAMP WHEN NEW AND AFTER 800 HOURS OF BURNING

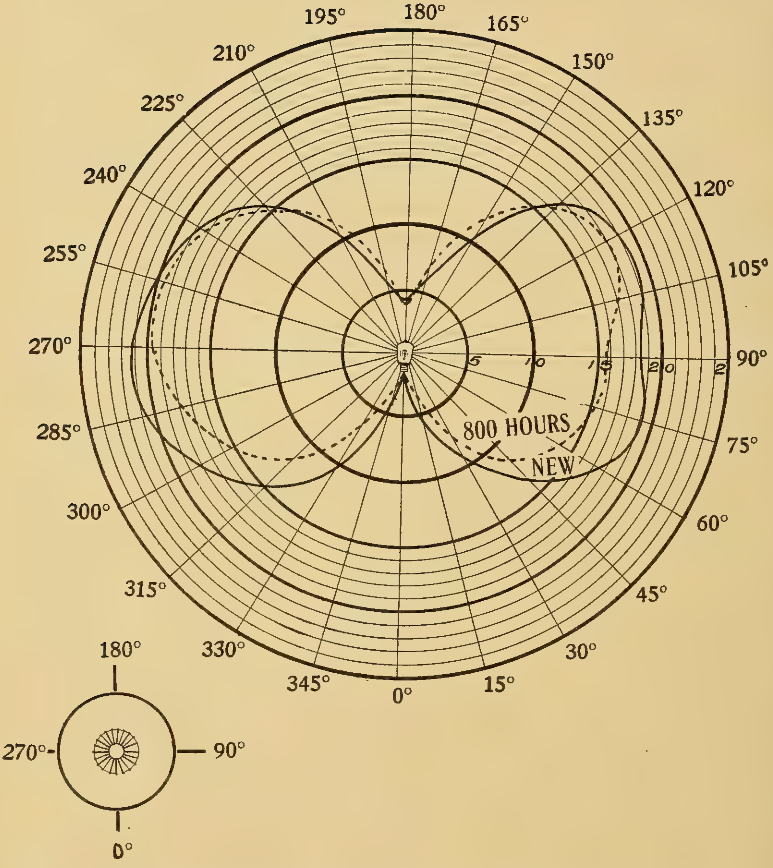

FIG. 14 VERTICAL DISTRIBUTION AT 0° AZIMUTH OF TANTALUM LAMP WHEN NEW AND AFTER 800 HOURS OF BURNING

tensity of the vertical distribution curves further from the horizontal. In the carbon and metallized filaments there is but little change in the character of the surface, and consequently the distribution in these lamps changes but little from this cause. The manner in which the bulb of the tantalum lamp discolors after use will also partly explain the cause of the change in distribution. This discoloration takes the form of a band of black deposit on the glass, equal in width to the distance between the top and bottom supporting spires. Outside of this band there is a deposit, but it is much lighter. It is as if the particles of the metal were projected from the incandescent filament only in directions normal to its surface. The density of this deposit in the band cuts down the horizontal intensity a great deal, but at the tip, where the deposit is thinner, the candle power is decreased by a much less amount.

LIFE TESTS

Life tests were made of the three kinds of lamps under two different conditions. Ten lamps of each kind were put through the life and efficiency test upon a steady, well-regulated voltage supplied by a storage battery. The battery was kept floating across nearly constant voltage mains and a large rheostat was put in series with the lamps with which to make the finer adjustments by hand. The maximum variation was probably not more than one volt and the greater portion of the time the voltage was as nearly correct as the portable voltmeter used would indicate. This was designated "Condition A" and represents the best conditions under which the lamps would ever be operated in practice. The same number of lamps were operated under adverse conditions. A badly fluctuating alternating current was supplied to the lamps and there was considerable vibration. This condition, designated "Condition B", is representative of very bad operating conditions. Fig. 15 to 20 show the candle power performance of each lamp in the test. The uniformity of the lamps under condition A is noticeable, none of the curves varying greatly from the mean.

Burn-outs and Failures.—In the 800 hours of the test under condition A only three lamps were lost. Of the two tantalum lamps lost, one failed by the breaking of the glass stem and the leading-in wire, probably due to the expansion of the latter. The other failed in 30 hours probably on account of a fault in the filament. It was repaired and it then burned at a high candle power for a short time and then burned out. The failure of metallized filament lamp No. 2 was due to the fact that the filaments be-

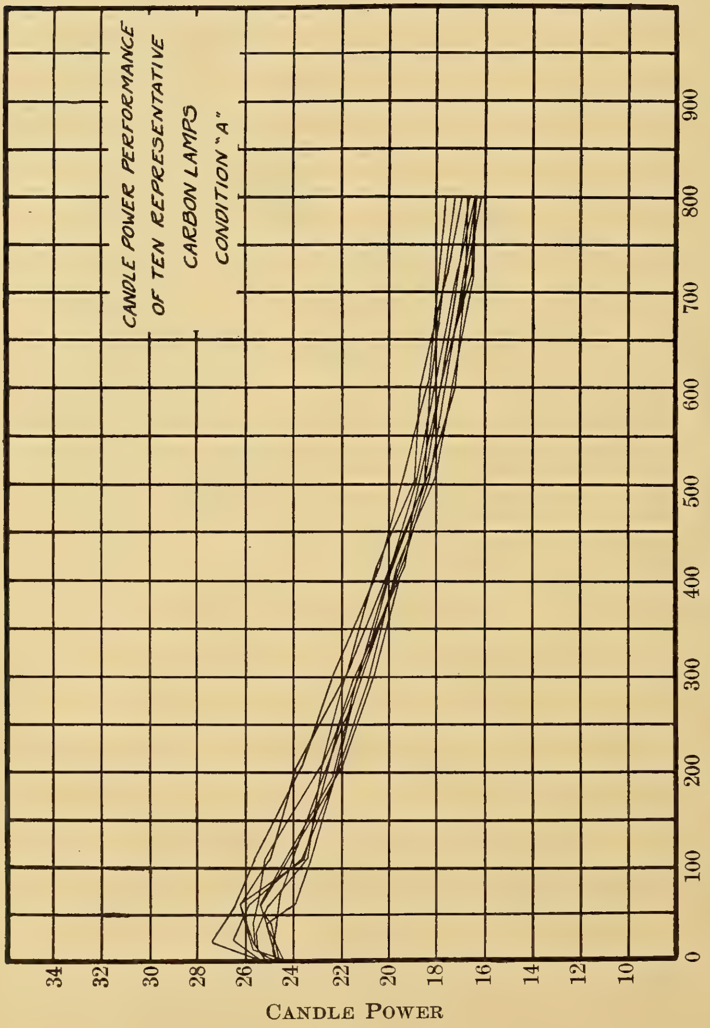

FIG. 15

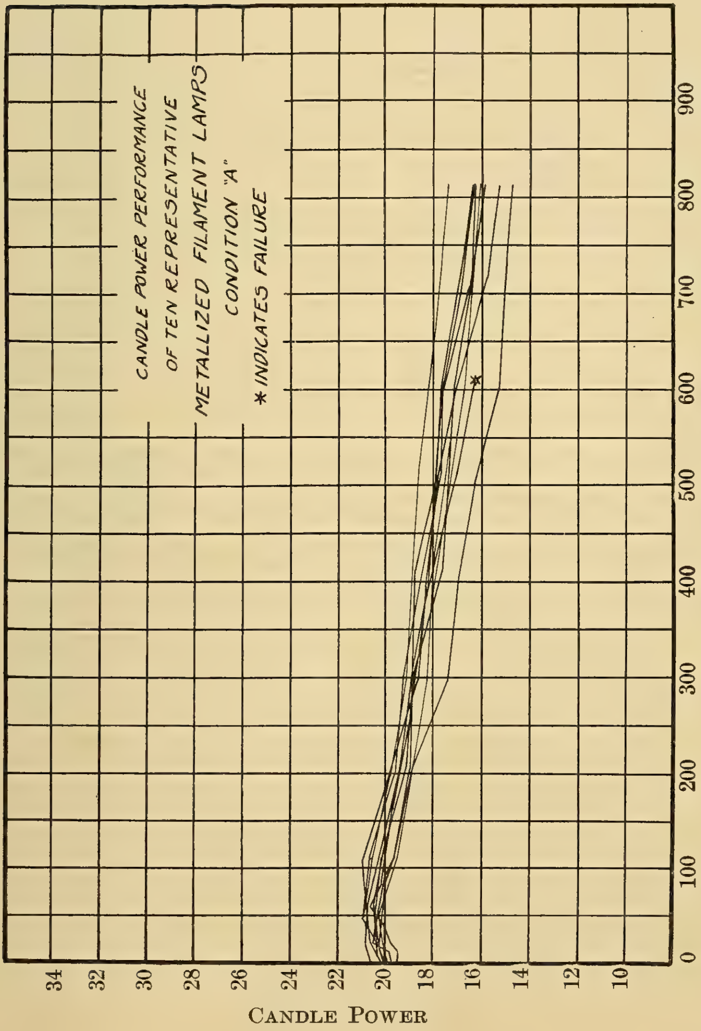

Fig. 16 Life in Hours

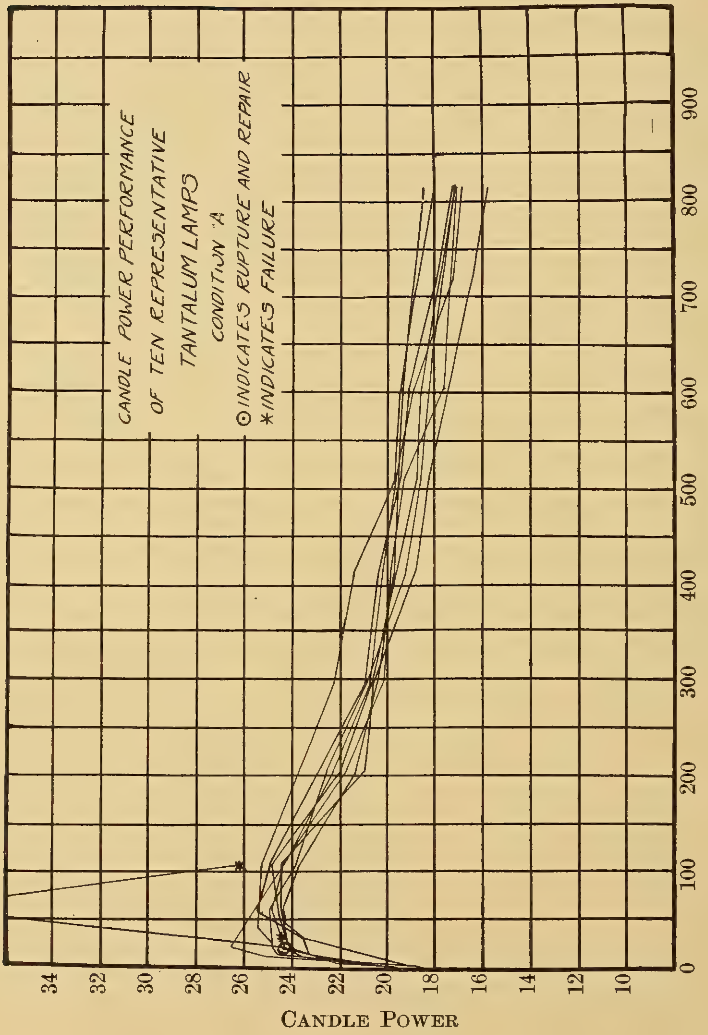

FIG. 17 LIFE IN HOURS

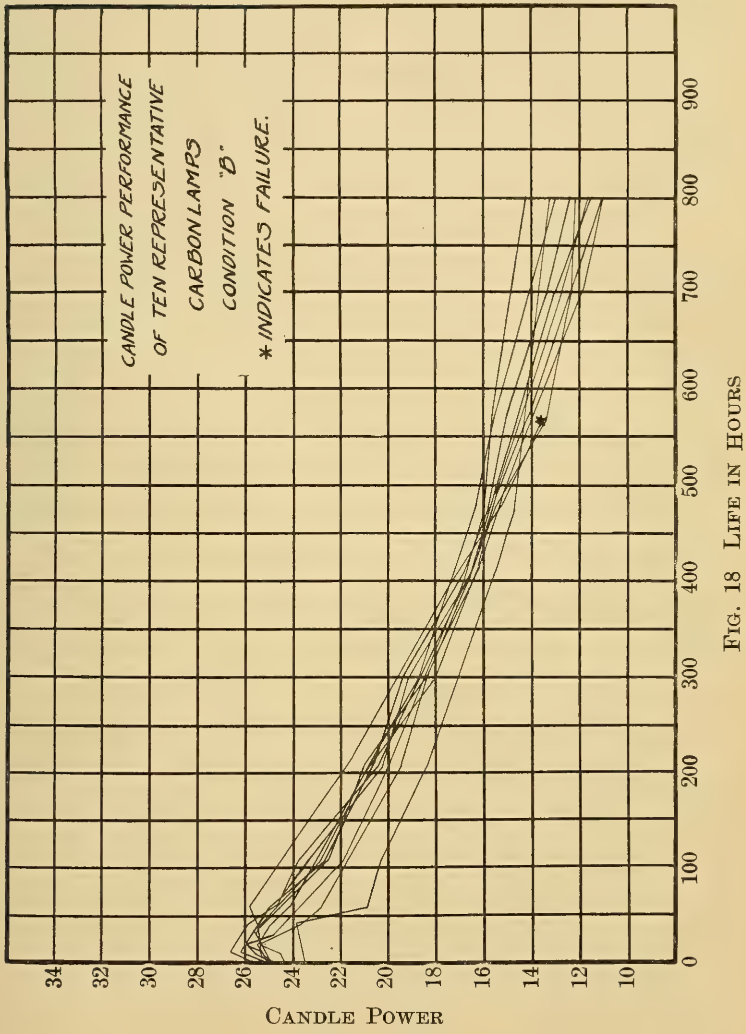
CANDLE POWER PERFORMANCE
OF TEN REPRESENTATIVE
CARBON LAMPS
CONDITION "B"
* INDICATES FAILURE.
0
100
200
300
400
500
600
700
800
900
LIFE IN HOURS
34
32
30
28
26
24
22
20
18
16
14
12
10
CANDLE POWER

FIG. 18

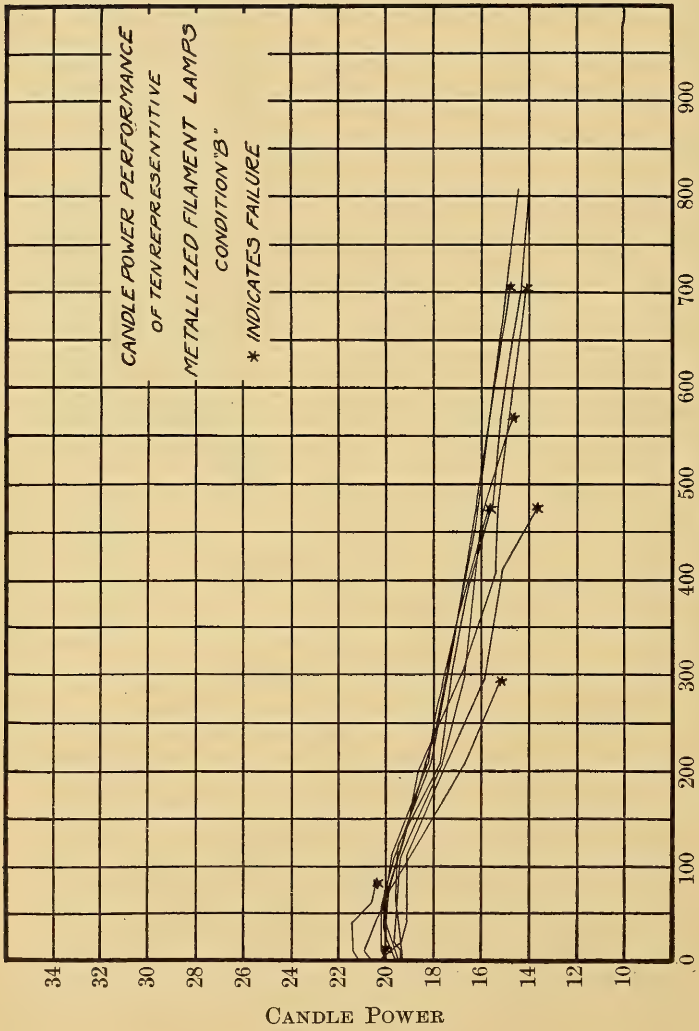

FIG. 19 LIFE IN HOURS

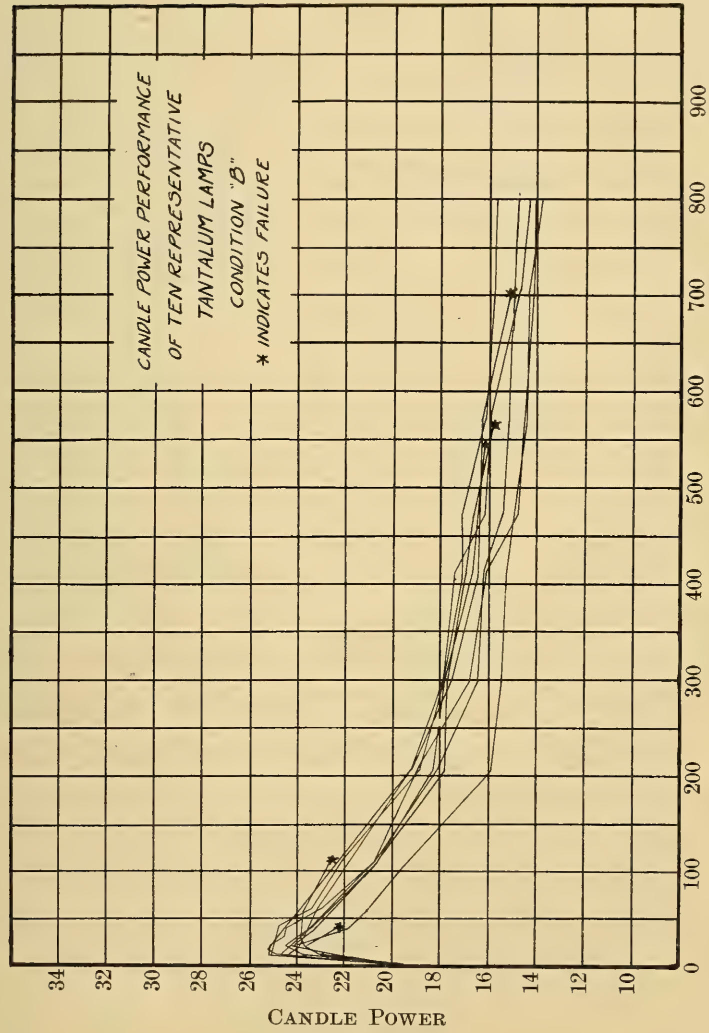

Fig. 20 Life in Hours

came crossed in placing the lamp in the socket. Naturally it burned at a very high brilliancy until the lamp could be removed from the socket and the cross shaken out. When this was done and the lamp returned to its place, it burned out in a very short time due to the weakening of the filament while at the high temperature. The operation under condition B shows less uniformity for lamps of any one kind and a great increase in failures during the 800 hours of the test. Only one carbon lamp burned out before the test was ended, however. Five of the tantalum and eight of the metallized lamps failed during the 800 hours of burning under this condition, thus showing the great superiority of the old style carbon lamp on poorly regulated circuits as far as reliability is concerned. The first two tantalum and the first two metallized lamps that failed did so early in their life and the failure was probably due to mechanical defects in the filament. The failures due to natural causes commence then at about 400 hours in the metallized and at about 550 hours in the tantalum under these conditions. The poor showing of the metallized filament is striking, it being much poorer than the tantalum, which is not claimed to give good life on alternating current circuits. The combination of poor regulation and vibration seems to be very detrimental to its long life. Possibly the frequency with which the lamps had to be handled had something to do with the large number of burnouts although great care was taken throughout the tests not to subject the lamps to shocks or jars.

CANDLE POWER MAINTENANCE AND CHANGE OF EFFICIENCY

The curves in Fig. 21 and 22 show the relative changes of candle power and efficiency for the three kinds of lamps, the curves representing the mean performance of the lamps tested. The carbon lamp under condition A starts out with a high candle power, increases comparatively rapidly for the first 50 hours or so then decreases steadily for the remainder of the test. The tantalum lamp rises very rapidly for the first 20 hours of the test, then more slowly until the end of the first 100 hours, after which the candle power decreases more slowly, after 400 hours its candle power being greater than that of the carbon. The metallized lamp changes less than either of the others. It rises during the early period of burning, remains almost constant for a time, decreasing slowly in candle power during the remainder of its life.

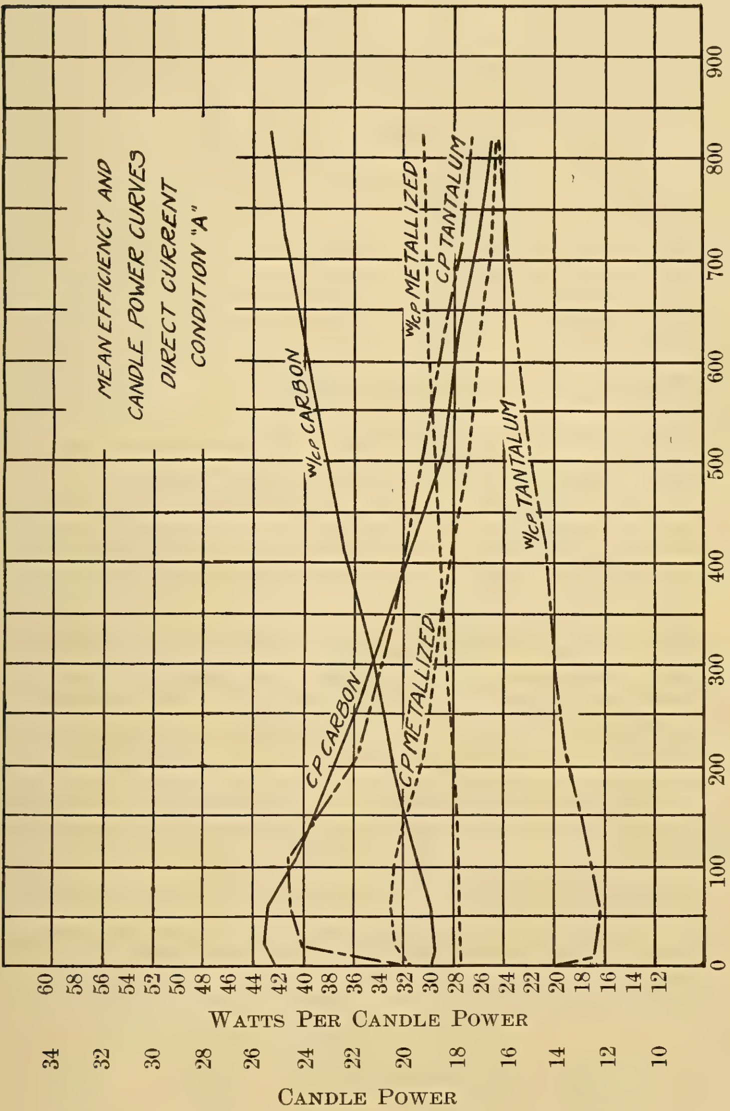

Fig. 21 Life in Hours

The metallized lamp changes no more in efficiency than it does in candle power. The carbon lamp changes rapidly, becoming less and less efficient with respect to the metallized filament. The tantalum lamp becomes relatively more and more efficient than the metallized lamp during the first 250 hours and then tends

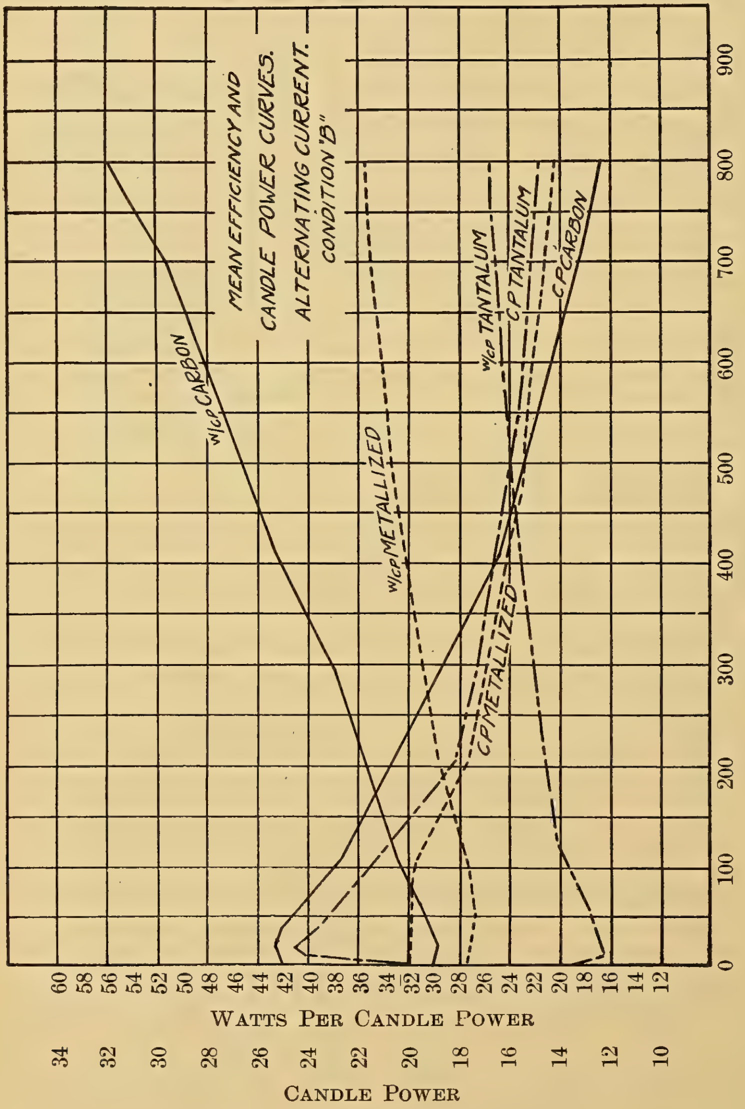

FIG. 22

to drop off and approach it in efficiency. When operating under condition B the changes in candle power and efficiency are much the same except that the changes occur more rapidly. It is noticeable that in this case the carbon lamp starts out with the highest candle power, but after 500 hours of burning it has the lowest. Its decrease in efficiency is correspondingly rapid. Under condition B the tantalum lamp no longer becomes less efficient relatively to the metallized lamp as the period of burning increases as it does under condition A. Its efficiency curve tends to fall further and further below that of the metallized lamp.

The reason why the tantalum and metallized filament lamps show a better efficiency than the carbon is of considerable interest. There is no doubt that the greater amount of the superior efficiency of the newer types of lamps is due to higher filament temperatures. As is well known, when a solid body is heated, at first only the long, low frequency heat waves appear, then the red light waves, and as the temperature is further increased, wave lengths corresponding to the other colors of the spectrum through the violet and ultraviolet appear. If for any temperature we measure by means of the bolometer the intensity of radiation at points throughout the visible and invisible portions of the spectrum and plot these values against the wave lengths, we get a curve similar to curve A in Fig. 23, having a maximum at some point, *m*. The visible, that is, the light-giving portion is shown unshaded. As the temperature is increased, the maximum of this curve moves toward the region of shorter wave lengths, as shown in the curve B. There is, however, an increase in the length of each ordinate so that the curve does not move bodily down the spectrum with an increase in temperature, but the ordinates of the energy curve move toward the shorter wave lengths by an amount such that the product of the corresponding abscissas and the temperature remains constant for each ordinate. It is seen then that the visible portion of the spectrum is a greater proportion of the entire spectrum than at the lower temperature and hence a better light efficiency results. If we remember that the velocity with which the molecules of a body are moving increases with the temperature, then we can in a general way see why it is that the point of maximum intensity in a continuous spectrum is shifted toward the violet as the temperature increases.

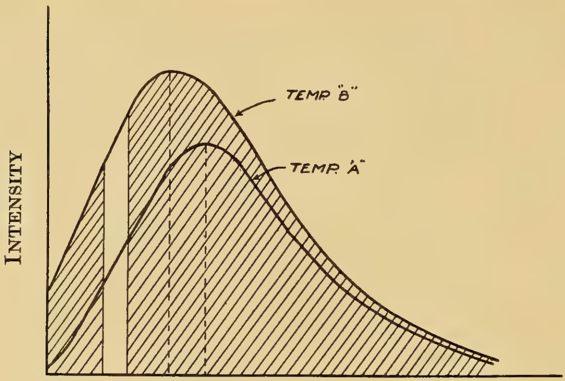

FIG. 23 WAVE LENGTH

The curves between temperature and candle power per square inch of filament area, or emissivity, shown in Fig. 24, indicate that the tantalum filament has a lower and the metallized a higher emissivity than the carbon filament. Since emissivity is vitally connected with the light efficiency of an incandescent body, the difference in the relative positions of the emissivity curves of the metallized and tantalum lamps with respect to the carbon seems to indicate that a part of the better efficiency of the two newer filaments is due to different causes.

The tantalum filament, having a lower emissivity than the carbon filament, requires less energy to maintain the same temperature than the carbon; that is, the lower emissivity of the tantalum filament gives it a better efficiency than the carbon at the same temperature. This is in addition to the fact that tantalum has a greater atomic weight and a higher vapor tension point than carbon, thus allowing it to be operated at a higher temperature with the consequent better efficiency.

Since the metallized filament has a higher emissivity than the carbon filament it must require a greater input of energy per square inch of surface to maintain a given temperature. Experiment shows that the input of the metallized lamp is about 460 watts and the carbon about 410 watts per square inch of filament area at a temperature 1720° C. To give, as it does even at equal temperatures, a better watt per candle efficiency it must then give off a greater proportion of light energy to heat energy than the carbon lamp. Since the carbon filament is approximately though

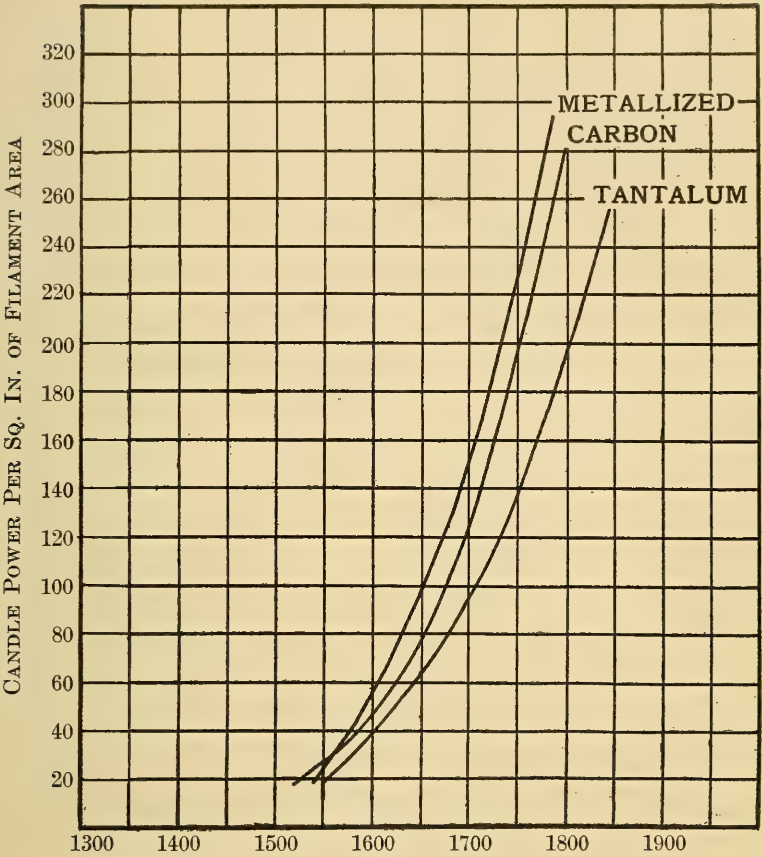

FIG. 24 FILAMENT TEMPERATURE, DEGREES CENT.

only approximately, equivalent to the theoretical solid black body this fact seems to show that the greater efficiency of the metallized filament must be due, at least in part, to a sort of selective radiation. That is, it radiates either a greater proportion of its energy within the range of the visible spectrum than a black body or a smaller proportion in the invisible range. In Fig. 25 and 26 the dotted curves show the radiation from a solid black body. Fig. 25 shows the curve for a body having at the same temperature almost the same radiation outside the visible spectrum but a much greater radiation within, while in Fig. 26 is shown the curve for a body having practically the same radiation within the

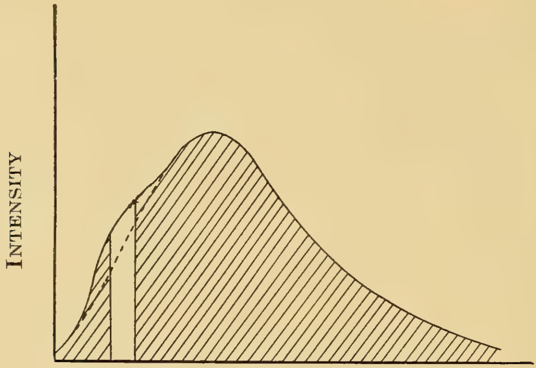

FIG. 25 WAVE LENGTH

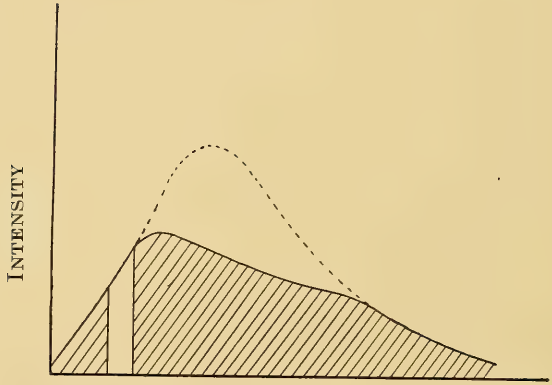

FIG. 26 WAVE LENGTH

visible portion of the spectrum, but a much less radiation without than in the case of a black body. In one of these cases the metallized filament, no doubt, falls. In both cases, however, there is an increase in efficiency due to selective radiation.

In the curves between watts per candle and filament temperature, there is another indication that the high efficiency of the metallized and tantalum lamps is due to different causes. It is seen in the curves of Fig. 27 that at ordinary efficiencies the tantalum is the lowest, the metallized next, while the carbon is the highest. However, at about 1830° C., the curves for the metallized and tantalum filaments cross and hence for higher temperatures the metallized filament has the better efficiency. When the

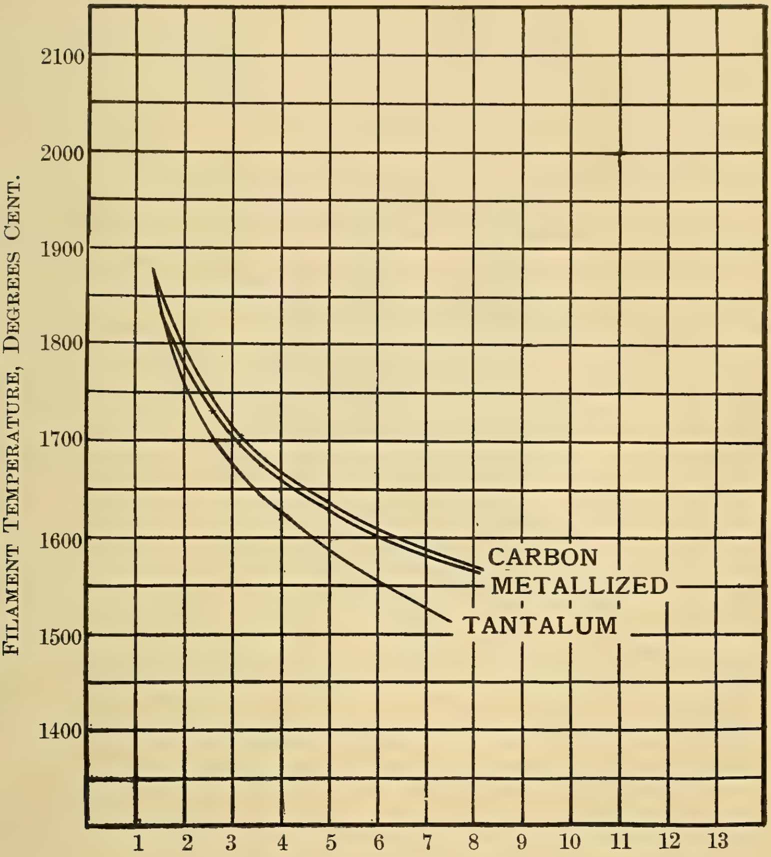

FIG. 27 WATTS PER CANDLE POWER

curves are produced to about 1875° C., the curve for the tantalum filament crosses that for the carbon so that beyond this point the latter would be more efficient if it could be operated at such temperatures. This too shows that it is due to the ability of the tantalum filament to withstand high temperatures without too rapid disintegration that it has so high an efficiency.

When we consider the curve for the metallized filament with respect to that of the carbon it is seen that it falls below that of the latter and tends to fall further below it at the higher temperatures; that is, as the temperature is increased, the metallized filament becomes relatively more and more efficient for any given temperature. Evidently then it is due not so much to its ability to withstand high temperatures that the metallized filament is the more efficient but rather to a selective radiation that becomes more and more pronounced as the temperature is increased.

Connected closely with the life of the lamps is the condition of the filaments after a period of burning. The carbon filament is shown in the micro-photographs*, Fig. 28, when new, after 1000 hours' burning under condition A and after 800 hours' burning under condition B. Little or no disintegration or breaking up of the filament is shown. It is almost as smooth and strong looking after 800 hours of the hard service as it was when it was new. Fig. 29 shows the micro-photographs of the metallized filaments when new, after 1000 hours under condition A and 800 hours under condition B. It is seen that after the different periods of

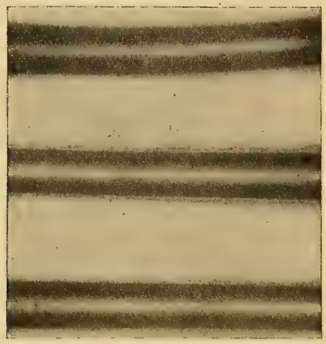

FIG. 28 CARBON FILAMENT

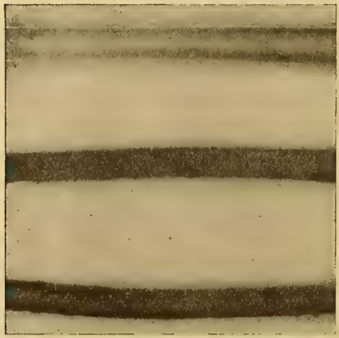

FIG. 29 METALLIZED FILAMENT

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*Taken by Mr. David Klein, Department of Chemistry of University of Illinois.

burning, the filament is not quite as smooth as when new, but is pitted somewhat and has decreased in size a little. The most remarkable change, however, is shown in the case of the tantalum filament in Fig. 30. When new it is smooth and cylindrical, showing slight pittings or markings. After 1000 hours under condition A it has roughened up a great deal, being covered with notches and ridges due perhaps to unequal evaporation of the filament. The filament burned on alternating current shows a greater change even than that burned under condition A. It appears to have a sort of a segmented structure; in fact, in places along the filament it appears as if small sections had fallen out a part of the way and had been caught and welded again. When the filament is in this condition even a slight jar will serve to shatter the entire filament. One lamp after burning for almost 800 hours under condition B was dropped a short distance. After being lighted it was found that the filament had been broken and welded together again in no less than eight places.

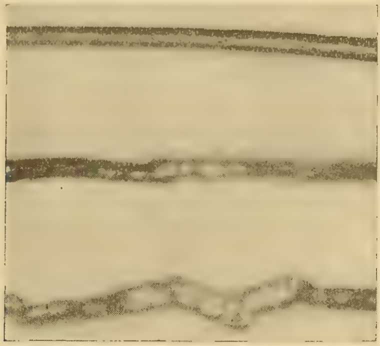

Fig. 30 Tantalum Filament

A summary of the performance of the lamps on the life and efficiency tests is shown in Table 4, together with a table of the costs of energy and renewals at different rates per kilowatt-hour. This latter table is shown graphically in the curves of Fig. 31 and 32. These curves are plotted between "Total cost in cents per candle power hour for lamps and energy" as ordinates and "Cost of energy per kilowatt-hour" as abscissas. Fig. 31 shows the curves for lamps operating under condition A, that is, upon a very well regulated direct current circuit, while Fig. 32 is for

TABLE 4

SUMMARY OF LIFE AND EFFICIENCY TESTS

OPERATING CONDITION	CARBON		METALLIZED		TANTALUM	
	A	B	A	B	A	B
Av. Mean Horizontal C. P.						
(a) (new)	24.9	25.2	20.6	20.5	19.8	19.8
(b) (400 hrs.)	19.7	16.4	18.1	16.1	19.9	16.5
(c) (800 hrs.)	16.5	12.3	14.9	14.2	17.2	14.8
Spherical Reduction Factor (a) (new)	.810		.803		.797	
(b) (400 hrs.)	.803		.805		.811	
(c) (800 hrs.)	.794		.801		.865	
Av. Spherical Candle Power (a) (new)	20.2	20.4	16.6	16.5	15.8	15.8
(b) (400 hrs.)	15.8	13.2	14.6	13.0	16.1	13.4
(c) (800 hrs.)	13.1	9.8	11.9	11.4	14.9	12.8
Av. Watts per Lamp	73.3	72.6	51.9	51.8	42.0	39.7
Av. Initial Watts per C. P.	3.00	3.1	2.62	2.62	2.02	1.99
Rated Watts per C. P.	3.1		2.5		2.0	
Total Lamp Hours	8180	7766	7963	4900	6779	6123
Total Candle Power Hours	165236	135905	144130	80905	137614	105928
Total Kilowatt Hours	599.6	563.8	413.3	253.8	284.7	243.1
No. Burnouts in 800 Hrs.	0	1	1	8	2	4
Cost of Lamps Each	$.17		$.25		$.51	
Total Cost Lamps & Renewals	$1.70	$1.87	$2.75	$4.08	$6.12	$6.83
COST OF POWER PER KW. HR. — Total Cost of Lamps and Energy Per C. P. Hr.						
$ .01	$ .00466	$ .00553	$ .00477	$ .00818	$ .00652	$ .0092
$ .03	$ .0119	$ .0138	$ .0105	$ .0144	$ .0107	$ .0133
$ .05	$ .0192	$ .0221	$ .0162	$ .0207	$ .0148	$ .0179
$ .07	$ .0264	$ .0305	$ .0220	$ .0270	$ .0189	$ .0225
$ .10	$ .0373	$ .0429	$ .0306	$ .0364	$ .0252	$ .0295
$ .12	$ .0447	$ .0512	$ .0363	$ .0427	$ .0293	$ .0343
$ .15	$ .0555	$ .0637	$ .0449	$ .0520	$ .0356	$ .0418
$ .20	$ .0736	$ .0843	$ .0593	$ .0678	$ .0459	$ .0524

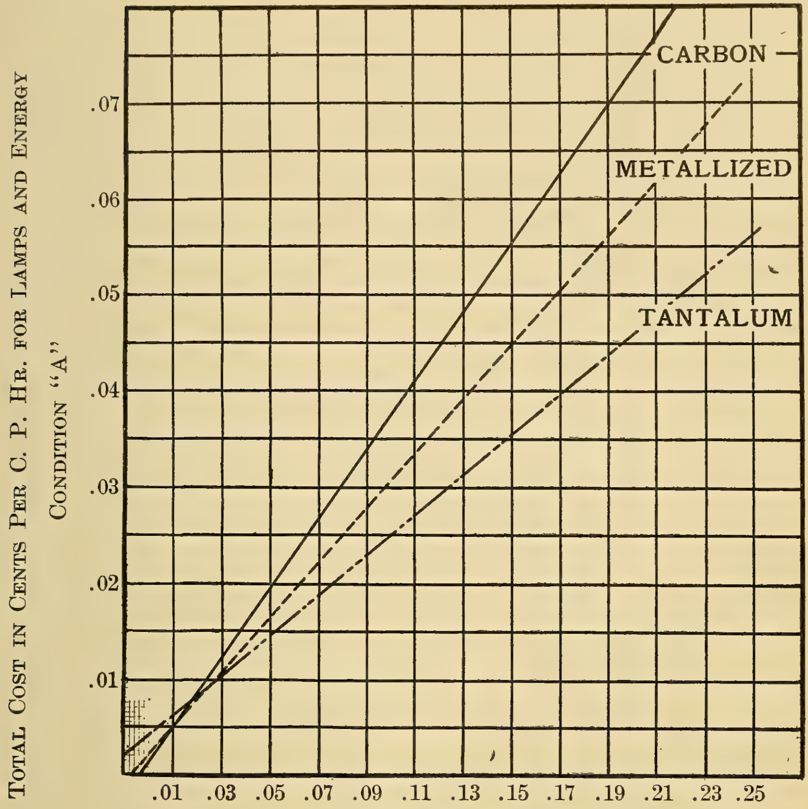

FIG. 31 COST OF ENERGY PER K. W. HOUR

lamps upon poorly regulated alternating current circuit with some vibration, that is, condition B.

Considering the curve for the carbon lamp working under condition A it is seen that for very low prices per kilowatt-hour for power, this lamp is the most economical on account of the small number of burn-outs and the low cost of the lamps. For costs of power from \$.011 to \$.022 per kilowatt-hour the metallized lamp gives the lowest cost, while for all the higher prices of energy the tantalum gives the best economy. For condition B the relative performance does not change a great deal, though on account of the large number of burn-outs with the metallized lamp at no time does it give the most economical results.

These results seem to show that so far as economy of oper-

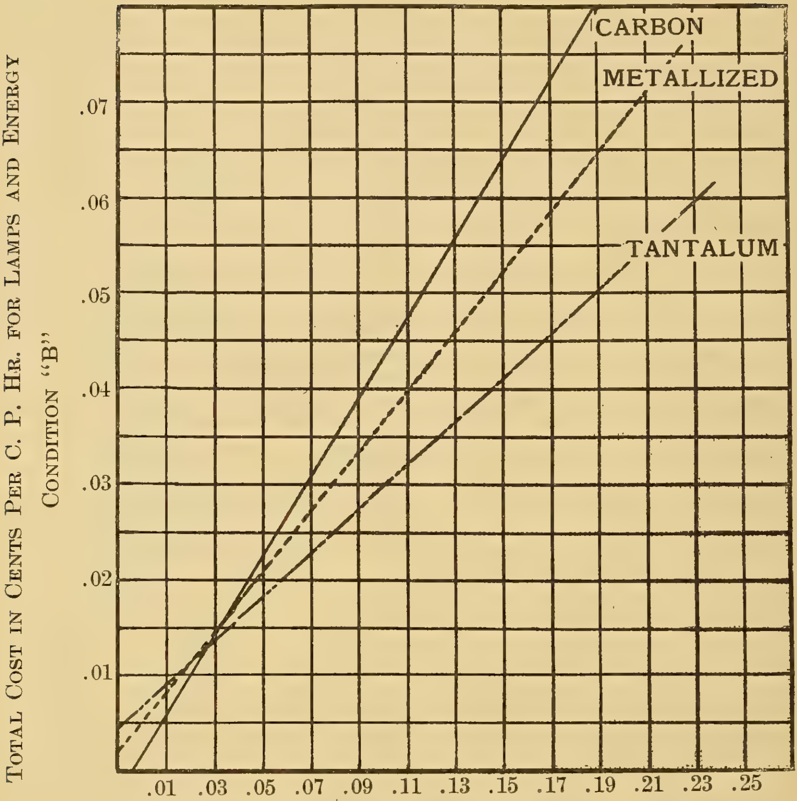

FIG. 32 COST OF ENERGY PER K. W. HOUR

ation goes, the metallized lamp has practically no field in incandescent lighting. From the standpoint of low cost of renewals, an important item with lighting companies that furnish free renewals, it cannot compete with the carbon or tantalum lamp, especially upon poorly regulated circuits and where there is vibration or rough usage. In cost of power consumption the carbon lamp leads for very low costs of power, and the tantalum for higher costs of energy. The metallized lamp seems to have a narrow field upon very well regulated circuits where the cost is between $.02 and $.03 per kilowatt-hour.

SUMMARY

As a summary it might be well to consider separately the three lamps and compare them with respect to the following eight considerations which determine the choice of an incandescent lamp.

1. Efficiency.
2. Cost of Operation.
3. Maintenance of Candle Power and Efficiency.
4. Life.
5. Quality of Light.
6. Distribution of Light.
7. Susceptibility to Voltage Variations.
8. Ability to Withstand Rough Usage.

1. *Efficiency*

In the matter of efficiency alone, this test as well as all other tests which have been made with these lamps shows conclusively that the metallized lamp is much superior to the carbon, and the tantalum is as much superior to the metallized. The difference between 3.1 watts per candle and 2.0 watts per candle, about 28 per cent, is sufficient to outweigh almost all other considerations. It means that a 20 candle power metallized or 22.5 candle power tantalum lamp can be operated with the same amount of energy as a 16 candle power carbon lamp. It means that a power plant which is running with a heavy overload of carbon lamps would, if the carbon lamps were exchanged for the same number of candle power of the newer lamps, be operating at about normal load with the consequent advantages. In the same way a method is provided to lighten overloaded feeders without any decrease in the candle power of light furnished.

2. *Cost of operation*

The curves of Fig. 31 and 32 show that upon well regulated circuits each type has a field of its own within which it is the most economical. For very low costs of power the carbon lamp gives the best economy. Hence particularly for persons who generate their own current it would not pay to change from carbon to the higher efficiency lamps, because in this case either the cost of power is low or else the fuel bill, the only item in which there would be a saving by using high efficiency lights, is not large compared with the other expenses such as attendance charges, taxes and interest. When the cost of energy is high, as it is in most cities, the tantalum lamp would be the best to use. The metallized lamp seems to be restricted to rather narrow limits of power cost and to good operating conditions. The newer types of lamps would have a great field in lighting railroad trains and steamships where the cost of power is always high, if filaments were robust enough to withstand the shocks and vibrations that are usually present. It seems that it might be possible and advisable for manufacturers to develop series tantalum lamps for this purpose. The filaments that would be used in a series lamp would no doubt be strong enough to withstand the vibrations and jars found in this service and their economy of current consumption would make them much preferable to the carbon lamp.

3. *Maintenance of candle power and efficiency*

In regard to maintaining candle power and efficiency the newer lamps make by far the best showing, the two being almost the same in this respect. The metallized and tantalum lamps have a drop of respectively 20 and 19 per cent in candle power in 1000 hours under condition A while the carbon drops 32 per cent in the same time and under the same conditions. The change in efficiency for the three lamps is in about the same proportion.

4. *Life*

Comparing the lamps upon the basis of average life to 80 per cent of the original candle power, which is standard for the carbon lamp, the following results are obtained.

TABLE 5

Condition	Life in hours		
	Carbon	Metallized	Tantalum
A	400	780	820
B	225	350	350

This method of comparison is if any thing unfair to the higher efficiency lamps, because, owing to their higher first cost, the smashing point should be after the lamps have reached a candle power considerably less than 80 per cent of the original. It serves, however, to show the superiority of the newer lamps in this respect.

5. *Quality of light*

The quality of the light from the metallized and tantalum lamps is much the same, both being considerably whiter, softer and more pleasing to the eyes than that of the carbon lamp. Being a whiter light, the newer lamps show more nearly the true values of colors than the carbon lamp and hence are superior for lighting dry goods and clothing stores, art and picture galleries and other places in which colors must be judged. The intrinsic briliancy of the three kinds of filaments is approximately

Carbon	140 C. P. per sq. in.
Tantalum	165 C. P. per sq. in.
Metallized	190 C. P. per sq. in.

On account of the great intrinsic brightness of the newer types of filaments, particularly the metallized, it is not advisable to use these lamps for interior lighting when they are placed low enough to be in the line of vision, unless they are provided with ground glass or opal globes.

6. *Distribution of light*

The distribution of the carbon and metallized lamps is so nearly identical as to admit of little choice between them in this particular. The tantalum lamp differs from these in having a low tip candle power which is a point in its favor when used with reflectors.

For use with reflecting globes the most efficient lamp for any given watt per candle consumption would be one with a long straight filament mounted vertically. This kind of an arrangement gives the condition where the minimum proportion of light is radiated downward and upward, but gives a distribution which can be changed to suit the requirements by means of reflectors, and is such that the minimum light is lost in the base. Getting a downward distribution by placing the greater part of the filament horizontal, as has been done in many of the "downward light" lamps on the market, is an inefficient method. It is true that these lamps throw the maximum of their intensity further down from the horizontal than an ordinary lamp, but in so doing just as much light is thrown upward where it is mostly lost in the base and on the ceiling by absorption and improper reflection. To get light where it is needed and do it most efficiently is accomplished by mounting the filament so that as nearly as possible the entire length of it is parallel to the axis of the base, and then using good reflectors. The tantalum lamp meets this requirement very well, as is shown by its low tip candle power which indicates a small loss of light in the base. The intensity at angles even up to 30° from the tip is low in this lamp. The same condition would be shown at the base if the distribution were not changed by its presence. Near the base all light that has its course changed downward by reflectors must strike the reflector at a large angle. This, of course, is a condition that favors absorption and losses. With the tantalum the radiation in these unfavorable directions is less than in the other two and is superior for that reason if reflectors are used. When the lamps are used bare the carbon and metallized give a greater candle power downward where it is generally needed than does the tantalum lamp.

7. *Susceptibility to voltage variation*

Table 2 gives a comparison of the way these lamps act in regard to the very important point of susceptibility to voltage variation. For use upon poorly regulated feeders or at the end of long feeders that are sometimes heavily loaded, the metallized and tantalum lamps will make a much more uniform and pleasing light than the sensitive carbon lamp.

8. *Ability to withstand rough usage*

It is in this particular that the carbon lamp stands supreme. Long experience in making them has enabled the manufacturers to make a carbon filament that will withstand almost any reasonable usage. The filaments of both the metallized and the tantalum lamps are easily broken, especially after they have been burned for a while. The filament of the former is so fine that jars such as would be caused by screwing the lamp into or out of the socket sometimes make the two halves of the filament cross each other near the top. This short-circuits about one-third of the filament, and if the current is turned on, the lamp then burns at about three times the normal candle power. This, of course, greatly reduces the life of the lamps if the filaments are not separated. They may be shaken apart by tapping the lamp, but usually not before the filament has been materially weakened by burning at the high candle power. The filament of the metallized lamp is easily set to vibrating in an annoying manner. Often while working with them the vibration of the filaments was such that the flicker was easily seen on a piece of white paper at a distance of four or five feet from the lamp. It was only in some positions about the lamp that this was noticeable, but in these positions it was very pronounced and disagreeable. It seems to be caused principally by the reflections from the bulb of the lamp. The motion was in this way magnified. No such effect could be obtained from the carbon or tantalum lamps. The carbon filaments were stiff enough to resist the vibrations and the way in which the tantalum filaments were mounted prevented any vibration in them.

CONCLUSION

From the study of these lamps it appears that the carbon filament and the tantalum filament lamps can cover adequately all the phases of incandescent lighting that are now covered by the three types. For low power costs and for rough or unusual uses and for small candle power units the carbon lamp is best and often the only one that can be used. For higher costs of power upon poorly regulated circuits and for lightening the load upon overloaded stations the tantalum lamp is best. It is not recommended by its manufacturers for use upon alternating current, yet the results obtained show that although it does not do so well upon alternating current as it does upon direct current circuits, it still gives better economy for the higher power costs than the carbon lamp. The principal fault of the metallized lamp is that of mechanical weakness, which probably does not exist in the larger sizes where a heavier filament is used, so that for units of 40 or 60 candle power or above, this type of lamps is very satisfactory.

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Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. (*Out of print*).

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Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. (*Out of print*).

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906.

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Bulletin No. 10. Tests of Plain and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907.

Bulletin No. 11. The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. (*Out of print*).

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BULLETIN NO. 20

TESTS OF CONCRETE AND REINFORCED CONCRETE COLUMNS

SERIES OF 1907

BY

ARTHUR N. TALBOT

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 20 DECEMBER 1907

TESTS OF CONCRETE AND REINFORCED CONCRETE COLUMNS; SERIES OF 1907

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY
ENGINEERING AND IN CHARGE OF THEORETICAL
AND APPLIED MECHANICS

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I. INTRODUCTION

1. *Preliminary.*—In structures of reinforced concrete, columns and other members subject to direct compressive stresses are used. Usually these members are reinforced with longitudinal rods, and quite generally light ties connect these rods at intervals, though tests indicate that with the size and spacing usually adopted these ties add little or no strength to the column. Much interest has been manifested recently in columns reinforced with steel hooping in such a way as to give lateral restraint to the concrete. A variety of opinions has been expressed on the action and on the value of this type of reinforcement. Tests which have been reported show that the hooped column has great strength, though the results of the tests have not been conclusive on its applicability to general construction. In Bulletin No. 10 of the University of Illinois Engineering Experiment Station, Tests of Concrete and Reinforced Concrete Columns, Series of 1906, there were given the results of tests made on plain concrete columns and columns with longitudinal reinforcement. Further tests of plain concrete columns have now been made, and also tests of hooped columns. This bulletin will give the results of these tests, and it is hoped that it will throw some light on the properties of both plain and hooped concrete. It may be added that further tests are now in progress.

2. *Acknowledgment.*—The investigations herein described were made in the Laboratory of Applied Mechanics of the University of Illinois as a part of the work of the University of Illinois Engineering Experiment Station. Much of the work of making the tests on the columns was done as thesis work by Sidney Grear, Charles E. Hoff, and Clyde L. Mowder, senior students in Civil Engineering, class of 1907. Credit is due them for the thought and care given to this work. The immediate supervision of the work of making the columns and of the work of conducting the tests was given by D. A. Abrams, Associate in the Engineering Experiment Station. Many of the tests included in this work were made by Mr. Abrams and Mr. W. R. Robinson, First Assistant in the Engineering Experiment Station, and their aid in this and in interpreting the results and assisting in the preparation of this bulletin has added much to the value of the work. The bands used for hooping were furnished by Mr. Robert A. Cummings, Pittsburg, Pennsylvania, and were made by

The Electric Welding Company of Pittsburg, Pennsylvania. The wire spirals used for reinforcement were furnished for these tests by The American System of Reinforcing for Concrete Construction, of Chicago.

3. *Scope of Bulletin.*—Nineteen plain columns and thirty hooped columns were tested. The columns were generally cylinders 12 in. in diameter and 10 ft. long, but a few of other dimensions were used. The ratio of length to diameter ranged from 6 to 13, all being short columns. Tests were usually made at an age of 60 days, but a few tests were made at 6 months. In addition to the above, two columns remaining from the series of 1906 were tested at ages of 12 and 16 months. In the plain concrete columns tested, the strength of the column, the relation between the load and the deformation and the resulting modulus of elasticity have been determined. An effort was made to measure Poisson's ratio for concrete. For the hooped columns, both bands and wire spirals were used for reinforcement. Columns reinforced with bands will be referred to as band-hooped columns and those reinforced with wire spiral as spiral-hooped columns. The bands were of three weights: No. 8, No. 12, and No. 16 gauge, all 1 in. in width. The wire used in the spirals was No. 7 and ¼ in.; both high-carbon and mild-steel wire were used. The concrete was generally a 1–2–4 mixture, but in order to determine the effect of the restraint of the hoops upon richer and leaner concrete, two columns were made with 1–4–8 concrete and two with 1–1½–3 concrete. A feature of the tests of the hooped columns is the exhibition of shortening and of toughness. In the tests of the hooped columns, measurements were made to determine the amount of shortening due to the load. An attempt was made to determine the amount of the stress in the steel hooping. To throw a little light on the general phenomena, the concrete from one of the columns was tested after it had been loaded and then stripped of the spiral, and one of the columns was loaded eccentrically.

4. *Notes on Hooped Concrete.*—While it has long been known that lateral restraint adds to the resistance of materials in compression, M. Armand Considère, Inspecteur Gènèral des Ponts et Chaussées of France, was the first to utilize, develop, and investigate the construction formed by restraining the concrete by means of bands and spirals now generally known as hooped con-

crete (béton fretté). Experiments were made by Considère in 1902 which were communicated to l'Academie des Sciences in August and September 1902. A more extended account of these experiments was published in Genie Civil.* The tests of 1902 and 1903 were made upon octagonal columns, 5.9 in. short diameter, reinforced with spirally wound wire and longitudinal rods. In November 1904 a bridge of 65.6 ft. span, built for experimental purposes, was tested at Ivry, Paris. The tests and analyses of hooped concrete were given by Considère in a work describing his researches in reinforced concrete.†

In 1907 Considère published a monograph, "Le Béton Fretté et ses Applications", in which were collected and discussed various experiments on hooped concrete. In this Considère gave his revised conclusions on the properties and applicability of this form of reinforced concrete. A principal table gives the results of a part of the experiments made by Bach at Stuttgart and the second and third sets of experiments made by Considère under the direction of the Commission du Ciment Armè. These columns were octagonal and square, 200 and 275 mm. in diameter, and 1 m. and 4 m. long. The amount of the added strength given by the hooping and longitudinal reinforcement, as shown in this table, ranges from 525 to 880 lb. per sq. in. for each 1% of total reinforcement, based on the gross section of the column. The average value was 710 lb. per sq. in. Based on the area of the hooped core this would be about 880 lb. per sq. in. Another test cited is that of a column 700 mm. in diameter and 1.4 m. in length, reinforced with hooping equivalent to 2.73% of the concrete core, and having longitudinal reinforcement equal to 1.44% of the cross section. This column, which was tested by Professor Guidi, of Turin, carried 1 620 000 kilograms, or 6000 lb. per sq. in. of the gross area of the column, and 7650 lb. per sq. in. of the hooped core. Tests made with columns reinforced with three concentric helices gave very high results.

Considère considers that between the molecules of hooped bodies there is produced a phenomenon analogous to the frictional resistance in sand boxes employed for carrying the centers of bridges during construction, but he adds that cohesion in hooped bodies persists under pressure in spite of large deformations. In order to produce the maximum resistance the hooping should fill the following conditions: The hoops should be near enough together and should form a net-work so close that the concrete cannot escape through the meshes. In the earlier writings, he gives the maximum allowable distance between the spirals as $\frac{1}{5}$ to $\frac{1}{10}$ of the diameter of the helix. The hooped core should be full or not present voids of importance. The projection of the hoops on a plane perpendicular to the axis of the prism should be convex. He modifies the conclusions given in 1902 by attributing the increased strength to an increased cohesion of the particles as well as to intermolecular friction.

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*Genie Civil, v. XLII. No. 1, 2, 3, 4, 5, 6, and 9.

†Experimental Researches on Reinforced Concrete, Considere, translated by Leon S. Moisseiff, McGraw Publishing Company, New York.

Considère gives the following rule for the resisting strength of hooped columns: The compressive resistance of hooping members exceeds the sum of the following three elements, (1) the compressive resistance of the concrete core without reinforcement increased by 50% of itself, (2) the resistance of the longitudinal reinforcement stressed to its elastic limit, (3) the compressive resistance which would be produced by longitudinal reinforcement at the elastic limit of the hooping metal and equal to 2.1 times as great a volume as that of the hooping metal. He considers the factor 2.1 as very low but adopts it in accordance with the instructions issued by the Minister of Public Works of France, October, 1906.* He finally gives the following formula for computing the strength of a short hooped column,

$$C = 1.5\,c + 2400\,p + 5100\,p'$$

where C is the unit strength of the hooped column, c is the unit strength of a plain concrete column, p is the ratio of the longitudinal reinforcement to the concrete core, and p' is the ratio of the hooping to the concrete core. The area of the column is taken as that of the concrete core. All loads are here given in pounds per square inch. The formula above gives results which agree closely with the experimental values cited in the monograph.

Considère considers that hooped concrete has many advantages for numerous constructions. The case of posts and columns is given as an important application and it is considered that hooping will permit the use of leaner concrete or will allow a reduction of cross section. In beams the hooping may be utilized to increase the resistance to compression. Piles, compression pieces in bridges, and other forms of struts will show points of advantage. A strong feature is stated to be its extreme ductility and toughness and the fact that it exhibits signs of weakness before failure. The extreme shortening found is not considered troublesome, especially where the structure is articulated. He recommends a rather low factor of safety and considers that the experiments made have established the utility and safety of hooped concrete as a form of building construction. As before noted, the Minister of Public Works of France issued instructions authorizing the use of this form of construction.

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*Genie Civil, v. 50, p. 177.

Important tests on hooped concrete were made by Professor C. Bach, Director of the Materialprüfungsanstalt der Kgl. Technischen Hochschule in Stuttgart in 1905. The results of these tests were published in Mitteilungen über Forschungsarbeiten, No. 29, and are discussed in Morsch's Eisenbetonbau, page 69. The columns externally were octagonal in form with a short diameter of 275 mm. and a height of 1 m. The circles of the spirals had a diameter of about 235 mm. Longitudinal reinforcement was also used. Two sets of tests were made in which the pitch of the spiral was more than one third of the diameter of the core, and the increase of strength of these columns over that of plain concrete was only about half as much per 1% of reinforcement as was found in columns having the pitch of the spiral less than $\frac{1}{6}$ of the diameter of the core. The tests of the columns reinforced with the small pitch gave an increase in strength over plain concrete amounting on an average to 485 lb. per sq. in. for each 1% of total reinforcement used, when based upon the gross area of the column and 755 lb. per sq. in. when only the area within the hooping is considered. The range in the latter case is from 412 to 1070 lb. per sq. in. The results do not show very definitely how much of the increased strength is due to longitudinal rods and how much to spirals. In the monograph referred to above, Considère considers that only one set (9 columns out of 57) followed the requirements given in the instructions of the Minister of Public Works of France, and he omits the others from his analysis. The remaining columns, however, give considerable information.

Tests of Metals, 1906, gives the results of tests of hooped columns at the Watertown Arsenal by James E. Howard. These tests include quite a variety of reinforcement, with bands, spirals, and longitudinal bars and angles, and seem to indicate that the strength added by 1% of spiral reinforcement ranges from 600 to 1000 lb. per sq. in., based on the area within the hooping.

II. MATERIALS, TEST PIECES, AND METHODS OF TESTING.

5. *Materials.*—The materials used in making the columns were similar to the best materials used for this class of work in this section of the country. The stone, sand, and nearly all the cement were bought in the open market. Universal portland cement, used in two columns, was furnished by the makers, but the Chicago AA portland cement used in the remainder was bought of a local dealer. The bands were supplied by Robert A. Cummings of Pittsburg, Pennsylvania, and were made by The Electric Welding Company of Pittsburg, Pennsylvania. The American System of Reinforcing for Concrete Construction, of Chicago, furnished the steel spiral reinforcement.

Stone.—The stone was crushed limestone from Kankakee, Illinois. It was ordered to pass through a 1-in. screen and over a ¼-in. screen. The stone came in two shipments, and there was an appreciable difference in the quality of the two. The first lot was finer, softer, and of a darker color than the second lot. The harder, coarser stone of the second lot made much the better concrete. Tests showed that this last lot had about 50% voids and weighed 85 lb. per cu. ft. loose. Table 1 gives the average of several mechanical analyses of stone from the second lot.

TABLE 1.

MECHANICAL ANALYSIS OF STONE.

Size of Mesh	Per cent Passing
1¼-in.	100
1 "	95
½ "	35
¼ "	5
⅛ "	2

Sand.—The sand was of good quality, sharp, well graded, fairly clean, contained 28% voids, as determined by pouring sand into a vessel partly filled with water, and weighed 103 lb. per cu. ft. loose. It came from near the Wabash River at Attica, Indiana. The result of two mechanical analyses of this sand is given in Table 2.

Cement.—In all except two columns, Chicago AA portland cement was used. In two columns Universal portland cement was used. Table 3 gives the results of tension tests of these cements made according to standard practice.

TABLE 2.

MECHANICAL ANALYSIS OF SAND.

Sieve No.	Separation Size inches	Per cent Passing	
		First Trial	Second Trial
5	.174	99.6	99.6
10	.091	83.1	79.8
12	.067	74.8	71.0
16	....	67.4	62.5
18	.043	55.0	50.6
30	.027	36.7	32.9
40	.019	25.5	22.4
50	.013	14.5	13.7
74	.009	7.2	6.5
150	....	1.8	2.0
200	....	1.3	1.0

TABLE 3.

TENSILE STRENGTH OF CEMENT.

Ref. No.	Ultimate Strength, lb. per sq. in.							
	Chicago AA Cement				Universal Cement			
	Age 7 days		Age 28 days		Age 7 days		Age 28 days	
	Neat	1-3	Neat	1-3	Neat	1-3	Neat	1-3
1	786	230	811	265	410	187	680	370
2	683	145	851	235	470	200	670	330
3	760	218	861	225	360	120	560	360
4	786	186	760	270	405	145	570	290
5	866	215	965	265	320	195	600	295
6	815	211	935	287	310	180	620	310
Av.	783	201	864	258	379	171	617	326

Concrete.—Men accustomed to mixing concrete made the test specimens and an effort was made to have the concrete as good as would be used in the best practice. All materials were pro-

TABLE 4.

Tension Tests of Steel Used in Columns.

The values given for "Weld not in tested section" are averages of several tests.

Nominal Size or Number	Area of Section sq. in.	Yield Point pounds	Maximum Load pounds	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Per cent Elongation in 8 in.	Remarks
¼-in.	.049	5650	7430	115 500	152 000	1.5	High-carbon spiral
7	.024	1430	2170	59 700	90 500	12.2	High-carbon spiral
¼ in.	.046	2500	3380	54 100	73 000	15.5	Mild-steel spiral
7	.025	940	1310	38 000	53 000	16.4	“ “ “
Weld not in tested section.							
16	.063	3160	4270	50 300	68 100	21.5	9-in. bands
12	.127	5920	8160	46 600	64 300	17.5	“ “
16	.064	3000	3830	46 700	59 700	18.0	12-in. bands
12	.127	5460	7710	46 900	60 800	17.5	“ “
8	.180	8930	11640	49 800	65 000	20.2	“ “
Weld in tested section.							
16	.064	3010	3040	46 700	47 100	1	9-in. band. Broke at weld.
16	.062	3150	4190	50 600	62 400	15	9-in. band. Did not break at weld.
12	.137	6250	8300	45 900	60 900	7	9-in. band. Broke at weld.
12	.123	6160	8150	50 100	66 300	14	9-in. band. Did not break at weld.
16	.065	....	2730	....	41 900	....	12-in. band. Broke at weld.
12	.121	5700	7080	47 300	58 700	7	12-in. band. Broke at weld.
8	.188	9300	12120	49 300	64 400	17	12-in. band. Did not break at weld.

portioned by loose volume and weights were taken as a check on the measurement. The mixing was done with shovels by hand. The sand and cement were first mixed dry. The stone, which had previously been thoroughly moistened, was then added and the mass turned until uniform in appearance. Water was added in such proportion as to give a fairly wet mixture. This consistency permitted tamping into the forms to good advantage.

Steel.—The steel used in the hoops had a yield point of about 48 000 lb. per sq. in. and an ultimate strength of from 60 000 to 65 000 lb. per sq. in. The hoops were 1 in. wide and of three thicknesses, corresponding to No. 8, 12, and 16 gauge. For the spiral reinforcement, both high-carbon and mild-steel wire were used, each being in two sizes, ¼-in. and No. 7. Tests showed that the ¼-in. high-carbon wire had a yield point of approximately 115 000 lb. per sq. in. The yield point of the No. 7 high-carbon wire was about 60 000 lb. per sq. in. Their ultimate strengths were about 155 000 and 90 000 lb. per sq. in., respectively. The ¼-in. mild-steel wire had a yield point of 54 000 lb. per sq. in. and the No. 7 wire 38 500 lb. per sq. in. Their ultimate strengths were about 73 000 and 54 000 lb. per sq. in. respectively. Table 4 gives the results of tension tests made on the reinforcement, some of the specimens being taken from tested columns and some from unused material. The hoops were electrically welded and as it was desired to know the strength of the weld, pieces containing the weld were tested. Several broke at the weld at a point much below the ultimate strength, but with one exception at a load higher than the yield point of the metal.

6. *Test Specimens.*—An effort was made to have the conditions of fabrication of the different test specimens as nearly uniform as possible. Usually, the test columns were made in sets of three. Individual columns of a given set were made at different dates throughout the season, the purpose being to make columns of different sets at the same time and thus distribute accidental variations of fabrication over a number of sets. To give a check test on the quality of the concrete, 12-in. cubes, 6-in. cubes, and an 8 x 16-in. cylinder were made in many cases from the batch for a column. The 12-in. cubes were made in sets of two, and the 6-in. cubes in sets of three. The mixture varied from 1–1½–3 to 1–4–8, by far the largest number being of a 1–2–4

mixture. As noted above, there was a difference in the stone which affected the strength of the test specimens, those made of coarser and harder stone being the stronger. General data on all test specimens are given in Table 5, page 14.

Plain Concrete Columns.—Nineteen columns containing no reinforcement were tested. These columns were cylindrical, 12 in. in diameter and 10 ft. long. In two columns Universal portland cement was used; in the others Chicago AA was used. Two columns not included in the nineteen named above, No. 21 and 22, were from the lot made in 1906 at the same time as those described in Bulletin No. 10, and the materials, method of making, and other conditions were given in that bulletin. These columns were 12 in. square and 6 ft. long, and were 12 and 16 months old when tested.

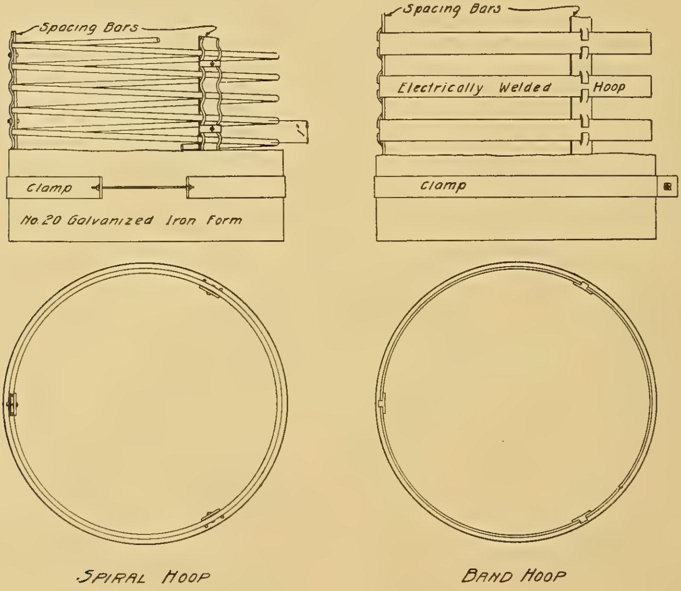

FIG. 1. FORMS AND REINFORCEMENT.

Band-hooped Reinforced Columns.—Twelve band-hooped columns were 12 in. in diameter and six were 9 in. in diameter. The length was 10 ft. The reinforcing consisted of circular bands of No. 8, 12, and 16 band steel 1 in. wide generally placed on 2 in.

centers. The spacing for one column was 3 in. centers and for two columns 4 in. centers. The bands were held in place by three longitudinal strips of thin metal suitably punched, as shown in Fig. 1.

Spiral-hooped Reinforced Columns.—Twelve spiral-hooped columns were tested. They were 12 in. in diameter and 10 ft. long. The reinforcement consisted of a single steel wire coiled in the form of a helix and spaced at a uniform pitch of 1 in. by means of heavy steel wires crimped as shown in Fig. 1. Both mild-steel and high-carbon-steel wire were used, the sizes being No. 7 and $\frac{1}{4}$-in. for each.

Cubes.—The cubes were made in two sizes, 12-in. and 6-in. These cubes were intended to aid in judging the quality of the concrete used in correspondingly numbered columns. The concrete for the cubes was taken from the middle of the batch for the correspondingly numbered columns and was thought to be representative. The concrete was well tamped in the forms and spaded around the edges to insure a good surface.

Cylinders.—The cylinders were all 8 in. in diameter and 16 in. high. The concrete was selected and tamped in the same way as for the cubes.

7. *Forms.*—The column forms were of galvanized sheet steel bent into a cylindrical shape and held in position by bands 1 in. wide and $\frac{3}{16}$ in. thick. The bands could be adjusted to the proper diameter by means of bolts. The forms were built in sections $2\frac{1}{2}$ ft. long and fitted together in stove-pipe fashion. The forms are shown with dimensions in Fig. 1. The forms for the 12-in. cubes were of the ordinary wooden type, two in a set; those for the 6-in. cubes were of steel, three in a set. The cylinder forms were of wrought iron with cast-iron bases, being the same as shown in Fig. 1 of Bulletin No. 10. All forms remained in place about ten days.

8. *Making of Columns.*—The concrete for each column and for the corresponding cubes and cylinders was mixed in one batch. The form was set up on a cast-iron base plate, 14 in. x 14 in. x $1\frac{1}{4}$ in., which was planed on both sides and served as a bearing plate in the column test. In making the plain concrete columns the forms were built up in $2\frac{1}{2}$-ft. sections, each section being filled before the next was added. The concrete was put in in layers of about 6

TABLE 5.

General Data of Test Specimens.

The length varied from 10 ft. 0 in. to 10 ft. 4 in. except for Columns No. 21 and 22, which were 6 ft. long.

Column No.	Diam. in.	Kind of Concrete	Kind of Stone	Reinforcement		Minor Specimens		
				Per cent	Size and Spacing	Cubes		Cyl.
						12-in.	6-in.	8-in. x 16-in.
101	12	1–2–4	Soft		None	2	...	....
102	"	"	Hard		"	...	...	....
103	"	"	"		"	2	...	....
104	"	"	"		"	2	...	....
105	"	"	"		"	...	6	1
108	"	"	"		"	2	3	1
109	"	"	"		"	2	...	....
110	"	"	"		"	2	...	....
111	"	1–1½–3	"		"	2	3	1
112	"	"	"		"	2	3	1
116	"	1–3–6	"		"	2	3	1
117	"	"	"		"	...	3	1
121	"	1–4–8	"		"	2	3	1
122	"	"	"		"	2	3	1
128	"	1–2–4	"		"	2	...	....
129	"	"	"		"	2	...	....
163	"	"	"		"	2	...	....
164	"	"	"		"	2	. .	1
168	"	"	"		"	1	...	1
21	12-in. sq.	1–2–3¾	....		"	...	...	....
22	"	"	....		"	...	...	....
131	12	1–2–4	Hard	1.08	No. 16 bands (mild steel) 2 in. cc.	2	3	....
132	"	"	"	1.08	do.	2	...	....
133	"	"	"	1.07	do.	2	...	....
136	"	"	"	2.08	No. 12 bands (mild steel) 2 in. cc.	2	3	....
137	"	"	"	2.08	do.	2	...	..
138	"	"	"	2.10	do.	2	3	1
141	"	"	"	1.02	No. 12 bands (mild steel) 4 in. cc.	2	...	...
142	"	"	"	1.03	do.	2	3	1
143	"	"	"	1.39	No. 12 bands (mild steel) 3 in. cc.	2	...	....
146	"	"	"	3.20	No. 8 bands (mild steel) 2 in. cc.	2	3	...
147	"	"	"	3.20	do.	2	3	1
148	"	"	"	3.20	do.	...	...	1
151	9	"	"	1.47	No. 16 bands (mild steel) 2 in. cc.	...	...	....
152	9	1–4–8	"	1.35	do.	2	3	1
153	9	"	"	1.41	do.	2	...	....
156	9	1–2–4	"	2.76	No. 12 bands (mild steel) 2 in. cc.	...	...	....
157	9	1–1½–3	"	2.94	do.	2	3	1
158	9	"	"	2.78	do.	2	...	....

TABLE 5.—*Continued.*

Column No.	Diam. in.	Kind of Concrete	Kind of Stone	Reinforcement		Minor Specimens		
				Per cent	Size and Spacing	Cubes		Cyl.
						12-in.	6-in.	8-in.×16-in.
171	12	1–2–4	Hard	0.85	No. 7 spiral (high steel)	2	...	....
172	"	"	"	0.85	do.	...	...	1
173	"	"	"	0.82	do.	...	...	....
176	"	"	"	0.84	No. 7 spiral (mild steel)	2	...	....
177	"	"	"	0.85	do.	2	...	....
178	"	"	"	0.84	do.	2	...	....
181	"	"	"	1.63	¼-in. spiral (high steel)	...	...	1
182	"	"	"	1.67	do.	2	...	.. .
183	"	"	"	1.68	do.	...	...	....
186	"	"	"	1.63	¼-in. spiral (mild steel)	2	...	1
187	"	"	"	1.71	do.	2	3	....
188	"	"	"	1.62	do.	...	...	.. .

in. and tamped or churned until water flushed to the top. In making the reinforced columns, the form was placed around the hoops or spirals for the full length of the column, after which it was set up vertically on the base plate. The concrete was put in from the top of the column and tamped or churned as described above. The reinforcement was usually covered by from $\frac{1}{16}$ in. to ¼ in. of mortar. As this outside coat spalled off early in the test and did not add appreciably to the strength of the column, the diameter of the column was assumed to be the same as the diameter of the band or spiral. Generally the column had a very good surface. In the band-hooped columns care was taken to place the bands so that welds were distributed over the circumference of the column. Three or four inches of the ends of the wire in the spiral-hooped columns were bent inward to form an anchorage. The metal in the reinforcement was gauged in several places and the average taken to compute the per cent reinforcement. The lengths of the columns were quite uniform, varying from 10 ft. 1 in. to 10 ft. 4 in.

9. *Storage of Test Specimens.*—The columns were built near the walls of the testing room of the Laboratory of Applied Mechanics and remained where made until time of the test. (See Fig. 2).

The forms were taken off 10 days after making and after that the columns were sprinkled with water twice daily until they were tested. The temperature of the room varied from 55° to 70° F. The 12-in. cubes were stored in the open air of the same room. The 6-in. cubes and cylinders were stored in damp sand.

10. *Summary of Test Pieces.*—Table 5 gives a list of all the test pieces. Specimens having corresponding numbers were made from the same batch of concrete.

11. *Testing Machines Used.*—The machine used in testing the columns and 12-in. cubes was the 600 000-lb. Riehle vertical screw machine of the Laboratory of Applied Mechanics. The slowest speed, 0.05 in. per min., was used, except on two columns where repeated loading was tried. Here a speed of 0.10 in. per min. was used. Fig. 3 shows a column in the machine in position for testing. The 6-in. cubes and the cylinders were tested in Olsen testing machines of 100 000 or 200 000 lb. capacity.

12. *Measuring Devices.*—The instruments used for measuring the shortening of the columns were the same as those used in the 1906 series of column tests and are very well shown in Fig. 3 and 4 of Bulletin No. 10. The yokes were usually placed to give a gauge length of 100 inches. The arrangement was such as to give practically two independent sets of readings. The measuring device consisted of a steel drum one-half inch in circumference, which rotated as the contact-rod moved. On the axis of the drum was a pointer which passed over a graduated dial. This instrument could be read directly to thousandths of an inch, and by a vernier to ten-thousandths of an inch.

The instruments used to measure the lateral expansion of the column were of two types. In one type, eight standards, each having four points of contact on the surface of the column, were distributed around the column and held together by elastic bands; a fine wire passed from standard to standard and was attached at either end to a dial extensometer from which readings were taken. Different devices were used to reduce the resistance to movement of the wire. This instrument gave the general expansion around the column. In the second type, contact was made at two points at the ends of a diameter of the column and the movement was magnified by transmission to the end of lever arms and there read on an extensometer dial. Fig. 3 gives a view of these instru-

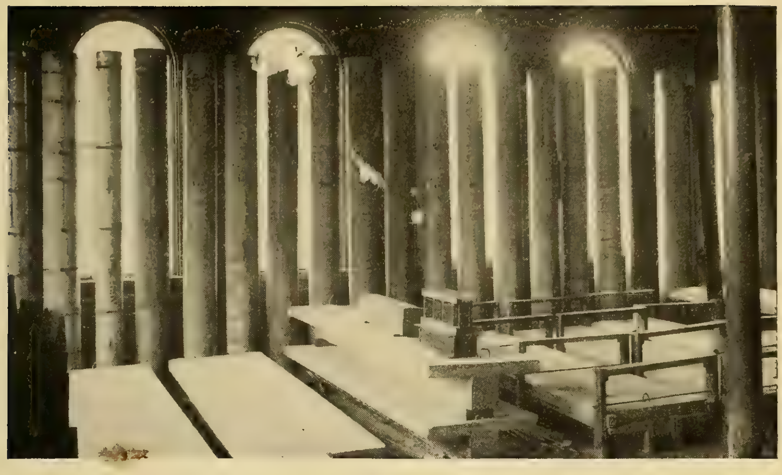

FIG. 2. VIEW SHOWING A NUMBER OF TEST COLUMNS.

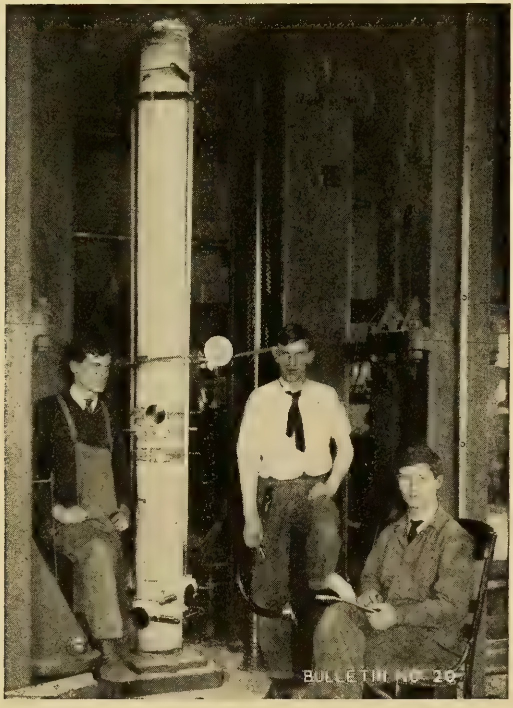

FIG. 3. VIEW SHOWING COLUMN READY FOR TEST.

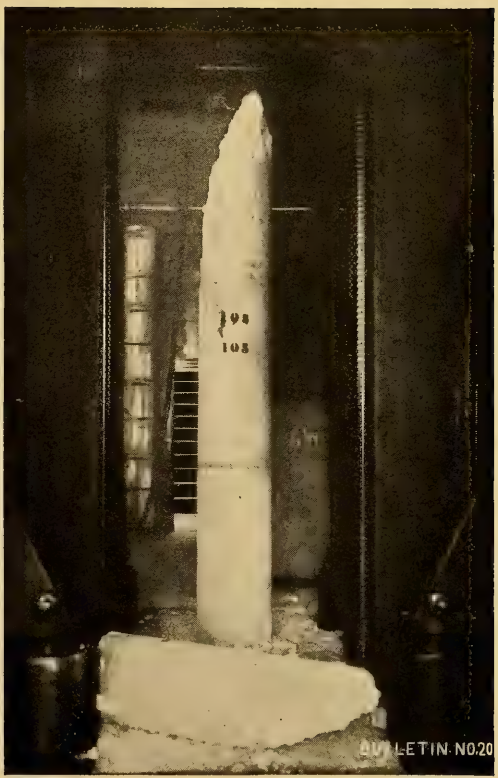

FIG. 4. VIEW SHOWING DIAGONAL SHEARING FAILURE OF PLAIN CONCRETE COLUMN.

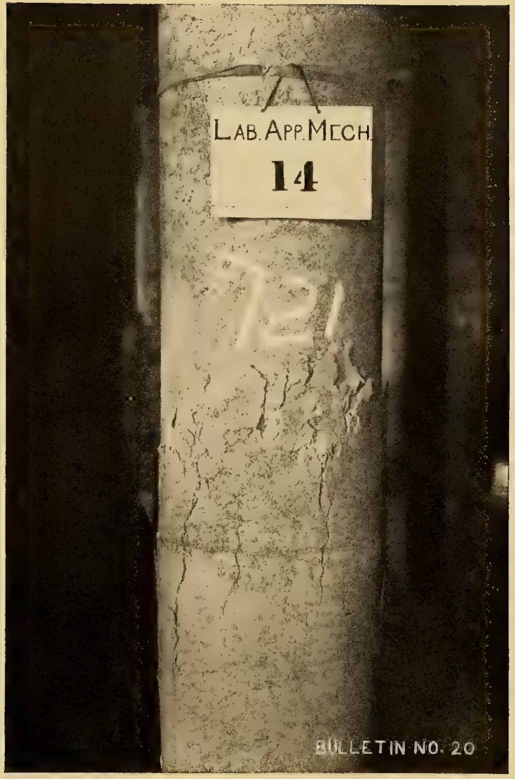

FIG. 5 VIEW SHOWING SIMPLE COMPRESSION FAILURE OF PLAIN CONCRETE COLUMN.

TABLE 6.

CUBE TESTS.

No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.	No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.
101_1	1–2–4	143.0	110	1890	122_1	1–4–8	144.0	79	1255
101_2	"	144.6	110	1875	122_2	"	146.2	79	1140
103_1	"	145.1	186	3120	122_3	"	36.0	288	1550
103_2	"	144.6	186	3560	122_4	"	36.0	289	1800
104_1	"	144.0	72	1960	122_5	"	36.0	303	1970
104_2	"	144.0	72	2110	128_1	1–2–4	144.0	200	1830
105_1	"	36.0	335	3750	128_2	"	144.8	200	1870
105_2	"	36.0	335	3200	129_1	"	142.5	200	1690
105_3	"	36.0	335	3840	129_2	"	139.8	200	1860
105_4	"	36.0	335	3320	163_1	"	144.8	185	2620
105_5	"	36.0	335	3280	163_2	"	144.0	185	2750
105_6	"	36.0	335	3970	164_1	"	145.1	184	2560
108_1	"	141.6	62	1880	164_2	"	144.6	184	2500
108_2	"	143.2	62	1850	168_1	"	144.8	185	2370
108_3	"	36.0	294	3180	131_1	"	144.0	104	2215
108_4	"	36.0	294	2780	131_2	"	144.0	104	2215
108_5	"	36.0	296	2570	131_3	"	36.0	330	3990
109_1	"	144.0	186	2950	131_4	"	36.0	311	4260
109_2	"	144.6	186	3170	131_5	"	36.0	311	4310
110_1	"	144.8	210	2250	132_1	"	144.0	105	2220
110_2	"	144.8	210	2530	132_2	"	143.5	104	2020
111_1	1–1½–3	141.6	90	2400	133_1	"	144.0	95	2180
111_2	"	141.6	90	2470	133_2	"	144.0	95	2170
111_3	"	36.0	311	3470	136_1	"	144.0	103	2200
111_4	"	36.0	312	4100	136_2	"	144.0	103	2170
111_5	"	36.0	325	4170	136_3	"	36.0	328	3110
112_1	"	144.0	69	2600	136_4	"	36.0	335	3140
112_2	"	144.0	69	2600	136_5	"	36.0	338	3510
112_3	"	36.0	292	4000	137_1	"	143.5	99	1440
112_4	"	36.0	295	4730	137_2	"	144.0	99	1570
112_5	"	36.0	295	4340	138_1	"	144.0	72	1830
116_1	1–3–6	145.8	84	1440	138_2	"	144.0	77	2200
116_2	"	145.8	84	1460	138_3	"	36.0	298	2960
116_3	"	36.0	305	2480	138_4	"	36.0	299	2920
116_4	"	36.0	306	2490	138_5	"	36.0	305	2880
116_5	"	36.0	306	2260	141_1	"	143.0	105	2570
117_1	"	36.0	292	2490	141_2	"	144.0	105	2140
117_2	"	36.0	294	2360	142_1	"	145.0	92	1710
117_3	"	36.0	294	2520	142_2	"	144.0	92	1700
121_1	1–4–8	144.6	89	795	142_3	"	36.0	311	3340
121_2	"	144.0	96	990	142_4	"	36.0	312	3320
121_3	"	36.0	305	1410	142_5	"	36.0	333	3820
121_4	"	36.0	306	1630	143_1	"	144.0	93	2900
121_5	"	36.0	315	1410	143_2	"	142.8	93	2570

TABLE 6.—*Continued.*

No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.	No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.
146_1	1–2–4	144.0	100	1610	157_5	1-1$\frac{1}{2}$-3	36.0	291	3000
146_2	"	144.0	100	1580	158_1	"	144.8	37	2630
146_3	"	36.0	326	2890	158_2	"	144.0	37	2510
146_4	"	36.0	336	2680	171_1	1–2–4	144.0	101	2810
146_5	"	36.0	336	2930	171_2	"	144.6	101	2640
147_1	"	142.5	92	2420	176_1	"	143.3	106	1990
147_2	"	144.5	102	2280	176_2	"	142.8	99	1500
147_3	"	36.0	332	3830	177_1	"	145.2	92	1840
147_4	"	36.0	321	3660	177_2	"	145.9	92	1780
147_5	1–4–8	36.0	312	3700	178_1	"	144.0	69	1410
152_1	"	145.8	59	1100	178_2	"	144.6	69	1530
152_2	"	143.0	60	950	182_1	"	144.0	93	2720
152_3	"	36.0	280	1620	182_2	"	144.0	93	2800
152_4	"	36.0	286	1670	186_1	"	144.6	99	1450
152_5	"	36.0	289	1630	186_2	"	144.6	102	1410
153_1	"	143.5	245	960	187_1	"	144.6	86	1630
153_2	1-1$\frac{1}{2}$-3	145.2	245	985	187_2	"	144.6	86	1730
157_1	"	145.0	57	1880	187_3	"	36.0	296	3390
157_2	"	144.0	55	2130	187_4	"	36.0	308	3800
157_3	"	36.0	286	3400	187_5	"	36.0	300	3900
157_4	"	36.0	289	3060					

TABLE 7.

CYLINDER TESTS.

No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.	No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.
105	1–2–4	50.3	338	3480	138	1–2–4	50.3	298	2130
108	"	50.3	281	2670	142	"	50.3	311	2320
111	1–1$\frac{1}{2}$–3	50.3	309	2620	147	"	50.3	311	2910
112	"	50.3	292	3700	148	"	50.3	295	2280
116	1–3–6	50.3	305	1540	152	1–4–8	50.3	279	935
117	"	50.3	294	1260	157	1–1$\frac{1}{2}$–3	50.3	276	2470
121	1–4–8	50.3	306	785	172	1–2–4	50.3	315	3140
122	"	50.3	288	1140	181	"	50.3	319	2990
168	1–2–4	50.3	185	1790	186	"	50.3	318	1950

ments. These instruments were placed near the middle of the length of the column with the contact points resting sometimes on the metal in the reinforced columns and sometimes on the concrete. The measurements taken are not as uniform or as trustworthy as could be desired, and the work must be considered as preliminary in character.

13. *General Method of Testing.*—A few days before testing, a plate similar to the base plate described above was set on a plaster of paris cushion on top of the column and served as the top bearing plate in the test. The top and bottom plates were connected with rods, and the columns were carried to the machine in a vertical position. The load was applied through a spherical bearing block which was carefully centered on the column. On the hooped columns, the load was applied in 20 000-lb. increments almost up to the maximum when 10 000-lb. increments were used. On the 9-in. columns the increments were made smaller. On the plain columns the load was run up from the start 10 000 lb. at a time. At each increment the machine was stopped while readings of the instruments were taken.

One 12-in. spiral-hooped column, No. 188, was tested under eccentric loading. A ½-in. square mild steel bar was placed under the bearing blocks at top and bottom, each 1¼ in. off center, to obtain this condition.

III. Experimental Data and Discussion.

14. *Cube and Cylinder Test Data.*—Tables 6 and 7, page 18, give data of the tests of cubes and cylinders made from the same batch of concrete as the columns having the corresponding numbers. It was found impracticable to make many of these tests at the age at which the columns were tested and the tests were made at various ages as shown in the tables. The data do not generally allow a comparison to be made between the strength of the cubes and cylinders and that of the corresponding columns, though in the instances where the age of test makes the data comparable it is seen that the strength of the cube is generally greater than that of the column and that the strengths of the column and of the cylinder more nearly agree. The results for cubes from the same batch of concrete show great uniformity. The variety of age at test throws light upon the additional strength gained with time. The gain in strength is quite marked and is perhaps more than the gain usually expected.

15. *Column Test Data.*—Table 8, page 20, gives data on the age of test, maximum load and manner of failure of the plain concrete columns. Table 10, page 34, gives data of the tests of band-hooped columns and Table 12, page 36, the data of tests of spiral-hooped columns. The plain concrete columns will be considered first.

16. *Phenomena of the Tests of Plain Concrete Columns.*—The columns generally failed in the middle half of their length and the position and manner of failure showed general freedom from defects. No. 106, however, failed at the top with a load of 677 lb. per sq. in., and an examination showed that the concrete at this point was quite defective, lacking in solidity and strength, and that the column was far from being representative. The upper end of this

TABLE 8.

SUMMARY OF TESTS OF PLAIN CONCRETE COLUMNS.

Col. No.	Kind of Concrete	Maximum Load		Age at Test days	Manner of Failure
		Total pounds	lb. per sq. in.		
111	1–1½–3	240 000	2120	68	Diagonal shearing without warning
112	"	281 000	2480	62	" " " "
101	1–2–4	132 000	1165	58	Diagonal shearing
102	"	218 000	2000	69	General crushing
103	"	250 000	2210	65	" "
104	"	180 000	1590	64	" "
105	"	220 000	1945	62	Diagonal shearing
108	"	165 000	1460	72	General crushing
109	"	205 000	1810	64	Sudden diagonal shearing
116	1–3-6	108 000	955	61	Slow general crushing
117	"	126 000	1110	62	General crushing
121	1–4–8	65 000	575	63	Slow general crushing
122	"	65 000	575	63	" " "
110	1-2-4	218 000	1925	203	General crushing
128	"	209 000	1845	194	" "
129	"	200 000	1770	181	" "
163	"	303 000	2680	187	Sudden diagonal shearing
164	"	245 000	2160	187	Diagonal shearing
168	"	200 000	1770	201	General crushing
21	1-2-3¾	381 500	2650	12 mo.	Diagonal shearing
22	"	389 000	2770	16 mo.	General crushing

column was afterwards cut off and another test made with results which showed that the remainder of the column was of normal composition. The results of No. 106 are not included in the data here given. In general, two somewhat distinct forms of failure were observed, (a) a failure which may be termed a diagonal shearing failure, and (b) a failure which may be termed simple compression. In the first the fracture was angular in nature, having the appearance of a diagonal shearing failure characteristic of the manner of failure in compression so common in brittle materials. In these failures the columns broke suddenly and without warning, some of them breaking even after the machine had been stopped and while no additional load was being applied. A loud report accompanied the failure. Column No. 112 failed suddenly and had given so little warning that the instruments had not been removed. In the second form of failure, here called "simple compression", the column shattered, cracking longitudinally for some distance, the cracks being quite well distributed over the faces of the column. In some of these, the failure was not noted until the weighed load began to decrease, and the position of the failure was not determined until the machine had produced a further shortening of the column. As will be seen in Table 7, nearly all the richer mixtures (1–1½–3 and 1–2–4) gave diagonal shearing failures, and the columns made with lean mixtures crushed and shattered throughout the whole fracture in what are here termed simple compression failures. Fig. 4 is a view of a typical diagonal shearing failure and Fig. 5 one of a simple compression failure. Of course, even the simple compression failures would be produced suddenly with a free dead load, but the action in the testing machine was quite distinct from the other. For both forms of failure, it may be stated in general that the approach of the ultimate strength of the columns might have been predicted from the increase in the rate of shortening and from the shape of the load-deformation diagram.

17. *Stress-deformation Diagrams for Plain Concrete Columns.*—In Fig. 16 to 20 following the text, the stress-deformation diagrams given represent the observed loads and the corresponding unit-deformations or shortenings for the columns tested. The method used in Bulletin No. 10 is followed. The ordinates represent the load or pressure in pounds per square inch on the col-

umn and the abscissas the unit-deformations or shortenings for a unit of length, as determined from the observed extensometer readings for the gauged length used. These values are taken from the average of the readings on the four faces of the column. The amount of deformation was calculated by using as the zero reading the extensometer reading taken at the original zero of load or load at which the first reading was taken; that is to say, the deformations shown are independent of any change or set which the load may have produced in the concrete. This is considered the proper method since one purpose of the determination of the shortening is for application to the case of a column with longitudinal steel reinforcement. This use of gross deformation and not of elastic deformation is discussed more at length in Bulletin No. 10.

18. *Strength of Plain Concrete Columns.*—In Table 8 in the column "Maximum Load" is given the ultimate load taken by the plain concrete columns. It will be noted that the richer mixtures give considerably higher strength than the leaner ones, the average at ages not far from 60 days being as follows: 1–1½–3 concrete, 2300 lb. per sq. in., 1–2–4 concrete, 1740 lb. per sq. in., 1–3–6 concrete, 1030 lb. per sq. in., and 1–4–8 concrete, 575 lb. per sq. in. In Fig. 6 this increase in strength with an increased proportion of cement is shown graphically, the weight of the cement being

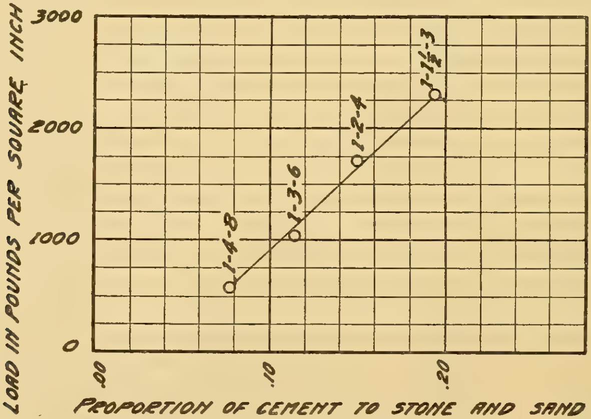

FIG. 6. RELATION BETWEEN STRENGTH OF CONCRETE COLUMNS AND AMOUNT OF CEMENT.

TABLE 9.

Properties of Plain Concrete Columns.

Col. No.	Age at Test days	Kind of Concrete	Initial Modulus of Elasticity lb. per sq. in.	Abscissa of Vertex of Parabola	Maximum Stress lb. per sq. in.	
					Parabola	Observed
111	66	1–1½–3	2 800 000	.0015	2100	2120
112	62	1–1½–3	3 890 000	.0013	2530	2480
101	58	1–2–4	2 230 000	.0011	1225	1165
102	69	1–2–4	3 350 000	.0012	2010	2000
103	65	1–2–4	3 440 000	.0013	2240	2210
104	64	1–2–4	3 390 000	.00095	1610	1590
105*	62	1–2–4	2 660 000	.0014	1860	1945
108*	72	1–2–4	3 380 000	.0009	1520	1460
109	64	1–2–4	2 830 000	.0013	1840	1810
116	61	1–3–6	....	....	....	955
117	62	1–3–6	....	....	....	1110
121	63	1–4–8	....	....	....	575
122	63	1–4–8	....	....	....	575
106	286	1–2–4	....	....	....	1460
110	203	1–2–4	3 270 000	.0012	1960	1925
128	194	1–2–4	3 140 000	.0012	1880	1845
129	181	1–2–4	3 600 000	.0010	1800	1770
163	187	1–2–4	3 000 000	.0017	2550	2680
164	187	1–2–4	3 100 000	.0014	2170	2160
168	201	1–2–4	2 770 000	.0013	1800	1770

*The columns marked (*) were made with Universal cement, all others with Chicago AA.

given in terms of the combined weights of sand and stone. The results for each mixture are the average of from two to five test columns. It is evident that cement is an effective reinforcing material and that as such it will compete very favorably with steel where compressive stresses only are to be taken. The addition of cement will give added strength quite out of proportion to the additional cost of the material. It is estimated that a mixture of 1–1½–3 would cost, say, 10% to 15% more than a 1–2–4 mixture, while the strength would be increased about 32%. A similar relation holds between the 1–3–6 and the 1–2–4 mixtures. The smaller sizes of columns necessary in the case of rich concrete may for the lower floors of a building effect a considerable saving in floor space, of itself an advantage.

A comparison of the strength of 1–2–4 concretes for the group tested at an age of about 60 days and for the group tested at an age of about 6 months shows that the strength of the concrete has increased in the interval something like 15%, though the variations in quality of stone and accidental variations make this comparison somewhat uncertain. The stone which is marked "soft" was much poorer in quality then that called "hard", and the effect of the quality of the stone used is quite apparent in the results of the tests.

It will be noted that the range of results is similar to that found in the plain concrete columns described in Bulletin No. 10. For the 1–2–4 concrete tested at an age of about 60 days the extreme range is 27% above and 34% below the average of the tests, and the average variation from the average strength is 12%. The strength of the 1–2–4 concrete was higher than the average for the 1906 tests, and this higher strength is due partly to the use of harder stone and partly to other changes in conditions. As in the 1906 tests, the ultimate strength was determinable with some accuracy by the location of the vertex of the parabola drawn to fit approximately the stress-deformation diagram. The loads corresponding to the vertex of the assumed parabola are given in Table 9 in the column headed "Parabola".

Attention has already been called to the fact that the strength of the columns is generally less than that of the cubes of the same mix and age and that the strengths of the columns and cylinders agree fairly well.

19. *Modulus of Elasticity.*—The treatment of modulus of elasticity of concrete will be the same as that given in Bulletin No. 10, Tests of Concrete and Reinforced Concrete Columns, Series of 1906. A discussion on the stress-deformation relation of concrete is also given in Bulletin No. 10. As there stated, modulus of elasticity is a term which has been used very loosely in connection with reinforced concrete. As a general property of materials, it is defined to be the ratio of the unit stress to the unit deformation within the elastic limit of the material. As applied in this way to materials having the property of proportionality of stress and deformation, the modulus of elasticity is a constant. For materials with a variable stress-deformation relation like concrete, the modulus of elasticity, as sometimes used, is still con-

sidered to be the ratio of the unit stress to the unit deformation regardless of the amount of the load carried relatively to the ultimate strength. As thus defined the modulus of elasticity of concrete will be a variable, and the value will decrease rapidly toward the ultimate strength of the concrete. For ordinary concretes, the stress-deformation relation may be considered to approach closely to a parabola with its vertex at the ultimate load, as was shown in Bulletin No. 10 and as is also apparent in the stress-deformation diagrams in this bulletin. The rate of change of stress and deformation at the beginning of loading is shown by the tangent to the parabola at the initial load and the value of the modulus of elasticity indicated by this tangent is called the initial modulus of elasticity. The values of the ratio between stress and deformation for the first one-third of the strength of the concrete do not vary much from this initial modulus of elasticity, but they are somewhat smaller for loads beyond this, the value of the ratio decreases rapidly and at the maximum load the ratio is only about half of the initial modulus of elasticity. It seems better to determine the initial modulus of elasticity, and then to obtain the relation between stress and deformation for other loads by means of a law which may be considered to express the changing relation. Formulas to show the relation between the stress and the deformation are given on page 27 of Bulletin No. 10, and a further discussion is also given on page 38 of that bulletin. It should be remembered that the value of the stress-deformation ratio for light loads on the concrete is somewhat smaller than the initial modulus of elasticity, so that if a straight-line stress-deformation relation (constant modulus of elasticity) is used for working loads, the value selected should be less than the initial modulus of elasticity. This must not be taken to mean that for relatively low loads the constant modulus of elasticity will give results which are greatly in error.

Comparison may then be made by means of the initial modulus of elasticity. The values of the initial modulus of elasticity for the several plain concrete columns tested are given in Table 8. The average for the 1–2–4 concrete at 60 days is 3 040 000 lb. per sq. in. This average is larger than that given in Bulletin No. 10 for the columns tested in 1906, which was 2 250 000. It was apparent from examination that the concrete in the 1907 tests was denser than that in the 1906 tests and, besides, the stone used was generally a harder material. This may explain in part the higher value of the modulus of elasticity.

The effect of richness of concrete upon the value of modulus of elasticity is of interest. The values for the richer mixtures are given in Table 9. The leaner mixtures are somewhat indefinite, but it is evident that the addition of cement gives much stiffer concrete.

The results for 1–2–4 columns tested at an age of 181 to 203 days give about the same value for the initial modulus of elasticity as those at 60 days.

20. *Poisson's Ratio.*—When a load is applied to a column or other test piece in compression, the column shortens longitudinally and at the same time expands laterally or transversely. The ratio between the lateral unit-deformation involved in this lateral expansion and the unit-deformation or the longitudinal shortening of the piece is termed "Poisson's Ratio". Poisson's ratio is of interest in connection with the effect of steel hoops in hooped concrete columns for loads within the compressive strength of plain concrete. It is also of interest in studying the relation between tensile and shearing stresses and the compressive strength of brittle materials. Little definite information on the value of Poisson's ratio for concrete is available. The values most frequently quoted are 0.25 and 0.3.

In the tests of the plain concrete columns, observations were made to determine a value for Poisson's ratio. Necessarily the apparatus and methods used were experimental in character. Refinement of measurement is essential, and any lack of rigidity in the apparatus or inertia of parts will interfere with the accuracy of the data. The results show inherent defects, but their general trend may serve to throw light on this property of concrete. Improvements in the instruments have now been made, and other tests are now under way from which a more definite value of Poisson's ratio for concrete may be expected.

From a study of the observations obtained, it would appear that Poisson's ratio for 1–2–4 concrete 60 days old for the lower loads is probably $\frac{1}{10}$ to $\frac{1}{6}$ and that near the ultimate load the ratio increases, probably reaching $\frac{1}{4}$. Tests were made with better apparatus on concrete columns 6 months old. For Column No. 163

the value is about 0.18 and for Column No. 164, 0.11 to 0.18. Still later tests give values considerably lower than these. The amount of the lateral deformations is shown in Fig. 20. It should be remembered that this concrete set in air.

21. *Repetitive Loading of Plain Concrete Columns.*—In most of the tests the column was loaded progressively to failure; that is, without releasing the load during the test. When a load is applied in any amount and released, the load then reapplied and this released, and this application and release of the load repeated, the longitudinal deformation of the concrete generally increases, and there is also an increase in the set when the load is released. This statement is true even for low loads. For the smaller loads there is a limit to the increase in deformation due to the repetition of the load. The total deformation and the resulting set soon become constant or nearly constant. For the higher loads the deformation and set continue to increase during a larger and larger number of applications, and failure will occur under repetitive

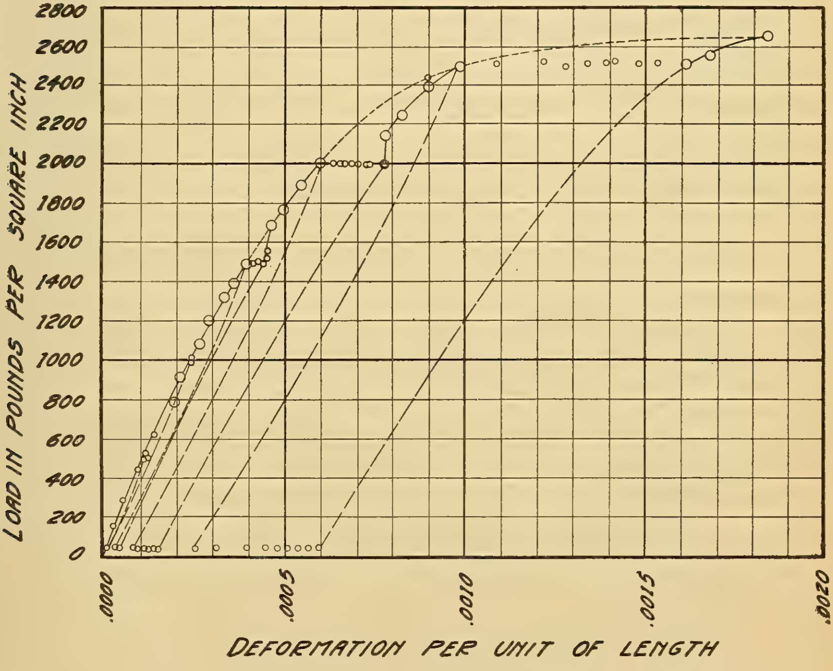

FIG. 7. STRESS-DEFORMATION DIAGRAM FOR CONCRETE COLUMN NO. 21.

loading at a load below the maximum strength for a single application. The number of repetitions necessary to produce a limit to the increase, and the amount of the increase of deformation, will of course vary with the quality and the age of the concrete. The resulting net deformation is generally called the elastic deformation, though it must be borne in mind that the proper-

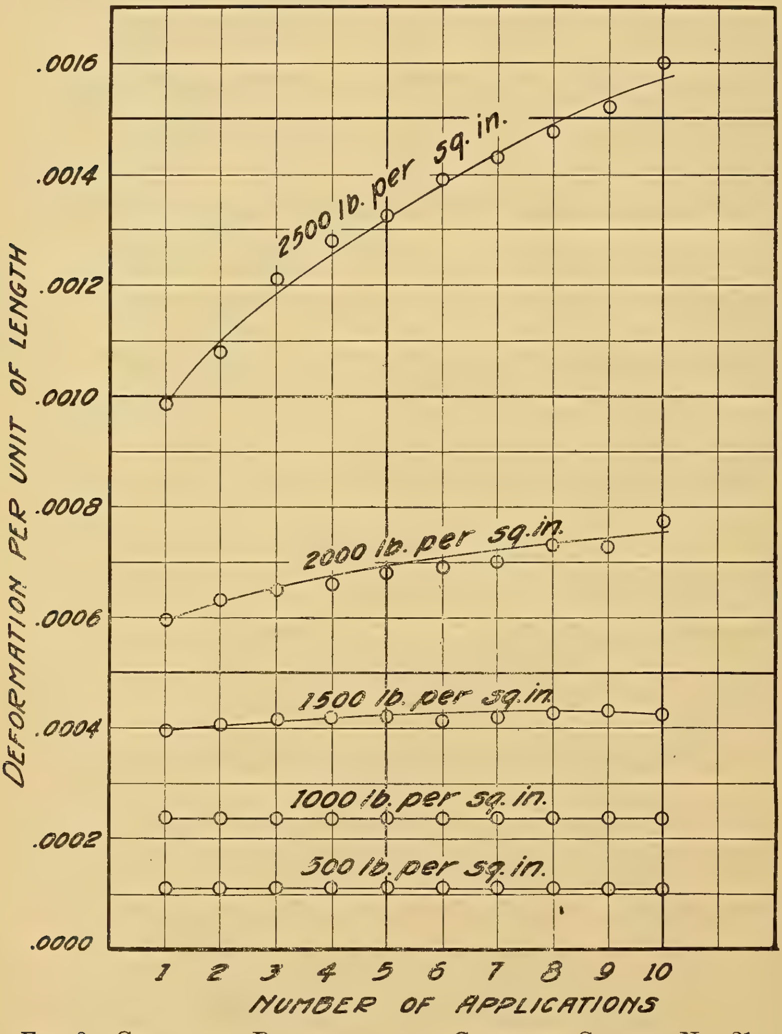

FIG. 8. CHANGE IN DEFORMATION IN CONCRETE COLUMN NO. 21.

ties of concrete found by using the elastic deformation are not generally applicable in the analysis of reinforced beams and columns. Whenever, after the repetition of a given load, the load is increased, the stress-deformation curve at once rises until it joins the general direction of the stress-deformation curve for a single application of the increased load. If the points for the last repetition of each load be joined, a curve having the same general form as the curve for single applications will be obtained, but with greater deformations. This general effect is illustrated graphically in Fig. 11, page 40, of Bulletin No. 10.

Columns No. 21 and 22 were tested by applying and releasing a load 10 times, then applying a greater load 10 times, and so on to the failure of column. The change in deformation and in set is shown in Fig. 7. The increase of the deformations with the repetition of the several loads for Column No. 21 is shown in Fig. 8. It will be noted that for the smaller loads the deformation increases very slowly after a few repetitions. For the higher loads the deformation continues to increase. Evidently at the upper loads a large number of repetitions would finally produce failure. The change in the set follows the same law.

22. *Phenomena of Tests of Hooped Columns.*—The general phenomena accompanying the tests of hooped columns are as follows: The earlier part of the test is much the same as for plain concrete columns. At a load equal to that which would cause failure in a plain concrete column, or a little above, the concrete over the spacing bars begins to scale, and this is soon followed with a scaling and shelling off of the surface of the column over the hoops everywhere. With added increments of load beyond the limit of strength of plain concrete the amount of shortening increases rapidly and the column expands or bulges laterally correspondingly rapidly. The maximum load in some cases may not be reached until the hooping has been stressed to its elastic limit, and in other cases this load may be reached without developing the elastic limit of the hooping, especially when large lateral deflection of the column and general distortion from the original shape occur. The lateral deflection of the column from a straight line is apparent before the maximum load carried is reached —in many cases considerably before. After the maximum load is passed, the lateral deflection rapidly increases. During the

last stage of the test the column usually forms a curve having the characteristics of the figure shown in text books for a column with fixed ends. Fig. 9 and 10 give views of hooped columns after the maximum load has been passed. Some columns finally bent out of line as much as 4 or 5 inches, although the load then carried was only a small proportion of the maximum and this condition could not obtain under a dead load.

There was a variation in the time of the first noticeable lateral bending in the tests. Generally speaking, in the case of companion columns, this bowing was perceptible first in the one which gave the lowest maximum strength. This load at which bowing became apparent ranged from 60% to 85% of the maximum load of the column in the case of the band-hooped columns and from 70% to 95% in the case of the spiral-hooped columns, though in the latter the average was nearly 90%. However, it was evident that slight bending began earlier than this. The amount of this bowing at the time of the maximum load was not large, but its presence, or the presence of the conditions which caused the bending, must have had much to do with the final failure of the column. In most of the tests, after passing the maximum load, the bending increased rapidly under loads smaller than the maximum and the column can not be considered to have failed by direct crushing.

The amount of shortening in the hooped column at the maximum load, as compared with plain concrete columns at their maximum load, was very great. The total amount of shortening in a hooped column 10 ft. long ranged from $\frac{3}{4}$ in. to $1\frac{3}{4}$ in., and the corresponding longitudinal unit-deformation was from 0.006 to 0.015. This is six to twelve times the shortening in a plain concrete column at its maximum load. The amount of set produced in the column is very great. In Column No. 173, as shown in Fig. 24, for a load well below the ultimate strength of the column, this set was 0.0025 per unit of length, or over two thirds of the total unit shortening of the column at the first application of the load.

The purpose of reinforcing concrete columns with hooping is to restrain the concrete laterally and to add to the strength of the column by preventing the disintegration of the concrete when that limit of lateral expansion has been reached for which in plain concrete rupture is produced by the internal tensile or shearing stresses developed. A little study will show that the effect of the hooping for loads below the ultimate strength of plain concrete must be relatively small. Consider that the load on the column is 800 lb. per sq. in., about one-half of the ultimate strength of the plain columns for the 1–2–4 columns 60 days old of the tests under consideration. The amount of shortening for a concrete column at half its ultimate load may be considered to be 0.0003 units per unit of length (inches per inch of length). If we consider that Poisson's ratio for this concrete is $\frac{1}{8}$, the lateral expansion for the column will be 0.0000375. Even if we consider that the steel hooping would receive this unit deformation the stress in the hooping would be only 1100 lb. per sq. in., and reasoning from this it seems evident that the effect of the hooping for lateral deformations of this amount is very slight. A comparison of the stress-deformation diagrams for plain columns and for hooped columns shown in succeeding pages bears this out. It seems probable that voids left in the concrete may interfere with early pressure on the hooping. For columns reinforced with steel rods placed longitudinally in the column, with a load on the column giving a shortening of 0.0003, the stress in the steel would be 9000 lb. per sq. in., a much larger amount proportionally. For hooped columns, if the value of Poisson's ratio remains at $\frac{1}{8}$ for higher loads, the stress in the hooping would increase somewhat proportionally. When the longitudinal shortening became 0.001, the steel in the hoops would be stressed 3750 lb. per sq. in. If the value of Poisson's ratio increases as the load at which plain concrete would fail is approached, considering it to be 0.25 at this point and considering also that the longitudinal shortening of the column at this load is 0.001, the stress in the hooping would become 7500 lb. per sq. in. Beyond the load which would produce failure in unrestrained concrete the conditions in the columns change materially, as is shown in succeeding pages, and the hooping receives additional stress rapidly and its function in maintaining the integrity of the concrete is important. It is to be expected that if other conditions were maintained the limit of strength of the column would be reached when the hoops become stressed to their yield point. It is also to be expected that if the hooping becomes pressed into the concrete, or if by reason of lack of homogeneity or other cause the column bends or deflects laterally and the pressure is unevenly distributed, the maximum load may be reached before the hoops can become stressed to their yield point. The tests indicate that such conditions exist.

23. *Stress-deformation Diagram for Hooped Columns.*—Stress-deformation diagrams for hooped columns are given in Fig. 21 to 25 at the end of the text. The same general notation is followed in these diagrams as is used in the diagrams for plain concrete columns. Fig. 11, page 32, shows Column No. 138, which was reinforced with steel bands and which may be considered to be typical of the banded columns. Fig. 12, page 33, shows Column No. 182 which was reinforced with spiral hooping and which may be considered to be typical of the spiral-hooped columns. An examination of all the diagrams shows that the stress-deformation diagram given by hooped columns is much the same as that given by plain concrete columns up to that load which would cause failure in a plain concrete column and up to an amount of shortening in the column, of, say, 0.001 or 0.0015, unit-deformation. That is to say, the comparison shows a very close relation between loads and shortenings for the two classes of columns up to the point of

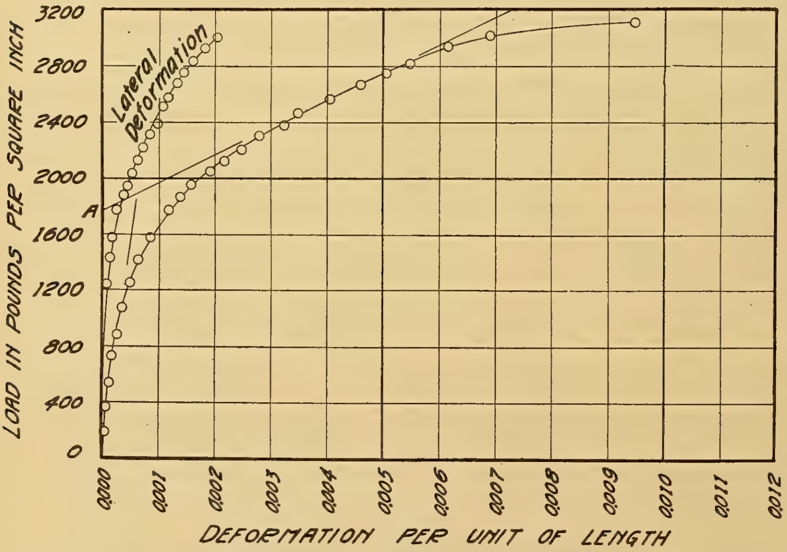

FIG. 11. STRESS-DEFORMATION DIAGRAM FOR BAND-HOOPED COLUMN No. 138.

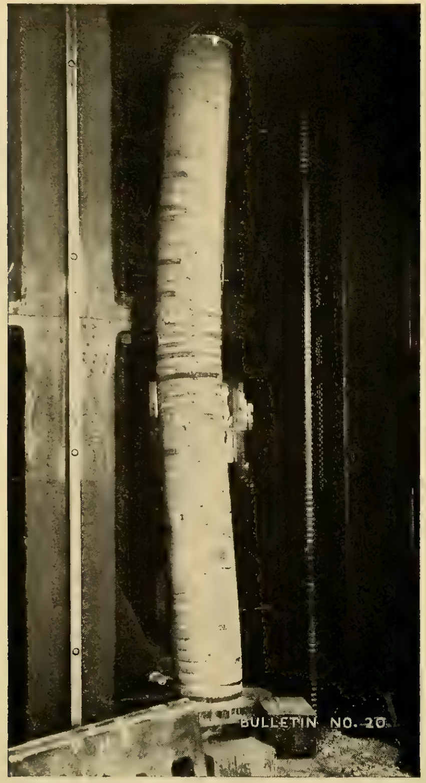

FIG. 9. BAND-HOOPED COLUMN NO. 136 AFTER TEST.

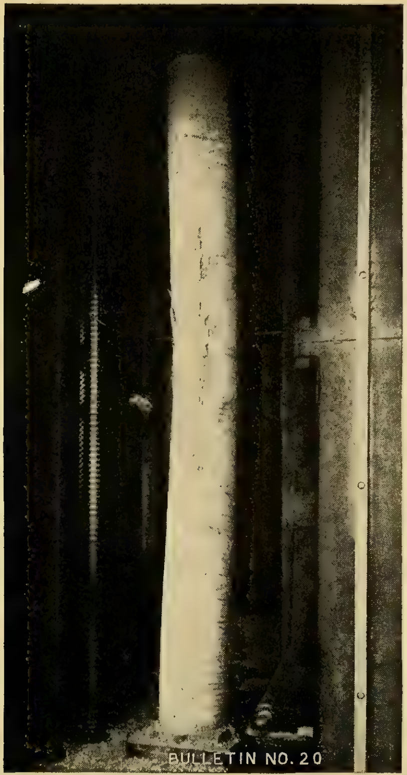

FIG. 10. SPIRAL-HOOPED COLUMN NO. 181 AFTER TEST

failure for plain concrete columns. For loads greater than the ultimate strength of plain concrete, the shortening of the hooped columns increases very rapidly, the ratio of the increment of load to the increment of longitudinal deformation (unit shortening) being only something like 100 000 to 200 000, varying according to the amount of reinforcement. The stiffness of a hooped column for the added load after the hooping has come into action is therefore very small as compared with that of a plain concrete column within its limit of strength. The stress-deformation line at this stage of the test is fairly straight and it gradually swings to a horizontal position as the maximum load of the column is reached. Beyond this horizontal portion, if the test is continued, the load falls off and the shortening is continued as the column gradually bends out of line. If the straight line which fits the part of the stress-deformation curve where the hooping is effective be produced back to the axis of ordinates, (as shown in Fig. 11 and 12)

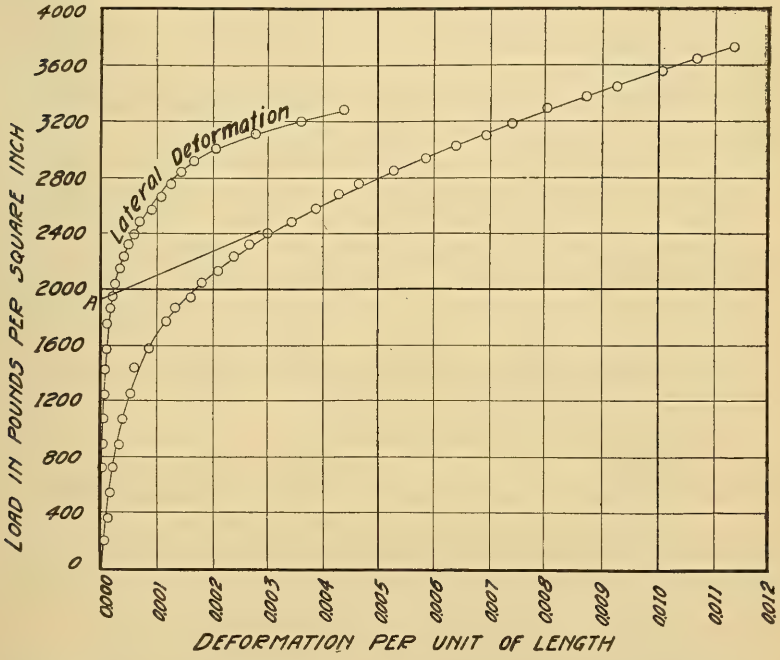

FIG. 12. STRESS-DEFORMATION DIAGRAM FOR SPIRAL-HOOPED COLUMN No. 182.

the intersection agrees closely with the load which produces a longitudinal shortening of 0.001 or 0.0015. As stated later, there is a good indication that this agrees closely with the strength that the column would have without hooping. The agreement of these values is noteworthy. The general slope of the stress-deformation curve above this point varies with the amount of the

TABLE 10.

SUMMARY OF TESTS OF BAND-HOOPED COLUMNS

Col. No.	Kind of Concrete	Reinforcement			Maximum Load		Age at Test days
		Size and Spacing of Bands	Section of Band sq. in.	Per cent	Total pounds	lb. per sq. in.	
		12-in. Columns.					
131	1–2–4	No. 16, 2 in. cc.	.065	1.08	270 000	2384	60
132	1–2–4	No. 16, 2 in. cc.	.065	1.08	243 000	2150	57
133	1–2–4	No. 16, 2 in. cc.	.063	1.05	247 000	2182	59
136	1–2–4	No. 12, 2 in. cc.	.125	2.08	323 000	2860	63
137	1–2–4	No. 12, 2 in. cc.	.124	2.07	300 500	2660	69
138	1–2–4	No. 12, 2 in. cc.	.127	2.12	351 000	3110	60
146	1–2–4	No. 8, 2 in. cc.	.193	3.22	339 000	3000	60
147	1–2–4	No. 8, 2 in. cc.	.192	3.20	420 000	3715	66
148	1–2–4	No. 8, 2 in. cc.	.192	3.20	327 000	2890	63
143	1–2–4	No. 12, 3 in. cc.	.125	1.39	309 000	2735	60
141	1–2–4	No. 12, 4 in. cc.	.122	1.02	256 000	2275	59
142	1–2–4	No. 12, 4 in. cc.	.122	1.02	246 000	2178	66
		9-in. Columns					
152	1–4–8	No. 16, 2 in. cc.	.061	1.35	86 000	1345	70
153	1–4–8	No. 16, 2 in. cc.	.063	1.41	80 000	1260	56
151	1–2–4	No. 16, 2 in. cc.	.066	1.47	137 000	2140	65
156	1–2–4	No. 12, 2 in. cc.	.123	2.73	190 000	2970	67
157	1–1½–3	No. 12, 2 in. cc.	.132	2.94	228 000	3561	67
158	1–1½–3	No. 12, 2 in. cc.	.126	2.80	234 000	3685	81

reinforcement. It will be seen that there is some difference in the amount of deformation for columns hooped with bands and with spirals. This is discussed in a succeeding paragraph. On some of the figures the lateral or transverse deformation diagram is given. It must be borne in mind that the values shown by these

TABLE 11.

PROPERTIES OF BAND-HOOPED COLUMNS

Col. No.	Per cent Rein.	Strength of Column in lb. per sq. in.		Additional Strength per 1% of Reinforcement lb. per sq. in.	ϵ'	Initial Modulus of Elasticity lb. per sq. in.	ϵ''	Ratio of Increments
		Concrete from Diagram	Maximum					
12-in. Columns								
131	1.085	1600	2384	725	.0012	2 670 000	.004	196 000
132	1.085	1400	2150	690	.0012	2 340 000	.005	150 000
133	1.050	1450	2182	695	.0011	2 640 000	.0045	163 000
136	2.081	1600	2860	605	.0011	2 910 000	.006	210 000
137	2.071	1200	2660	705	.0012	2 000 000	.0085	172 000
138	2.12	1750	3110	640	.0012	2 920 000	.007	194 000
146	3.22	1250	3000	545	.0011	2 280 000	.0103	170 000
147	3.20	1650	3715	645	.0013	2 670 000	.009	230 000
148	3.20	1900	2890	310	.0012	3 160 000	.0055	180 000
143	1.39	1650	2735	780	.0011	3 000 000	.0055	197 000
141	1.02	1600	2275	660	.0012	2 670 000	.0045	150 000
142	1.02	1500	2178	665	.0010	3 000 000	.0045	150 000
9-in. Columns								
152*	1.35	650	1345	515	.0012	1 080 000	.008	87 000
153*	1.41	600	1260	470	.0010	1 200 000	.008	83 000
151	1.47	1200	2140	640	.0010	2 400 000	.004	235 000
156	2.73	1600	2970	500	.0012	2 670 000	.0063	218 000
157*	2.94	1950	3561	550	.0011	3 550 000	.006	269 000
158*	2.80	2150	3685	550	.0011	3 910 000	.0058	265 000

*All the columns were of 1-2-4 concrete, except 152 and 153, which were 1-4-8, and 157 and 158, which were 1-1½-3.

lines are not very accurate, but they serve to give an idea of the relation existing. Attention is called to the fact that these deformations are all based upon the original length of the column, since that is the condition which would exist in construction, and that the effect of set is not here considered.

TABLE 12.

SUMMARY OF TESTS OF SPIRAL-HOOPED COLUMNS.

In all cases pitch of spiral was 1 inch.

Col. No.	Kind of Concrete	Reinforcement			Maximum Load		Age at Test days
		Size of Wire	Section of Wire sq. in.	Per cent	Total pounds	lb. per sq. in.	
			High-carbon steel				
171	1–2–4	No. 7	.025	0.85	283 000	2503	56
172	1–2–4	No. 7	.025	0.85	283 000	2506	63
173*	1–2–4	No. 7	.024	0.82	277 000	2000*	57
181	1–2–4	¼-in.	.052	1.73	307 000	2718	57
182	1–2–4	¼-in.	.050	1.67	429 000	3800	60
183	1–2–4	¼-in.	.050	1.68	428 000	3793	63
			Mild steel				
176	1–2–4	No. 7	.025	0.84	235 000	2080	60
177	1–2–4	No. 7	.025	0.85	249 000	2203	64
178	1–2–4	No. 7	.025	0.84	251 000	2220	61
186	1–2–4	¼-in.	.049	1.64	234 000	2068	58
187	1–2–4	¼-in.	.051	1.71	385 000	3404	62
188	1–2–4	¼-in.	.048	1.61	147 000	....	59

*This column was not tested to destruction as a hooped column, but after a load of 2000 lb. per sq. in. had been applied, the spiral was stripped from the concrete and the resulting plain concrete core tested as noted in the text.

24. *Modulus of Elasticity and Compressive Deformations.*—If the hooping is effective in any marked degree at the beginning of the test it may be expected that the initial modulus of elasticity will be greater for hooped columns than for plain concrete columns and that the stress-deformation diagram will show that the col-

TABLE 13.

PROPERTIES OF SPIRAL-HOOPED COLUMNS

All the columns were made of 1–2–4 concrete.

Col. No.	Per cent Rein.	Strength of Column, lb. per sq. in.		Additional Strength per 1% Reinforcement lb. per sq. in.	ϵ'	Initial Modulus of Elasticity lb. per sq. in.	ϵ''	Ratio of Increments
		Concrete from Diagram	Maximum					
High-carbon Steel								
171	0.85	1600	2503	1060	.0012	2 670 000	.005	180 000
172	0.85	1700	2506	950	.0012	2 840 000	.005	161 000
173	0.82	1400	2010	....	.0010	2 800 000	......	....
181	1.73	1400	2718	760	.0014	2 000 000	.010	127 000
182	1.67	1900	3800	1140	.0014	2 720 000	.011	173 000
183	1.68	1900	3793	1130	.0015	2 540 000	.011	172 090
Mild Steel								
176	0.84	1350	2080	870	.0012	2 250 000	.0045	162 000
177	0.85	1500	2203	830	.0012	2 500 000	.0045	156 000
178	0.84	1450	2220	920	.0012	2 420 000	.005	154 000
186	1.64	1150	2068	560	.0012	1 920 000	.008	115 000
187	1.71	1800	3404	940	.0011	3 280 000	.0065	247 000
188*	....	....	....	....	......	....	......	....

*Not included in this table on account of being loaded eccentrically. (See p. 45).

umn is stiffer for loads up to the ordinary compressive strength of concrete. It is worth while to make an examination of the portion of the stress-deformation curve for the first stage of the test. To do this the point where the first marked change occurs in the direction of the line from that for plain concrete, or which corresponds with the ultimate longitudinal deformation found in plain concrete columns (ranging from 0.001 to 0.0015) indicated in the tables by ϵ', is determined. A study of the diagrams makes it seem probable that the load causing this shortening corresponds closely with the ultimate strength of the concrete of the column. Tables 11 and 13, p. 35 and 37, give the results found. The variation in these values is not greater than that found in plain concrete columns, but the average strength is somewhat less than the average for the plain concrete columns of the same mix and age. The tests of cubes and cylinders from the same mix, so far as these were made, confirm the belief that the values thus found are representative of the strength of the concrete. It is interesting to note that the load thus obtained agrees fairly well with that found by taking the straight portion of the stress-deformation curve as it averages beyond the elbow and extending it backward to an intersection with the axis of ordinates

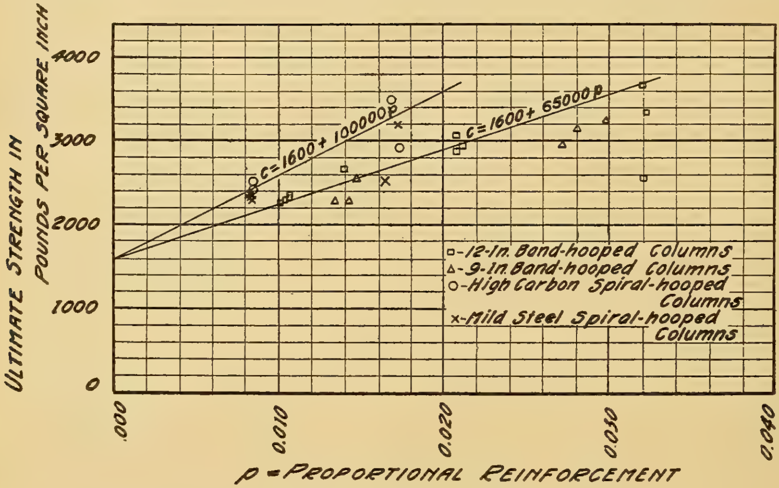

FIG. 13. ULTIMATE STRENGTH OF HOOPED COLUMNS.

(A in Fig. 11 and 12). After determining the critical point above described, the calculation of the initial modulus of elasticity may be made by the method used for plain concrete columns. These results are also given in Tables 11 and 13, p. 35 and 37.

The initial modulus of elasticity for the first stage obtained in this way averages about 2 610 000 lb. per sq. in. for the hooped columns made of 1–2–4 concrete. The average initial modulus of elasticity for the corresponding plain concrete columns is 3 040 000 lb. per sq. in. As a matter of fact the determination of this initial modulus of elasticity for hooped columns is subject to variation of judgment, but it appears then that the hooped columns are less stiff through the first stage than are the plain concrete columns.

Using the parabolic relation to determine points for lower loads than the assumed ultimate strength of the concrete (the method found satisfactory for the plain concrete), the calculated points so obtained show a very close agreement with the observed stress-deformation curve. No reason is known for the apparent decrease in stiffness, though the presence of the hooping may affect the fabrication of the columns or their shrinkage in setting and the density of the concrete. At any rate, it seems evident that there must be very little restraint by the hoops through this stage. An analytical treatment of the stresses which may be developed in hoops, especially with the low value of Poisson's ratio then acting, leads to similar conclusions.

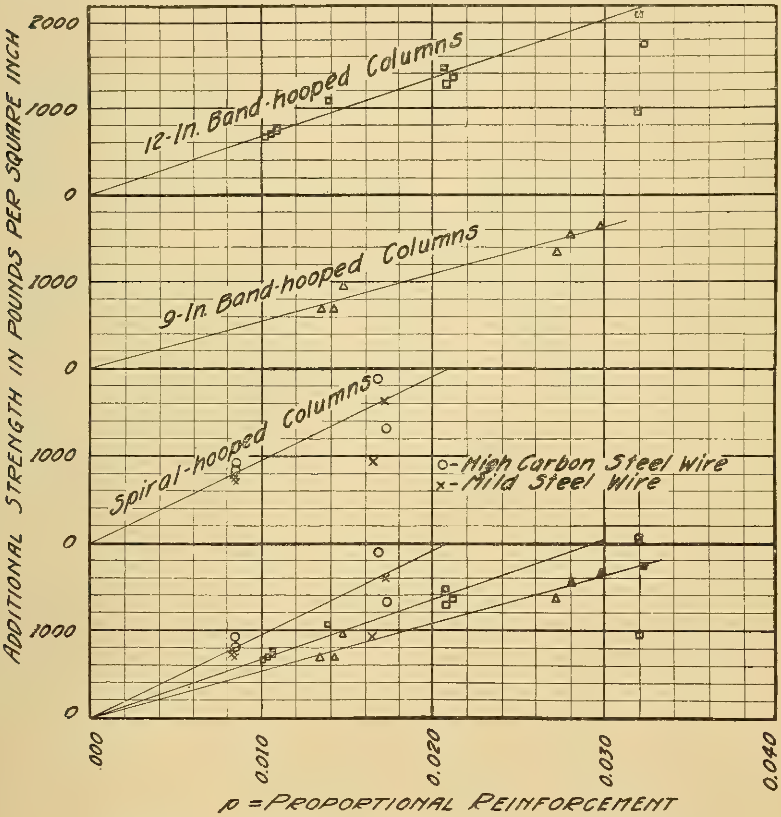

FIG. 14. ADDITIONAL STRENGTH DUE TO HOOPING.

When loads are applied which give shortenings larger than 0.001 to 0.0015, the second stage begins and a considerable stress is brought into action in the hooping. As the load is added the restraint of the hooping becomes marked and there is a very rapid increase in the amount of shortening produced by the added load. The stress-deformation diagram, through this stage of the test, approximates a straight line for the banded columns. For those with spiral reinforcement only a small portion is straight, but the main part of the curve has one general direction. The ratio of the increment of load to the increment of deformation, however, is comparatively small. It is well to compare this ratio with the modulus of elasticity of concrete. The values for this ratio, determined from the stress-deformation diagrams are given in Tables 11 and 13, in the columns headed "Ratio of Increments", ϵ'' being the unit-deformation corresponding to the maximum load. The average value for this ratio for columns with band reinforcement is 184 000; for those with spiral reinforcement, 165 000. This ratio is very much less than the modulus of elasticity of concrete. The values for Columns No. 152 and 153 show that the ratio becomes much smaller for leaner concrete; but those for Columns No. 157 and 158 indicate that it is higher for the richer mixture. Attention is called to the fact that the deformations used refer to the original length and not to the length after the set is deducted.

It will be seen that the amount of shortening in the column before the maximum load is reached depends upon the amount of reinforcement, and that it is more for columns with spiral reinforcement than for columns reinforced with bands. The average value for the banded columns given in terms of the excess of the unit shortening of the hooped column over the shortening for a plain concrete column per 1% of reinforcement ranges from 0.0031 to 0.0059 for the columns with spiral reinforcement, and 0.0013 to 0.0035 for columns reinforced with bands. Attention should be called to the fact that the columns reinforced with spiral will sustain higher loads, as well as shorten more, than the columns reinforced with bands. It is not known whether the concrete bulged out more between the spirals than between the hoops, but this condition seems probable.

25. *Strength of Hooped Columns.*—In Table 10 are given the

loads carried by the columns reinforced with bands, and in Table 12 those with spiral reinforcement. Fig. 13 shows the results graphically. It is evident that hooping adds strength and that the excess of strength over that of plain concrete columns is proportional to the amount of reinforcement. The full effect of the hooping is not clearly shown in Fig. 13 since the concrete in each column has its individual strength. A better comparison is made by first estimating the strength of the concrete for each column as found in "23. Modulus of Elasticity and Compressive Deformations". If the increase in strength over that of a plain concrete column be divided by the per cent of reinforcement, the additional strength per 1% of reinforcement is obtained. This is shown in Table 11 and Table 13 in the columns headed "Additional strength per 1% of reinforcement". It is also shown graphically in Fig. 14. The strength assumed for the concrete of the individual columns involves errors of judgment, even if the method of determining it is considered to be a proper one, but a careful study of the diagrams shows that the variation due to judgment may not, for average of the results, amount to more than 50 lb. per sq. in. above the values assumed, and this variation will not seriously affect the values found for the added strength of the column due to hooping. The strength of the 1–2–4 concrete given for the hooped columns is less than the average for the 1–2–4 concrete columns. Part of this may possibly be due to difference in materials and part to the effect of the presence of the hooping upon the density of the concrete.

For the columns reinforced with bands there is a fair agreement in the value of this additional strength per 1% of reinforcement for all the 12-in. columns regardless of the amount of reinforcement. The average is 669 lb. per sq. in., if we except Column No. 148 which appears by the stress-deformation diagram and other data to be erratic, probably the result of poor concrete or of defective hooping. The 9-in. band-hooped columns are also fairly uniform, the average additional strength being 537 lb. per sq. in. per 1% of reinforcement. It is noticeable that the additional strength per 1% of reinforcement in these 9-in. columns is much the same for the 1–4–8, 1–2–4 and 1–1½–3 concrete, and that the difference in the strength of these columns seems to show up mainly through a difference in the strength of the concrete itself. It should be recalled, however, that the hooped column with 1–4–8 concrete is much less stiff through the second stage than is that with 1–1½–3 concrete. The reader should also keep in mind the low degree of stiffness in hooped columns, as discussed elsewhere.

For spiral hooping there is a greater additional strength per 1% of reinforcement, and also more variation in the results. Omitting No. 186 which seems to be poor material, the average additional strength per 1% of reinforcement for these columns is 955 lb. per sq. in., which is about 50% greater than the additional strength found in the columns reinforced with bands. There seems to be little difference between the strength given by the high-carbon wire and the mild-steel wire, especially when it is remembered that the No. 7 high-carbon wire has a yield point of 60 000 lb. per sq. in. and the ¼-in. high-carbon steel wire of 110 000 lb. per sq. in. The results with the mild-steel wire are somewhat lower, but not so much as might be expected when the difference in elastic limit is taken into consideration.

What the cause is of the greater added strength per 1% of reinforcement with the spirals than with the bands, cannot be stated. The observations indicate that the lateral deformation of the concrete between hoops at the maximum load is greater in columns with spirals than in those with bands. Considering this and also that the columns with spirals compress more before final failure, as shown by the stress-deformation diagrams, and that the stress-deformation curve bends out of a straight line into a long curve, it would seem possible that the concrete squeezes out between the hooping, so to speak, in a larger proportional degree with the spiral reinforcement and thus that the spiral hooping was not stressed up to its elastic limit. The measurement of the stretch in the bands indicates that in some of the columns this material reaches its elastic limit. It seems evident that the hoops are stretched more at their edges than at their center. It is likely that in the placing together of the band reinforcement by the workmen in the laboratory, the structure formed was more uneven and less perfect than was the spiral reinforcement. The whole question is complicated by the bulging or lateral deflection of the columns, and this appeared at a relatively earlier stage with the bands than with the spirals. A study of the readings of the individual extensometers indicates that there may be quite a difference in the amount of shortening at points around the column before lateral deflection of the column is noticeable. As this part of the problem was unexpected, the observations were not made carefully enough to state definite results.

A general form of formula for hooped columns may be given as follows:

$$c = c' + pc''$$

where c is the maximum strength of the hooped column, c' is the strength of the plain concrete column, c'' is a coefficient for the hooping and p is the ratio of reinforcement found by dividing the total volume of the hoops by the volume of the core within the hoops, in other words, by considering the hooping to be distributed as a thin cylinder continuously along the hooped column.

The values given by the writer in the Proceedings of the American Society for Testing Materials, for 1907, are not far from the average results for the 12-in. columns. Using these, the expression for the columns reinforced with bands may be given as

$$c = 1600 + 65\,000\,p$$

and that for the spiral as

$$c = 1600 + 100\,000\,p$$

The latter equation may be considered to give values too large for the mild-steel wire and smaller than the average for high-carbon steel wire. Considerable variation from these values may be expected.

26. *Lateral Pressure and Stress in Hooping.*—The expansion of the column under load is accompanied by a lateral pressure and this pressure develops stresses in the hooping. It seems from the stress-deformation diagrams that the lateral pressure is proportional to the excess of the load over the strength of the plain concrete. The observations made on the expansion of the bands also indicate that the stress in the hooping is proportional to the increments of load beyond the load which plain concrete would resist. No analytical treatment of the stresses produced by the concrete as it expands will be gone into, but it may be worth while to see what stresses are possibly developed in the hooping and what the relation between the longitudinal compressive stress and the lateral expansive stress may be. It is evident from the tests that many of the band-hooped columns failed by the bands becoming stressed

beyond the yield point of the metal. This gives a clue to the amount of stress existing.

The following nomenclature is used: t' the thickness of band, b the breadth of one band, a the distance from center to center of bands or spirals, t'' the diameter of the wire of the spiral, p the ratio of the volume of the hooping to the volume of the core or part of the column within the hooping, and d the diameter of the hooped core. Then the equivalent thickness of an enveloping cylinder of metal is for the band-hooped columns

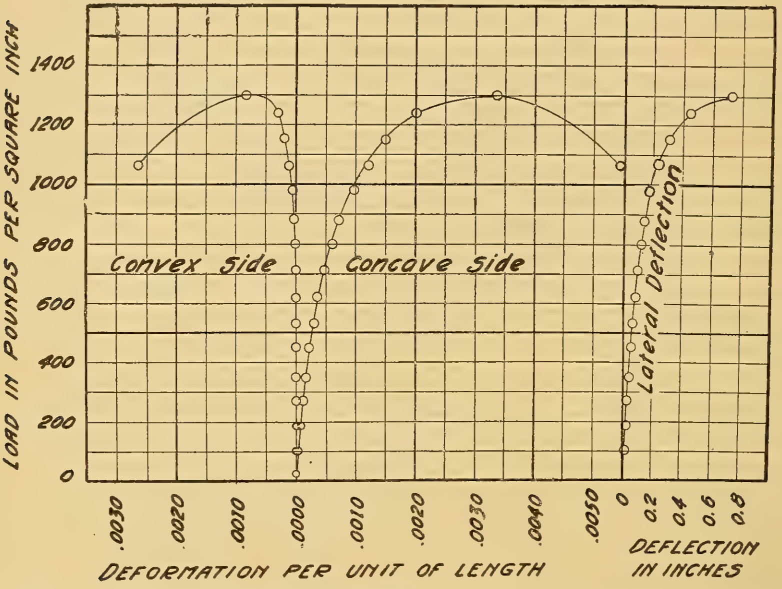

FIG. 15. CURVES FOR SPIRAL-HOOPED COLUMN NO. 188, ECCENTRICALLY LOADED.

$$t = \frac{b}{a} t'$$

and for the spiral hooping

$$t = \frac{t''^2}{a d^2}.$$

For either case we have

$$t = \frac{p d}{4}.$$

If we consider by analogy that the lateral pressure is caused by a lateral pressure in the concrete similar to hydrostatic pressure, using P as this lateral pressure in pounds per square inch, we shall have from the ordinary formula for thin cylinders

$$P = \frac{pf}{2}$$

where f is the stress in the hooping in lb. per sq. in. If now the stress in the hoops for the maximum load on the column be taken at the yield point of the material, 48 000 lb. per sq. in., this gives

$$P = 24\,000$$

or 240 lb. per sq. in. equivalent lateral pressure for each 1% of hooping. The formula already used considers that 650 lb. per sq. in. added compressive strength is given for 1% of band hooping. From this it would seem that the ratio of the equivalent lateral pressure to the added strength given by the reinforcement in the case of the band-hooped columns is 0.36. The probability that many of the columns failed by lateral bending and without stressing the bands to the elastic limit should be kept in mind in this connection. While this may not be an accurate value it may give some indication of the ratio between the lateral stress and the longitudinal stress in such columns.

27. *Condition of Hooped Concrete after Testing.*—An experiment of some interest was made by first loading a hooped column and then stripping the wire from it and testing the naked column. Column No. 173 (No. 7 wire, 0.82% reinforcement) was loaded to 2000 lb. per sq. in. with a resulting unit shortening of 0.0036. The load was then released and a set of 0.0025 was found. A load of 1945 lb. per sq. in. was then applied when the unit shortening became 0.0041. Upon the releasing of this load the set was found to be 0.0027. The spiral was then stripped from the column without taking it from the machine and a load was applied to the naked column. The column failed at 1080 lb. per sq. in. The indications of the stress-deformation diagram are that the plain concrete had a strength originally of about 1400 lb. per sq. in. It seemed likely that the stripped column sustained as much as 75% of its original strength as plain concrete. A comparison with the other columns and the form of the stress-deformation diagram indicates that the hooped column might have held 2400 lb. per sq. in. before failure.

28. *Eccentric Loading.*—Column No. 188 was tested with the

pressure applied both top and bottom along a line of 1¼ in. away from the center of the column. This column was reinforced with ¼-in. wire giving 1.62% of reinforcement. The phenomena accompanying the test were similar to those for central loading except that the lateral deflection was apparent at an earlier load and increased rapidly. The lateral deflection amounted to 0.72 inch when the maximum load was reached. Fig. 15 shows the deformations on two sides of the column and also the amount of lateral deflection. When the eccentricity of the load is considered and also the amount of eccentricity due to the lateral bending, the final loading carried agrees very closely with what would be expected with a homogeneous column under the same conditions. It is evidently not necessary to have the internal pressure uniformly distributed over the cross section of the column in order to utilize the strength of the hooping as would be the case if the internal pressure were considered to act in a manner similar to hydrostatic pressure. The column tested showed considerable toughness, a property which would not be present in plain concrete eccentrically loaded.

29. *Summary.*—The writer feels that there is yet much to be learned about concrete and reinforced concrete columns. After making the tests described he has had to give up several preconceived notions of the action of hooped concrete, notions based upon meager statements of tests and analyses made from assumed conditions. The tests here given are not altogether conclusive, but they may help to clear up uncertainties and give an idea of the action of concrete when restrained by hooping. The tests on plain concrete columns also bring out instructive results. The following observations on the tests may be made:

1. The average ultimate strength of the plain concrete columns at ages not far from 60 days is as follows: 1–1½–3 concrete, 2300 lb. per sq. in.; 1–2–4 concrete, 1740 lb. per sq. in.; 1–3–6 concrete, 1030 lb. per sq. in.; 1–4–8 concrete, 575 lb. per sq. in. The effect of the amount of cement used in the concrete is very marked, and cement is shown to be an economical reinforcing material for compression members. The average variation of individual results from the average strength in the case of 1–2–4 concrete about 60 days old is 12%, ranging from 34% below to 25% above the average. The tests made at an age of 6 months indicate an

increase of something like 15% over the strength at 60 days.

2. The average initial modulus of elasticity of 1–2–4 concrete columns at an age of about 60 days was 3 040 000 lb. per sq. in., as compared with 2 250 000 lb. per sq. in. given for the columns in Bulletin No. 10. The larger value is due in part to the denser and harder concrete used.

3. Poisson's ratio for concrete in compression is a variable quantity, increasing considerably just before the ultimate failure of the concrete. The value varies with the age and nature of the concrete. A value of 0.1 to 0.16 for 1–2–4 concrete 60 days old may be tentatively given for the lower loads, with values as high as 0.25 or 0.3 near the crushing load. This concrete set in air.

4. The repetition of a load gives an increased amount of shortening and an increased set, the amount of this increase varying with the amount of the load. For the lower loads the increase becomes small after a small number of repetitions. For the higher loads the effect of continued repetition is much more marked. The phenomena attending repetitive loading of columns are important and merit much fuller investigation.

5. In hooped columns the hooping does not come into action to any great extent before a load equivalent to the ultimate strength of plain concrete, or a little below, is reached. The longitudinal deformation and the lateral deformation are not modified by the hooping to any great extent before this load is reached.

6. Beyond this load the column shortens rapidly and the hooping receives additional stress correspondingly and its function in restraining the concrete laterally becomes important. If the column maintains a straight position, the limit of strength may be expected to come when the stress in the hooping exceeds the yield point of the metal. If the column deflects laterally through eccentric loading or lack of uniformity in the concrete or in the distribution of the hooping, a lower strength will be found.

7. The additional strength of the hooped column over that for an unreinforced concrete column of the same quality, as determined from the stress-deformation diagram, averages for each one per cent of hooping 955 lb. per sq. in. for the spiral-hooped columns and 669 lb. per sq. in. for the band-hooped columns having a diameter of 12 in. The additional strength per one per cent of reinforcement is much the same for 1–4–8 concrete, 1–2–4 con-

crete, and 1–1½–3 concrete, but it must not be overlooked that the 1–4–8 concrete is much less stiff than are the columns with richer mixtures, especially after the hooping is brought into action. The difference between the strength of columns with high-carbon wire and with mild-steel wire is not marked enough to warrant drawing conclusions.

8. For loads which bring the hooping into full action, the increase in the longitudinal deformation and in the lateral deformation are approximately proportional to the added load. The ratio of lateral to longitudinal deformation is more nearly constant than for plain concrete. Through this stage the ratio of increment of load to increment of longitudinal shortening is low, the averages for columns with band-hoops being 184 000 and with spiral-hoops 165 000. This ratio is very much less than the modulus of elasticity of concrete and the hooped columns through this stage of loading are correspondingly less stiff than concrete columns. The amount of the set produced by the higher loads is very great.

9. The total amount of shortening before failure occurs is very great, averaging something like six to twelve times that for plain concrete and fifty times that for the ordinary working stresses in concrete. At the maximum load it is somewhat less than this, say five times as much as plain concrete and perhaps five times that of mild steel at its elastic limit. Cracking and peeling of the concrete are apparent at loads corresponding to the ultimate strength of concrete.

10. The excessive amount of compression before failure affects the problem of combining hooping and longitudinal reinforcement very unfavorably, if the stresses are to be kept within the elastic limit of the latter.

11. The lateral deflection of hooped columns is large and may seriously affect the ultimate strength which is available for the column. For continued application of stress beyond the maximum load, the column deflects enormously. Scaling of the surface of the concrete and lateral deflections are warning signs given well before danger of failure exists.

12. The concrete itself retains a considerable element of its strength even after it has been shortened in a hooped state four or five times as much as would produce failure in unhooped concrete columns.

13. Columns of rich and lean concrete exhibit phenomena of similar characteristics, the hoop stress becoming effective at the ultimate strength of unhooped concrete.

14. The one experiment indicates that hooped columns will resist eccentric stresses in somewhat the same way as will other material.

15. Light hooping offers security against sudden failure and unevenness of concrete and will enable higher working stresses to be used. In combination with rich concretes and longitudinal reinforcement, using low stresses in hoops (i. e., basing the strength upon an assumed ultimate strength but little beyond the average ultimate strength of plain concrete), a satisfactory column may be made. It is suggested that a column of this character may be designed in such a way that the longitudinal reinforcement may carry the load during construction and still not be overstressed later with the final loading, provided the basal point for use with the factor of safety in determining the working strength of the column is somewhat below the ultimate strength for plain concrete.

16. Heavy hooping gives added strength, but in utilizing the full strength of such columns the column shortens unduly, deflects laterally, and will strain longitudinal reinforcement many times beyond the deformation which exists at the elastic limit of the metal. It may be applicable where a large limit of safety is desired or where large variations in shortening are unobjectionable, as where the structure is articulated. So far as ultimate strength is concerned, hooping adds two to three times as much strength to the column as does an equal amount of longitudinal reinforcement, but with the extreme amount of shortening and the liability to lateral deflection it may be doubted whether this increase of strength may be utilized to any great extent in ordinary construction.

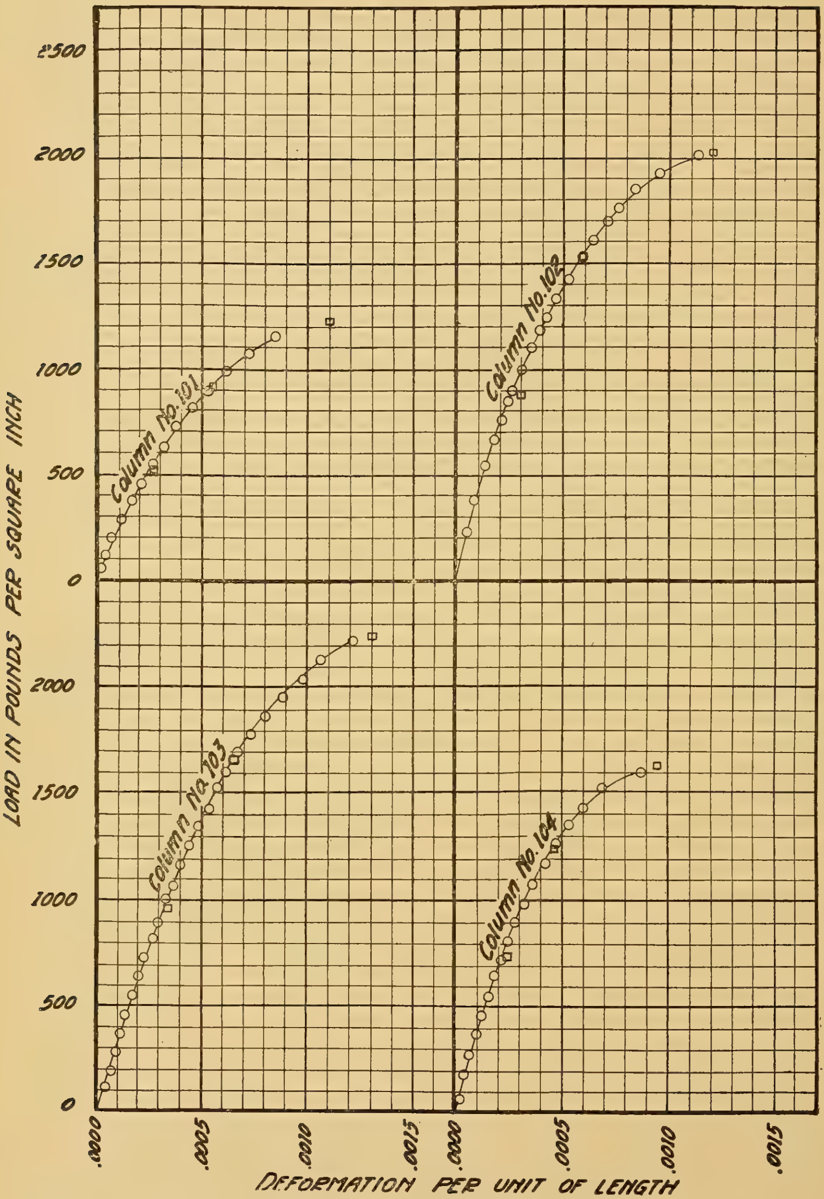

FIG. 16. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

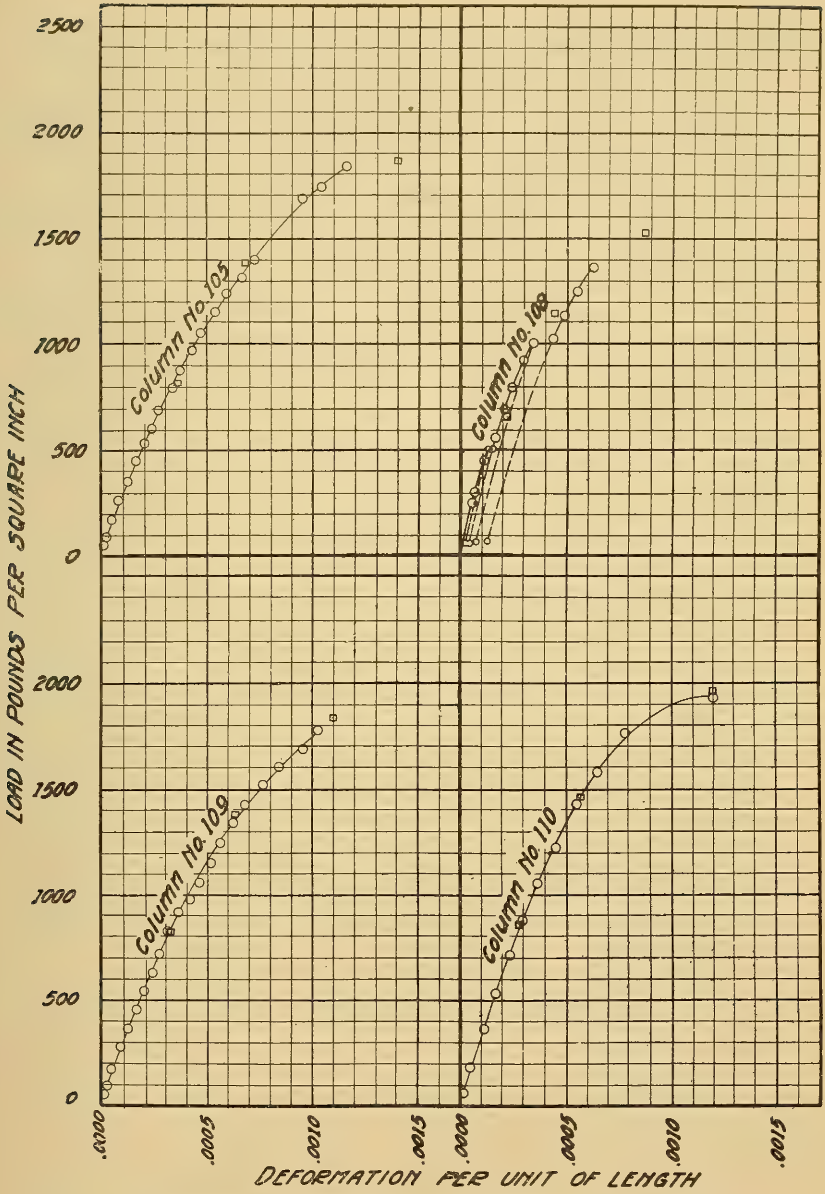

FIG. 17. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

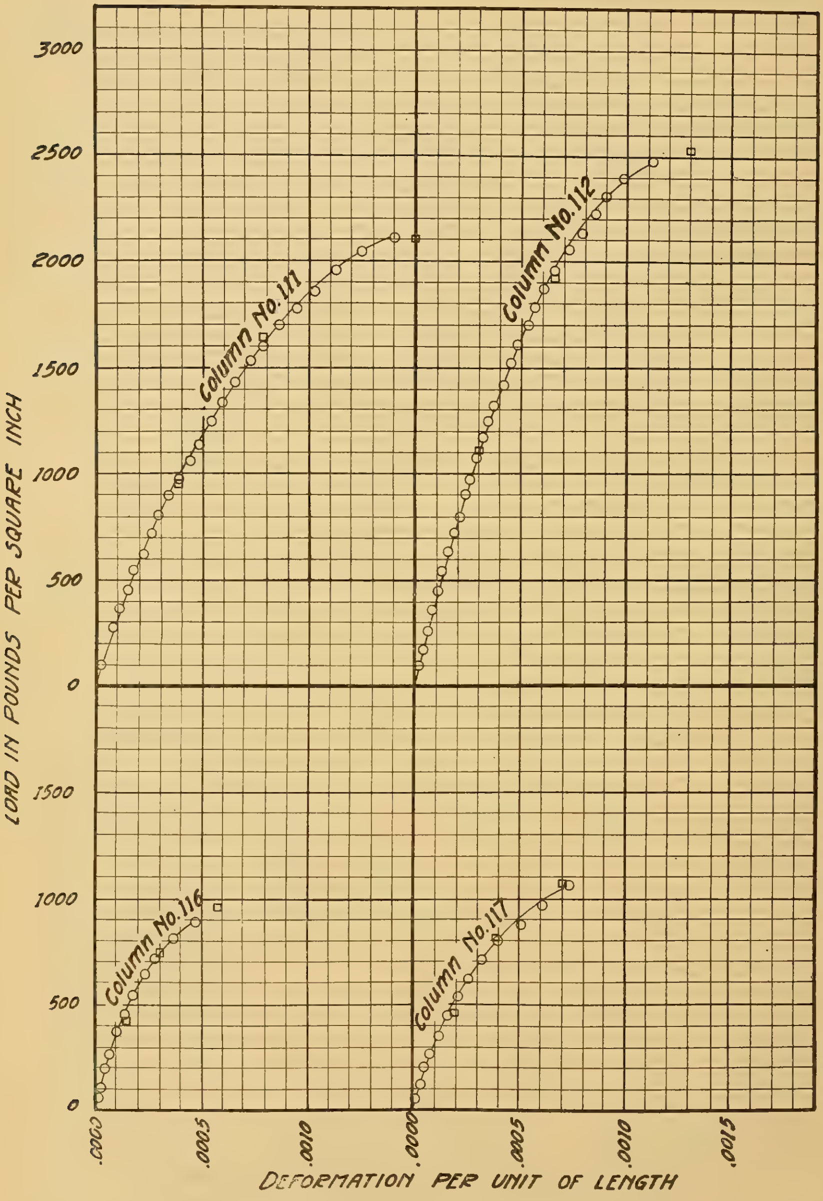

FIG. 18. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

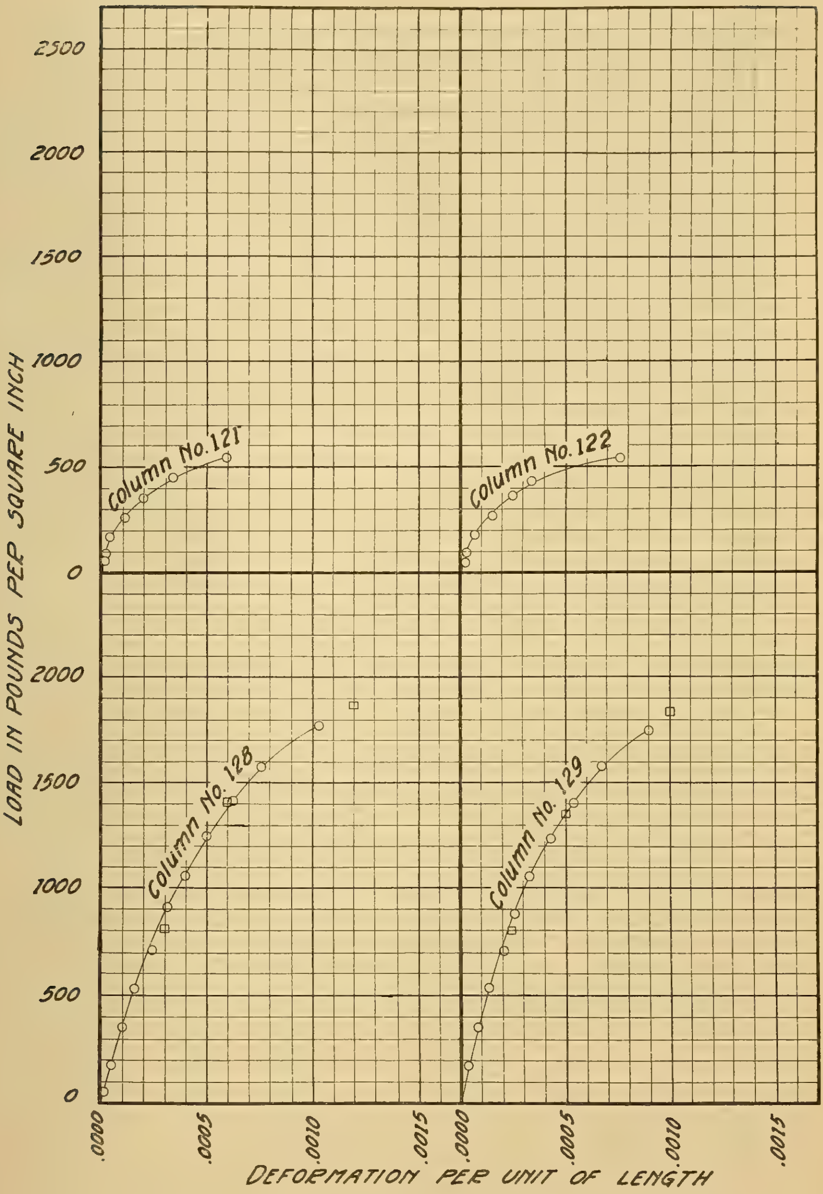

FIG. 19. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

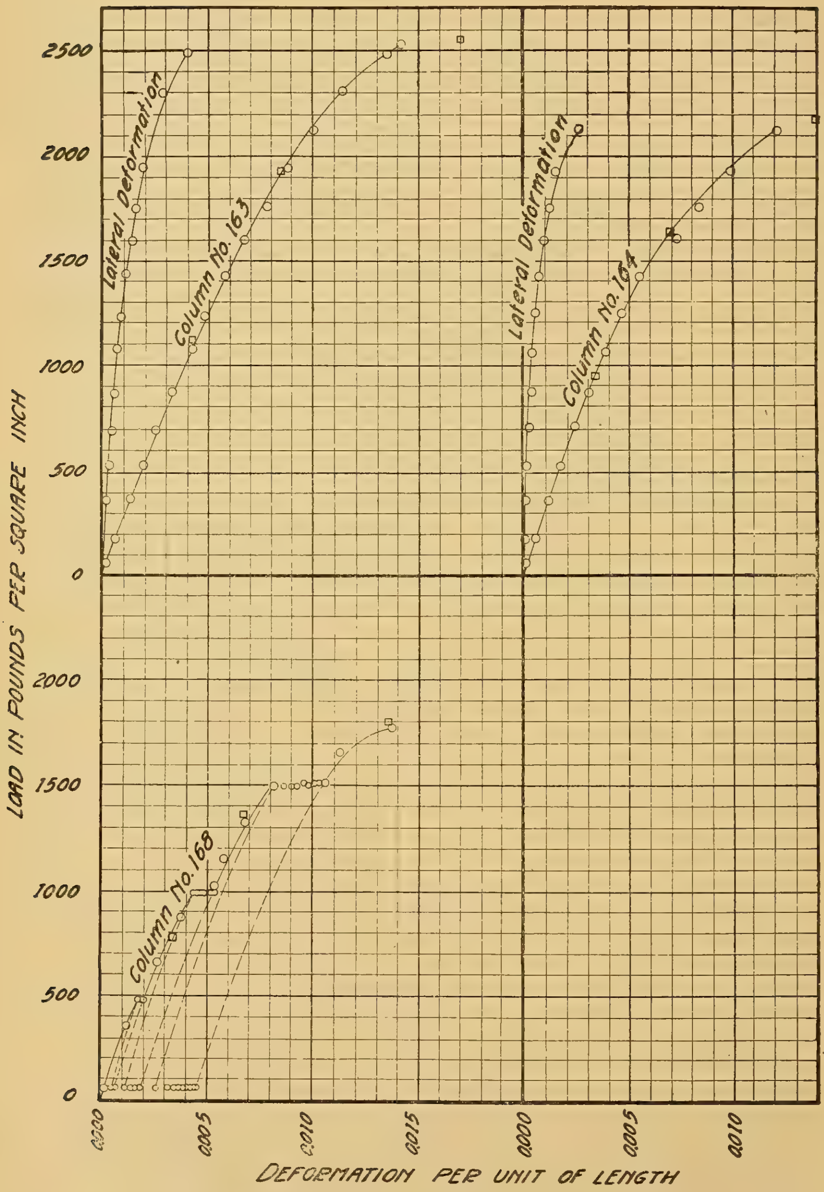

FIG. 20. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

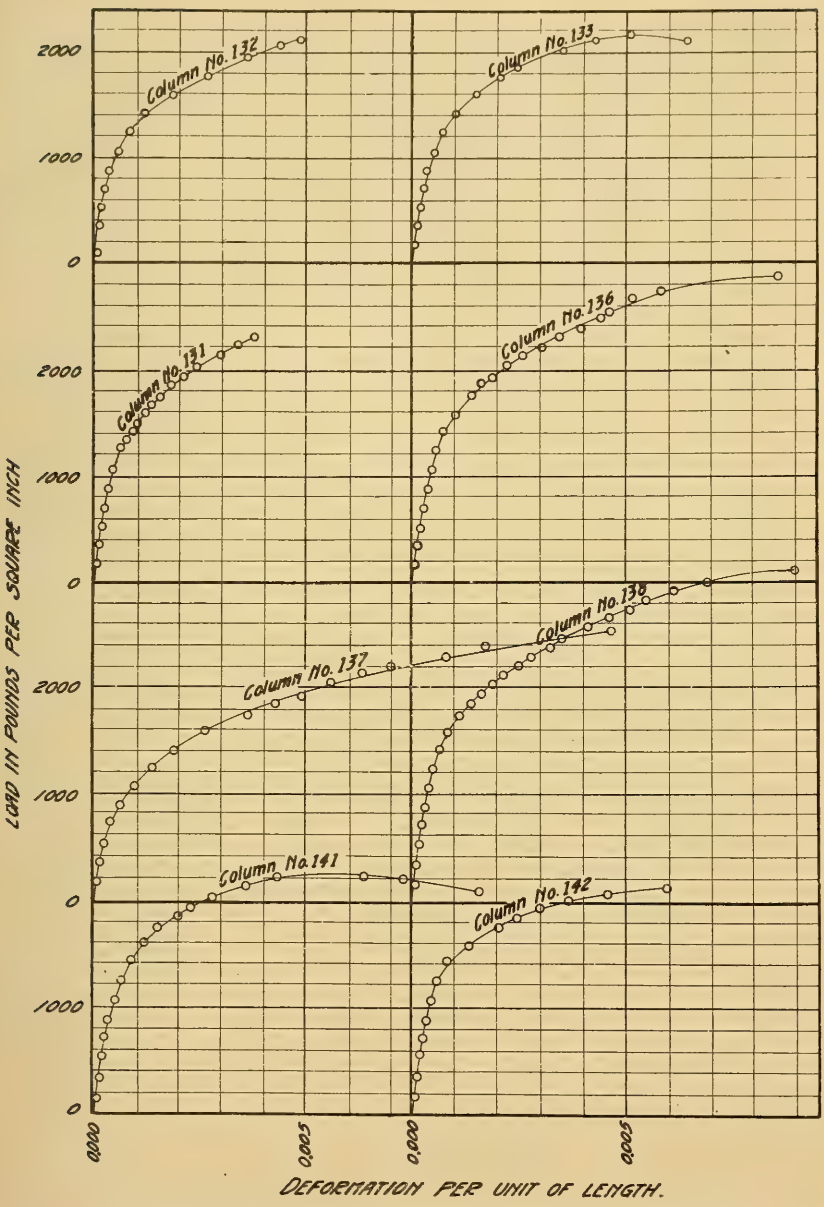

FIG. 21. STRESS-DEFORMATION DIAGRAMS FOR BAND-HOOPED COLUMNS.

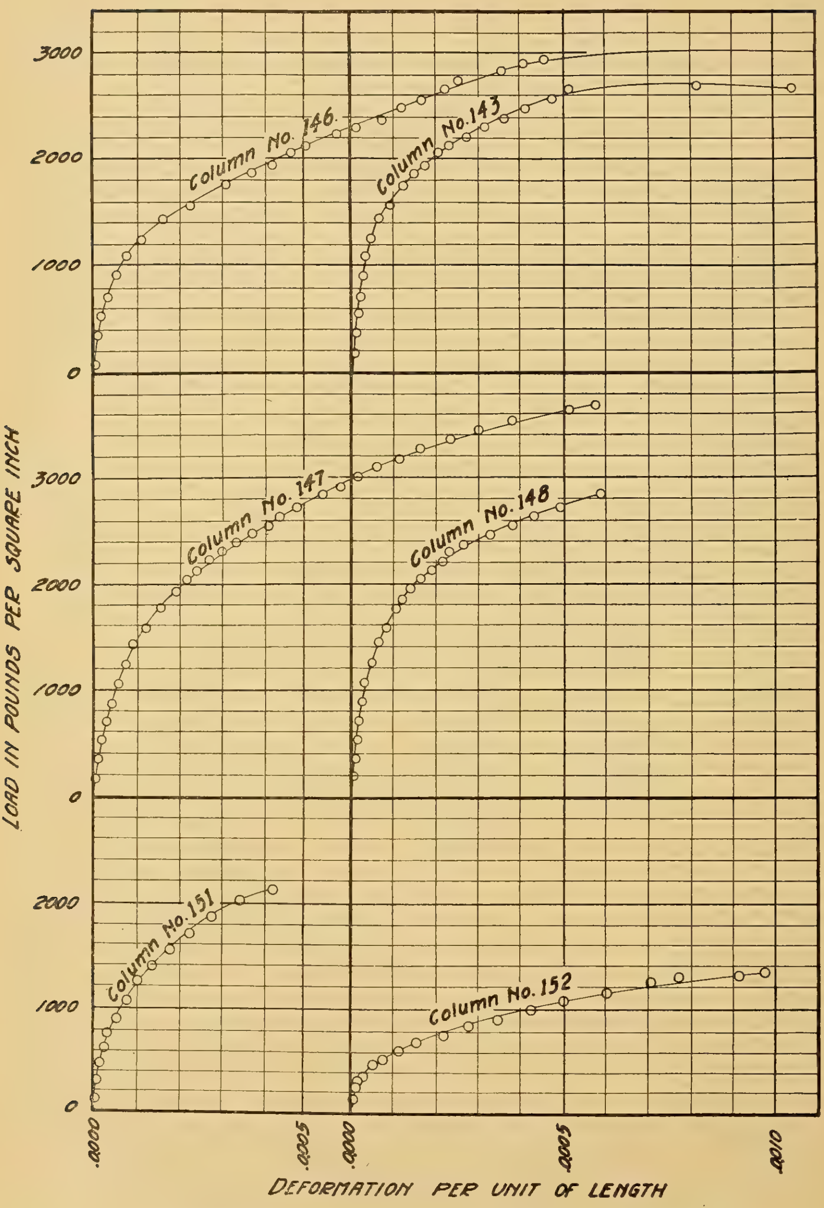

FIG. 22. STRESS-DEFORMATION DIAGRAMS FOR BAND-HOOPED COLUMNS.

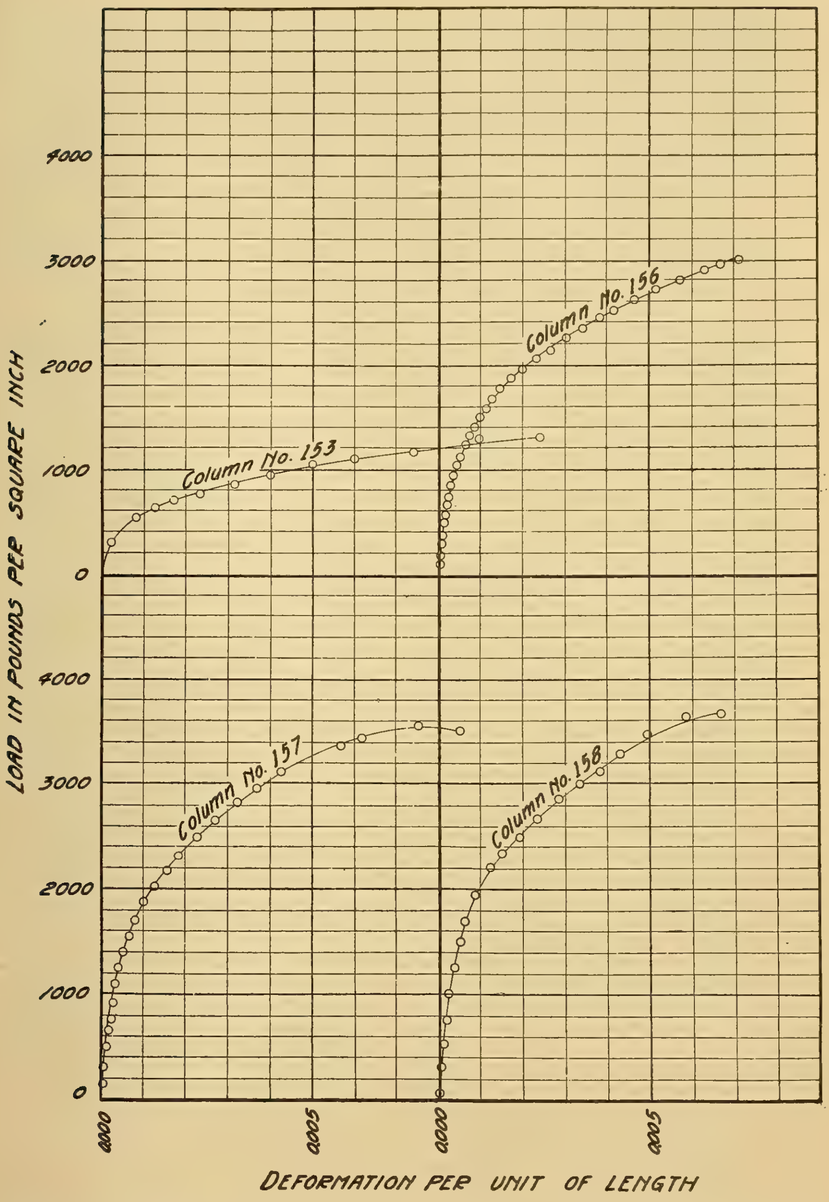

FIG. 23. STRESS-DEFORMATION DIAGRAMS FOR BAND-HOOPED COLUMNS.

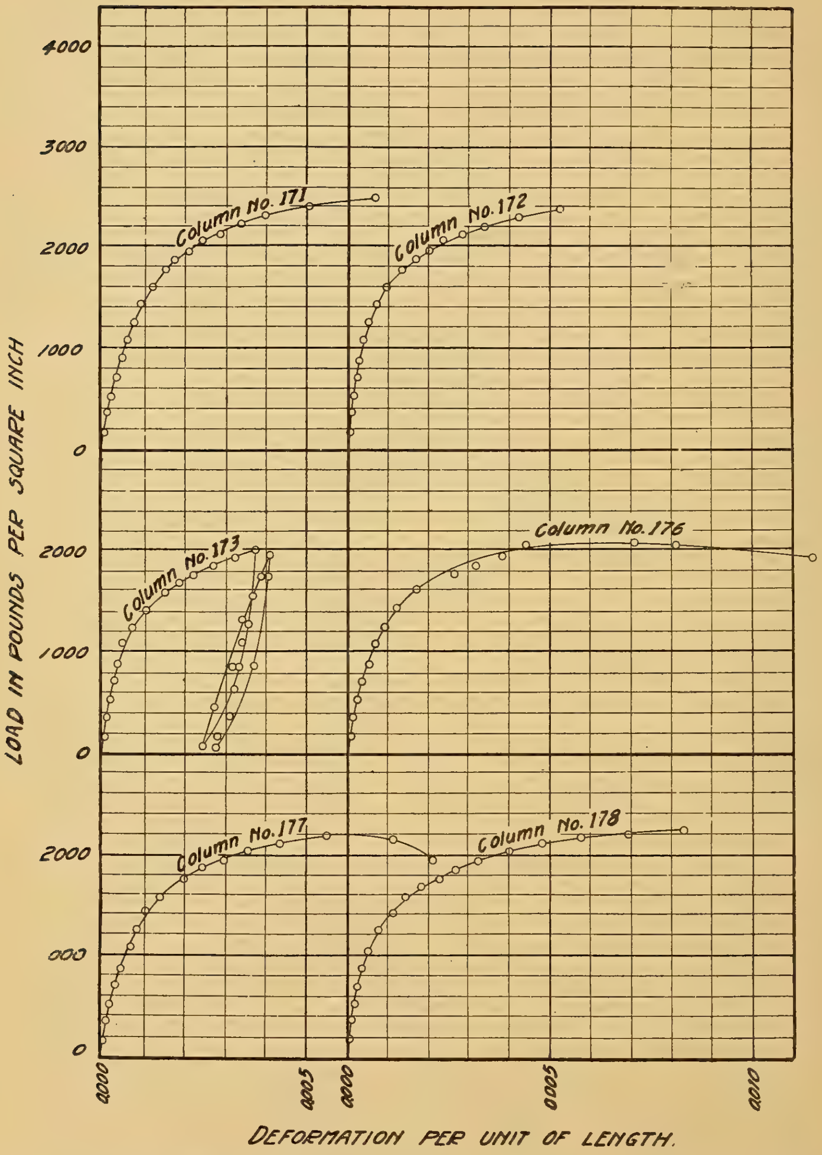

FIG. 24. STRESS-DEFORMATION DIAGRAMS FOR SPIRAL-HOOPED COLUMNS.

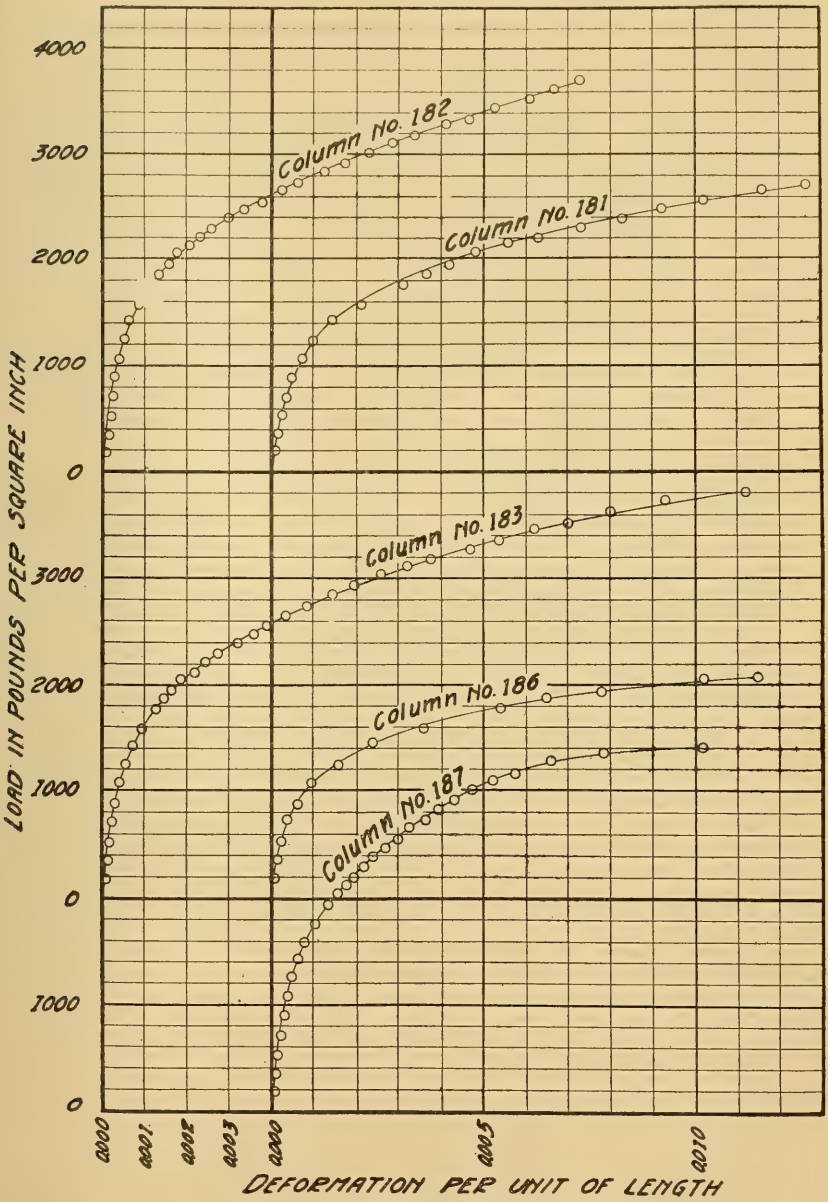

FIG. 25. STRESS-DEFORMATION DIAGRAMS FOR SPIRAL-HOOPED COLUMNS.

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